Endogenous Liquidity in Asset Markets

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ABSTRACT

This paper analyzes a model in which long-term risky assets are illiquid due to adverse selection. The degree of adverse selection and hence the liquidity of these assets is determined endogenously by the amount of trade for reasons other than private information. I find that higher productivity leads to increased liquidity. Moreover, liquidity magnifies the effects of changes in productivity on investment and volume. High productivity implies that investors initiate larger scale risky projects which increases the riskiness of their incomes. Riskier incomes induce more sales of claims to high quality projects, causing liquidity to increase.

*Kellogg School of Management, Northwestern University. This paper is a revised version of my thesis at the University of Chicago. I thank my advisors John Cochrane, Douglas Diamond and Lars Hansen, as well as Adriano Rampini, and Ronel Elul, Beatrix Paal, Nancy Stokey, Dimitri Vayanos, Jaume Ventura, Annette Vissing-Jørgenson; an anonymous referee; the editor, Rick Green; and seminar participants at Berkeley, Columbia, Harvard, Northwestern, Stanford, UCLA, University of Chicago, University of Illinois, University of Michigan, Wharton, the SED Annual Meeting, The Accounting and Finance conference in Tel Aviv, The Information, Financial Markets and the Business Cycle Conference in Rome, the AEA annual meeting, the SAET Biannual Meeting, and the WFA Annual Meeting, for helpful comments. Financial support from the Margaret Reid Dissertation Fellowship and the University of Chicago is gratefully acknowledged. All errors are mine.

Market liquidity appears to vary with the state of the economy. This is evident in the variation in spreads between liquid and illiquid assets over the business cycle and in the fact that liquidity crises are associated with economic downturns.[1] Moreover, in the cross section highly productive industries and economies are associated with more liquid asset markets.

This paper analyzes a model economy in which long-term risky assets vary in quality and are illiquid due to adverse selection. Investors can realize part of the value of payoffs from long-term assets early by issuing claims against the assets in a pooled market. The equilibrium claims price is determined by the average quality of claims sold. I use "liquidity" to describe the cost of transferring the value of expected future payoffs from long-term assets into current income. A lower cost implies higher liquidity. The claims price is increasing in the fraction of high quality claims traded thus the market is more liquid when there is less adverse selection.

Adverse selection causes markets to be illiquid because claims sold are likely to be low quality. However, trade for other reasons, such as consumption, investment, or portfolio rebalancing, can mitigate the adverse selection problem. How should we expect the relative fraction of adverse selection motivated trades to vary with the state of the economy? Is adverse selection driven liquidity consistent with procyclical liquidity? The answer is not obvious. Consider an environment where assets are sold either if they are low quality or if the owner of the asset has an exogenous negative income shock. If more investors received the negative income shock in bad times then more high quality assets would trade in bad times, making liquidity countercyclical.[2]

Instead, in this paper the state of the economy is determined by the standard business cycle fundamental, productivity. Liquidity is endogenously determined as a function of productivity in the following way: When productivity is high the relative return on risky investment is high and investors initiate larger scale risky projects at

[1]See Chordia, Roll, and Subrahmanyam (2001), Huberman and Halka (2001), and Longstaff (2001) for evidence of a systematic time varying component of liquidity. See also Pulvino (1998) and Eisfeldt and Rampini (2002) for evidence that real assets are less liquid in recessions.

[2]Gibbons and Katz (1991) study this type of effect on labor markets noting that workers displaced by plant closings should be less affected by adverse selection. Thus, if there were more plant closings in bad times adverse selection would have a smoothing effect on wages.

all income levels. Investors also self insure less with riskless assets. Consequently, the outcome of risky projects has a larger impact on investors' income and more claims to high quality projects are sold in order to supplement current income for use in consumption and new investment. The claims price, and hence market liquidity, increases.

Liquidity also magnifies the effect of changes in productivity on investment and volume. We expect investment to increase with productivity, but investment increases by more when liquidity also improves since higher liquidity makes long-term risky investment even more attractive. Similarly, liquidity magnifies the effect of higher productivity on volume. Higher productivity and investment alone can lead to a larger claims market but a correspondingly more liquid claims market (a higher claims price) further increases the number of claims issued.

Why is it important to study endogenous liquidity? Empirically, there seem to be states of the world in which "too few" trades occur. These are states of the world in which the popular press reports that "liquidity dries up"– sellers cannot find buyers at what they believe to be fair (risk adjusted) prices. Too few trades implies that there is a market friction. Too few trades only in some states implies that this friction varies with the state of the economy. Moreover, from a quantitative theoretical standpoint, if we calibrate the level of asset illiquidity to what we think is a reasonable unconditional level it is difficult to get liquidity to matter much for equilibrium asset prices and quantities.[3] In particular, we seem to observe a larger demand for liquid assets than our models predict. However, if liquidity in fact varies positively with fundamentals and assets are less liquid in bad times, liquidity can have a much larger effect.[4]

Much work has been devoted to understanding the role of liquidity in asset markets. A large literature beginning with Diamond and Dybvig (1983) and including

[3]For the effects of a fixed level of illiquidity (i.e., transactions costs) on asset prices and turnover, see, for example, Constantinides (1986), Vayanos (1998), and the introduction to Huang (1999). For a quantitative analysis of the demand for liquid assets in this context, see Aiyagari and Gertler (1991) and Eisfeldt (2000b).

[4]Accordingly, Pàstor and Stambaugh (2002) construct liquidity betas and find liquidity to be a significant factor for asset pricing.

Diamond (1997) and Holmström and Tirole (1996, 1998, 2001) studies how institutions may improve upon markets for liquidity provision when there are information asymmetries. Related work on the financial accelerator by Bernanke, Gertler and Gilchrist (1996, 1999) and others studies the effects of agency costs on the availability of external financing. Illiquidity is often motivated by adverse selection. Myers and Majluf (1984), Lucas and McDonald (1990), and Korajczyk, Lucas, and McDonald (1992) study the effect of adverse selection on equity issues, focusing on separating equilibria. Stiglitz and Weiss (1981) study a model in which the intensity of the adverse selection problem in loan markets varies with the interest rate, holding investment constant. Hendel and Lizzeri (1999) and House and Leahy (2000) study the effects of adverse selection in dynamic durable goods markets. Finally, Shleifer and Vishny (1992) show that liquidity can vary with whether the average purchaser is an industry insider or outsider; in contrast this model focuses on the average seller's reason for trading.

Market liquidity is also a central focus of the market microstructure literature. In the model presented here claims risk is idiosyncratic and agents trade simultaneously in a competitive market taking prices as given. This is in contrast to microstructure models where sellers have private information about a common payoff and agents trade sequentially, sometimes in an imperfectly competitive market. See, for example, Glosten and Milgrom (1985) for a model of a competitive market with sequential trade, or Kyle (1985) for a model with imperfect competition. Sequential trade and imperfect competition allow for a price impact dimension of liquidity not featured here. In this model investors in long-term assets expect to incur a cost if they sell the asset early. Claims purchasers, however, do not incur a cost. They diversify claims risk away and purchase a risk-free claims portfolio which earns a risk-free return. Sellers of high quality assets in this model are like the liquidity traders of the microstructure literature in that they bear the cost of the asymmetric information and it is their trades which improve liquidity. In this model, the amount of such liquidity enhancing trades is endogenous.

The paper proceeds as follows: In Section I I describe the model economy, the claims market, and the problem faced by an investor in this economy. Section II defines the stationary equilibrium of the model and discusses how to solve for such an equilibrium. Section III describes the stationary equilibrium of a particular economy. In Section IV I describe the effects of changes in productivity on liquidity through comparative statics across economies with varying productivities of the risky asset. Section V discusses results for aggregate fluctuations and Section VI concludes.

I. Model

A. The Model Economy

There is measure one of identical investors with preferences over consumption ordered by

$$E\left[\sum_{t=\tau}^{\infty}\beta^t(1-\delta)^t\frac{c_t^{(1-\sigma)}}{1-\sigma}\right] \tag{1}$$

where β is the discount factor and δ is the probability that an investor dies in any given period t. Each period a fraction of investors dies and the same measure of investors are born.[5] Investors have a constant endowment stream and access to a riskless storage technology and a long-term risky technology. The riskless storage technology has a zero depreciation rate and thus bears a risk-free rate of return equal to one.

The risky technology has an initial investment cost of I per unit. Risky projects pay off only after two periods and have a payoff $Y \in \{0, 1\}$ per unit invested. In the period following investment, investors receive a signal which indicates the conditional probability $q \in \{q_L, q_H\}$, where $q_H > q_L$, that the project will succeed (i.e., that the payoff will be one). The signal is common to all ongoing projects an investor owns and therefore we can think of investors owning one project, of variable scale. The probability of receiving the high signal, q_H, is q_0 and hence the unconditional expected payoff of a risky project is $(q_0q_H + (1-q_0)q_L)$. The timeline for a risky project is illustrated in Figure 1. Since this risky technology is linear, productivity is the ratio of the expected payoff relative to the initial investment.

[5]Death is used both to ensure the existence of a unique stationary distribution for a given claims price (see, for example, Stokey and Lucas (1989) Theorem 11.2) and to prevent investors from reaching the upper bound of the state space in the numerical solution discussed below.

[INSERT FIGURE 1 ABOUT HERE]

Since agents are long lived they hold overlapping projects. Investors begin each period with both projects that are ongoing (corresponding to the $t+1$ node) and completed (corresponding to the $t+2$ node). Investors are *ex ante* identical, but differ in terms of their current income realization (from storage and completed projects), the scale of their ongoing projects, and the quality of those projects, due to idiosyncratic shocks to their risky investments. The interaction of shocks to and decisions about projects at different horizons is a crucial feature of the infinite horizon model. In particular, the decision to issue claims to ongoing projects depends on the scale and quality of those projects relative to the success of completed projects and the prospects for new projects.

B. The Claims Market

In addition to access to the riskless and risky technologies, each period investors also have access to an anonymous competitive market on which they can sell claims to their ongoing risky projects. In terms of Figure 1, each period investors can sell claims to the $t+2$ output of ongoing projects at the $t+1$ node for a price of P. Hence, investors can supplement their current period income by selling claims to the future output of their ongoing projects. The claims price is equal to the price of the average claim sold and is discussed further in Section II. Since the quality of ongoing projects is private information, the claims market is illiquid due to adverse selection.

The information structure in the model is as follows: Investors have private information about the quality of their ongoing projects. In Figure 1 this corresponds to private information at the $t+1$ node. Project qualities are independent across investors and over time for any particular investor. Moreover, and importantly, the claims market is anonymous. The decision to sell claims to a project of a particular quality depends on the investor's income and project scale so that knowledge of these quantities along with the investor's selling decision could potentially reveal the quality of their ongoing projects.[6]

[6]See Leland and Pyle (1977) for a model where agents can signal quality by retaining partial

Payments on claims to future output are enforcable.[7] While investors do not know who the owner of any particular project is and moreover they cannot keep track of other investors over time, investors can keep track of individual projects on which they have claims. An alternative assumption is that investors in fact sell fractions of projects instead of claims on their output in which case there is no question of enforcement. Note that since the low payoff is zero, enforceability rules out claims which promise a positive amount with certainty.

Although reputation and some knowledge of an investor's other decisions are certainly mitigating factors, in practice some degree of information asymmetry remains. However, there is clearly pressure in this economy with exogenously incomplete markets to develop mechanisms or institutions to alleviate the information asymmetries.[8] The simplest way for investors to alleviate the adverse selection problem would be to develop a market for claims to initiated projects. In such a market investors could diversify away all idiosyncratic risk since there is no private information upon initiation. One way to justify the fact that these claims are ruled out is to assume that there is no way for buyers of such claims to ensure investment.

The cost of an equilibrium framework is that market incompleteness is exogenously imposed. The benefit of an equilibrium as opposed to an optimal contracting approach is that we are able to study a market where investors can trade any number of claims at the equilibrium price instead of essentially treating each size contract as a unique asset. Clearly, the optimal contract would insure investors against some of the idiosyncratic risk they face; how much would depend on the specific assumptions about the information structure. Note that here the secondary market provides partial insurance since the distortion of the claims price by adverse selection implies that investors are *ex ante* partially insured against the low quality realization. In fact, pooling in the claims market provides some insurance that a market with known

ownership in their project.

[7]The mortgage market, for example, has similar features to this market. Banks securitize mortgage loans and sell them to a pool of agents. The market is anonymous, with claims enforced by the bank.

[8]See, for example, Nachman and Noe (1994) and DeMarzo and Duffie (1999) for how optimal security design can alleviate *ex ante* information asymmetries.

qualities could not provide.

C. *The Individual's Problem*

Each period investors allocate funds to consumption and to investments in the riskless and risky technology, and choose how many claims to their ongoing project to issue. The optimization problem for an individual investor is:

$$\max_{\{c_t, x_{t+1}, y_{o,t+1}\} \in \Re^3_+, y_{s,t} \in [0, y_{o,t}]} E_\tau \left[\sum_{t=\tau}^{\infty} \beta^t (1-\delta)^t \frac{c_t^{(1-\sigma)}}{1-\sigma} \right] \tag{2}$$

subject to

$$c_t + x_{t+1} + y_{o,t+1} I \leq e + x_t + (y_{o,t-1} - y_{s,t-1}) Y_t + y_{s,t} P \tag{3}$$

given initial ongoing projects $y_{o,\tau}$, an associated quality signal q_τ, and initial income from riskless assets and projects, $x_\tau + (y_{o,\tau-1} - y_{s,\tau-1}) Y_\tau$. Consumption in period t is denoted by c_t, x_{t+1} is riskless storage for next period, $y_{o,t+1}$ is the number of risky projects initiated this period, i.e., the number of ongoing projects for next period, and $y_{s,t}$ is the number of claims to ongoing projects sold. For reasons discussed in Section II.A., it is not necessary to distinguish investment in riskless storage from investment in claims to other investors' ongoing projects. Intuitively, since investors can diversify away the risk from these claims by holding a perfectly diversified portfolio, they will do so. The initial investment cost for risky projects is I, $Y_t \in \{0, 1\}$ is the payoff of projects completed at time t, and P is the price of claims to the future output of ongoing projects.

The expectation is over the pair (Y, q) which follows a Markov chain. The univariate processes $\{Y_t\}$ and $\{q_t\}$ are independently distributed across investors and over time. Furthermore, Y_t and q_t are independently distributed. However the current period signal q_t is the conditional probability that next period output Y_t equals one.

In words, each period the investor chooses this period's consumption, how much storage and ongoing projects to initiate, and how many claims against the future output of currently ongoing projects to sell, to maximize expected discounted utility subject to the per period budget constraint, a constraint that prevents investors from selling unbacked claims, and nonnegativity constraints on storage, ongoing projects and claims sold. The nonnegativity constraint on claims sold is crucial because it prevents investors from creating ongoing projects, i.e., it makes the risky technology a long-term technology. The budget constraint in words states that an investor's consumption, plus the amount of storage for next period, plus the number of projects initiated times the cost of new projects, must be less than the income from storage from last period, ongoing projects from last period to which claims were not sold, and income from claims sold against currently ongoing projects plus the investor's endowment.

The optimization problem for an individual investor can be stated in recursive form by defining the individual state vector as (w, y_o, q). Using "$^-$" to denote previous period values, $w = e + x + (y_o^- - y_s^-)Y$, i.e., an investor's current period income is their endowment plus storage from the previous period plus income from completed projects; y_o is the number of ongoing projects in the investor's portfolio (projects initiated in the previous period); and q is the signal about the quality of those projects.

The Bellman equation for an individual investor's problem is:

$$v(w, y_o, q) = \max_{\{c,x',y_o'\}\in\Re^3_+, y_s\in[0,y_o]} \frac{c^{(1-\sigma)}}{1-\sigma} + \beta(1-\delta)E\left[v(e + x' + (y_o - y_s)Y', y_o', q')|q\right] \quad (4)$$

subject to

$$c \leq w + y_s P - y_o' I - x'. \quad (5)$$

The budget constraint holds with equality because of nonsatiation. We can verify the following properties of the function v which solves the problem in equations (4) and (5).

Proposition 1 *There exists a unique fixed point v to the functional equation in (4). This function v is increasing in each of its arguments, w, y_o, and q. In addition, v is concave and the associated policy functions are continuous and single valued.*

The proof is standard and is hence omitted.[9]

[9]The standard proof (see, for example, Stokey and Lucas (1987)) that v is increasing in the state

Taking first order and envelope conditions and combining equations, we find the Euler equations for storage, new investment, and claims sold. These are just the familiar equilibrium asset pricing Euler equations, modified by the multipliers on the constraints:

$$u'(c) = \beta(1-\delta)E\left[u'(c')|q\right] + \nu_x \tag{6}$$

$$Iu'(c) = \beta^2(1-\delta)^2 E\left[Y''u'(c'') + \nu_s^{y_o\prime}|q\right] + \nu_o \tag{7}$$

$$Pu'(c) = \beta(1-\delta)E\left[Y'u'(c')|q\right] + \nu_s^{y_o} - \nu_s^0. \tag{8}$$

In words, equation (6) states that the cost of storing one more unit equals the return next period. If the cost exceeds the return then $x' = 0$. Equation (7) states that the cost of investing in an additional unit of the risky technology equals the return on the investment. This return can be decomposed into two parts: First, the return from having the option to sell claims to that project next period (thereby relaxing the multiplier, $\nu_s^{y_o\prime}$, on the constraint that $y_s' \leq y_o'$) and the return from receiving the payoff of the project after two periods. If the cost exceeds the return then $y_o' = 0$. Finally, equation (8) states that the return from selling an additional claim equals the cost in foregone payoffs tomorrow. If the return is less than the cost then $y_s = 0$, and if the return is greater than the cost then $y_s = y_o$.

The Euler equations illustrate the magnification effect liquidity has on investment and volume. Market liquidity feeds back on the investment decision by making the option of selling claims to the long-term technology more valuable and thereby increasing the return to investing an additional unit. Liquidity feeds back on the selling decision by increasing the return to selling, thereby increasing volume.

II. Stationary Equilibrium

Solving the general equilibrium problem amounts to finding the endogenous distribution over the individual state vector and the associated policy functions for

variable q needs to be modified slightly since q does not enter the one period return function directly, but instead affects value by determining the expectation over payoffs in the following period. The fact that v is increasing in w along with the fact that $q > \hat{q}$ implies that $E[w'|q] > E[w'|\hat{q}]$ for all policies for storage, initiated projects and claims sold, replaces the condition in the standard proof that the one period return function is increasing in q.

storage, new investment, and claims sold as a function of the state. Given this distribution and the policy functions we can compute all endogenous prices and quantities. In particular we can compute P, the claims price to ongoing projects, and determine the level of liquidity in the stationary equilibrium. Before defining the stationary equilibrium of the model described above, I first discuss how P is determined in equilibrium.

A. Equilibrium Claims Price

The following argument establishes that the market claims price P is equal to the price of the average claim sold. Define the "no adverse selection" price of a quality q_i claim to be p_i. One can think of this price as the "true value" of a claim of quality q_i. The expected payoff of a claim to a particular project is $\kappa p_H + (1-\kappa)p_L$, where κ is the fraction of claims on the market which are high quality. The certain payoff of a diversified portfolio of claims is $\kappa p_H + (1-\kappa)p_L$, or the same as the expected payoff of any one claim. Since investors are risk averse, they prefer the certain payoff of the diversified portfolio. The return on storage is one, thus the claims price is equal to the payoff of a diversified portfolio of claims, i.e.,

$$P = \kappa p_H + (1-\kappa)p_L, \tag{9}$$

and there is no need to distinguish storage from claims to risky projects. Investors take the equilibrium price P as given, i.e., they know the average quality of claims on the market.

To ensure that the market for risk-free assets clears at a risk-free rate of one, the equilibrium per capita quantity of riskless assets, which is the sum of per capita storage plus per capita claims sold, must be greater than or equal to the per capita number of claims sold. Since this is not always the case for storage held by investors who have access to the risky technology, I use a second class of "buffer stock" agents to clear this market at a risk-free rate of one. These agents also have measure one and preferences described by (1). We can think of these agents as young agents who become investors with probability δ each period. They bear idiosyncratic risk in their endowment streams and have access to the storage technology and the claims market. They use these riskless assets to insure their endowment streams through buffer stocks. Alternatively, one can view the model as representing a small open economy which takes the risk-free rate as given.

Thus, the equilibrium claims price will be a weighted average of the "no adverse selection" prices of high and low quality claims. In an economy with only diversifiable idiosyncratic risk, if there were no private information and if pooling risk were costless, claims would sell for their expected value and all projects would be sold off in their entirety. In other words, the entire return from long-term risky investment would accrue to the investor during the first period, and at the end of the first period ownership of risky assets would be sold off as riskless pooled claims.

I define the "no adverse selection" price of a quality q_i claim to be a fraction ϕ of the conditional expected payoff of a quality i project. Thus $p_i = \phi E[Y'|q_i]$, where $\phi \in (0, 1)$. The parameter ϕ is a reduced form representation of the fact that even without adverse selection, perfect risk sharing is not costless. One can think of ϕ as the result of the following informational friction: If claims to a project are sold, the project must be monitored at a proportional cost of $1 - \phi$. Claims to projects which are not monitored have a zero payoff and thus all projects which are sold are monitored. Thus, the conditional expected payoff net of monitoring costs of a claim to a project of quality i is $\phi E[Y'|q_i]$, which is the price specified above.

The parameter ϕ captures the fact that pooling risk is not costless even in the absence of adverse selection, and has the realistic effect in the economy with adverse selection that low quality projects are not all immediately sold off to outsiders. Note also that investors who die do not complete their projects, and setting $(1 - \phi) \geq \delta$ accounts for the fact that these projects "fail" with certainty. The qualitative results of the model carry over to the case where $\phi = 1$, however, I will focus on the case where $\phi < 1$ throughout the paper since it allows for more robust numerical examples. A detailed discussion of this and the effect of the parameter ϕ on the equilibrium in the model appears in Section III.D.

B. *Definition of a Stationary Equilibrium*

Denote the individual state vector (w, y_o, q) by z. An economy $\mathcal{E}$ is described by the risk aversion parameter, discount factor, death rate, constant endowment, initial investment cost for risky projects, probability of receiving the high and low signals, the high and low payoffs of risky projects, prices at which claims to high and low projects could be issued in an economy with no adverse selection, and risky endowment processes for the second class of agents $\{\hat{e}\}$. Thus, an economy $\mathcal{E}$ is defined as: $\mathcal{E} \equiv \{\sigma, \beta, \delta, e, I, q_0, q_H, q_L, p_H, p_L, \{\hat{e}\}\}$. Define a stationary equilibrium of this economy as:

Definition 1 *A* **Stationary Equilibrium** *for an economy $\mathcal{E}$ consists of: Policy functions $\{c(z), x'(z), y_o'(z), y_s(z)\}$, a price P, quantities $\bar{x}$, $\bar{y}_o$, and $\bar{y}_s$, and a probability distribution $\lambda(z)$, such that:*

1. *The policy functions $\{c(z), x'(z), y_o'(z), y_s(z)\}$ solve the investor's optimization problem given P.*
2. *The probability distribution $\lambda(z)$ is the stationary distribution of the transition matrix $\Lambda(z'|z)$ induced by investors' policy functions and the Markov process for payoffs and signals, $(\{c(z), x'(z), y_o'(z), y_s(z)\}$, $\Pi(Y, q))$.*[10]
3. *The per capita quantities of storage, ongoing projects, and claims to projects sold $\bar{x}, \bar{y}_o$, and $\bar{y}_s$ are implied by the average consumer behavior given the policy functions and the stationary distribution.*
4. *The per capita quantity of claims to projects sold $\bar{y}_s$ is less than or equal to the per capita quantity of storage $\bar{x}$ plus the per capita quantity of storage $\hat{x}$ demanded by the buffer stock agents.*[11]

[10] The transition matrix $\Lambda(z'|z)$ is formed by setting $\Lambda(w', y_o', q'|z) = \Pr(q')\Pr(Y'|q)$ if the policy functions for x, y_o, and y_s imply that tomorrow's state will be (w', y_o', q') when Y' is the payoff realization. Otherwise, $\Lambda(z'|z) = 0$.

[11] I assume that the risky endowment stream of the buffer stock agents ensures that this is the case, and do not explicitly consider these agents further, here or below.

5. *P is consistent with the quality distribution of claims sold, i.e., $P = \kappa p_H + (1-\kappa)p_L$, where κ is the endogenous fraction of claims sold which correspond to high quality projects.*

C. *Solving for a Stationary Equilibrium*

To solve for a stationary equilibrium, we need to find a fixed point in P, where the price of claims is consistent with the quality distribution. I solve for the equilibrium numerically using discrete state space dynamic programming and the following algorithm:[12]

(1.) Fix the parameters for an economy, $\mathcal{E}$, i.e., fix preference (σ, β, δ), endowment (e) and technology (I, q_0, q_H, q_L, p_H, p_L) parameters.

(2.) Initialize κ, the fraction of projects sold which are high quality. Then, $P = \kappa p_H + (1-\kappa)p_L$.

(3.) Solve the investors' maximization problem. Find the policy functions and calculate the implied stationary distribution over individual states, $\lambda(z)$.

(4.) Update κ and P by finding the fraction of claims sold which are high quality. In other words, compute $\kappa^+ = \frac{\sum_{(W\times Y_o)}\lambda(w,y_o,q_H)y_s(w,y_o,q_H)}{\sum_Z \lambda(z)y_s(z)}$, where $\lambda(z)$ is the measure of investors in state z. Then, $P^+ = \kappa^+ p_H + (1-\kappa^+)p_L$.

(5.) Iterate on steps (3) to (4) to convergence.

(6.) Use the implied stationary distribution $\lambda(z)$ and the policy functions $\{c(z), x'(z), y'_o(z), y_s(z)\}$ to find the equilibrium per capita quantities $\bar{x}$, $\bar{y}_o$, $\bar{y}_s$, and $\bar{c}$. The equilibrium claims price is the price P from the last iteration.

[12]See, for example, Lungqvist and Sargent (2000). It is straightforward to show that the problem here satisfies the properties necessary for value function iteration to work, i.e., standard proofs for convergence of the value function v and the policy functions $\{c(z), x'(z), y'_o(z), y_s(z)\}$ apply (see Stokey and Lucas (1989)). Moreover, as noted above, the existence and uniqueness of the stationary distribution for a given claims price are guaranteed by the fact that all investors face a positive probability of dying and being reborn into a common state.

The problem defined by equations (4) to (5) in Section I is difficult to solve analytically. As in models with transactions costs, the solution is characterized by action (selling) and inaction (not selling) regions.[13] Moreover, the investment choice problem involves allocating funds between two risky investments (new and ongoing projects) with different horizons and different payoff distributions.

I provide an analytical characterization of the solution to a finite horizon analogue to this model in the Appendix. However, the finite horizon analogue cannot deliver on the following dimensions: First, finite versions which accommodate analytical solutions must leave out the crucial interaction between projects of different vintages which arises in the infinite horizon case from having long-lived investors with overlapping projects.[14] It is important to include the effect of the risky income from completed projects on the selling decision. Part of the reason liquidity increases with productivity is precisely that when risky investment increases, pre-trade income streams are riskier. It is also important to include the effect of the demand for income for new investment on the selling decision. Liquidity increases with productivity partly because the optimal scale for new investment in the issuing period is larger. Second, solving for the stationary equilibrium of the infinite horizon model endogenizes the distribution over income and project scale as a function of productivity. With other modeling strategies this distribution would be a free parameter, and one with important implications for liquidity since conditional on project quality these variables exactly determine how many claims are issued. Finally, solving the infinite horizon problem shows that long lived consumers do not circumvent the liquidity problem by saving in the riskless asset.[15]

[13]See, for example, Constantinides (1986).

[14]See also Hendel and Lizzeri (1999) for a model of adverse selection in the used car market where the interaction between the market for new and used goods is key to their results.

[15]This is in contrast to the standard result in the literature on consumption and savings behavior with liquidity constraints starting with Deaton (1991). This literature finds that long-lived agents display "buffer stock" behavior and self insure shocks through savings in the riskless asset. Here, although investors are long lived and have access to a riskless liquid asset, they also use the claims market to insure shocks.

III. Characterization of a Stationary Equilibrium

In this section, I present the parameterization and results for what I will refer to as the "baseline" economy. I use these results to discuss the numerical solution to the individual's problem, paying particular attention to the policy function determining claims sold.

A. Parameterization

Table I summarizes the preference, endowment, and technology parameter values for the baseline economy. The analysis here is intended to provide insight into the qualitative properties of endogenous liquidity due to adverse selection and though most parameters are assigned standard values, they are not based on data from specific markets.

[INSERT TABLE 1 ABOUT HERE]

The value of the risk aversion parameter σ is two. The discount rate $\beta = 0.99$ and the death rate $\delta = 0.03$ together imply an effective rate of time preference of 0.9603, thus a time period is taken to be one year. The death rate is used to ensure that few investors will be at the upper bounds of the discretized state space. The projects of an investor who dies are not completed; as discussed in Section II.A. this loss is accounted for by setting $(1 - \phi) \geq \delta$. When investors are born they are initialized with the smallest scale ongoing project and income equal to endowment. This slightly positive scale initial project ensures that positive investment occurs even in economies with low productivity.[16] The endowment $e = 0.2$ is chosen relative to the technology input and output parameters for the same reason, i.e., to ensure that positive investment occurs in equilibrium.

The conditional probabilities that a project succeeds are centered around .5. The low quality signal $q_L = 0.45$ and high quality signal $q_H = 0.55$ are equally likely so

[16] Alternatively I could use a finer grid and give investors no initial project, or a slightly positive initial income. I have verified that the results do not change using either specification. Moreover, in all of the examples studied here, only investors with the low quality signal sell claims to their initial projects.

$q_0 = 0.5$. The timing of uncertainty resolution for the risky project is important. If too much information about quality is revealed by the signal the market with adverse selection breaks down and only low quality claims are sold or in extreme cases no investment is undertaken, similar to the standard result of Akerlof (1970). Therefore, I focus on parameters which load the risk of the risky technology toward the second period of the project.

The unconditional expected payoff of the risky technology is $E[Y] = q_0 q_H + (1 - q_0) q_L$. The net return on investment in the risky technology is $E[Y/I] - 1 \equiv r_2$. The choice for the required input to the technology, $I = 0.4167$, relative to its output, $Y \in \{0, 1\}$ implies a two period expected net return of 20 percent, or a one period expected return of 9.5 percent on projects held to maturity. The one period return on projects to which claims are sold $P/I \equiv r_1$ is determined by the equilibrium claims price. The fraction of expected payoffs which yields the "no adverse selection" prices of high and low quality claims is $\phi = 0.9$, which implies that $p_H = 0.495$ and $p_L = 0.405$.

To discretize the state space for income w, storage x, and risky projects y_o, I first choose an upper bound for y_o. I set the upper bound on risky projects to 0.75, and allow investors to initiate risky projects in increments of 0.05 up to a maximum scale of 0.75. I set the grid for x such that the upper bound is not binding in states with significant mass in the stationary equilibrium.

B. *Equilibrium Level of Liquidity*

The results for the baseline economy are shown in Table II. The fraction of claims sold which are high quality, κ, is 6.05 percent in this example, which implies that the claims price, P, is slightly higher than the no adverse selection value of low quality claims. I define liquidity as the cost of transferring the value of expected future payoffs from long-term assets into current income and measure this cost in two ways. First, liquidity can be measured by the tradeoff between the timing of income ("maturity") and expected return.[17] Investors who sell claims to their

[17]See Hirshleifer (1971, p. 2) for a similar description of liquidity: "Illiquid assets are those characterized by a relatively large discount for "premature" realization; this corresponds to a relatively

ongoing projects realize a one period return of about -1.5 percent on those projects, whereas they earn a one period expected return of 9.5 percent on projects to which they retain ownership. Alternatively, liquidity can be measured by the discount incurred by extracting income early. Claims to high quality projects sell at a discount of about 17 percent from their true value. In terms of quantities, investment in the risky project is about four times the amount of storage, and claims are sold to 30 percent of all ongoing projects.

[INSERT TABLE II ABOUT HERE]

C. Equilibrium Policy Function for Claims Sold

The equilibrium policy function for claims sold as a function of individual state, $y_s(w, y_o, q)$, along with the stationary distribution over individual states is all we need to determine the equilibrium claims price. The policy function tells us how many claims are issued by investors with each combination of current income, ongoing project scale, and project quality. The distribution over individual investors tells us how many of each of these "types" of investors there are. The claims price is simply determined by computing how many claims are issued by investors with high quality projects relative to the total number of claims issued and taking the implied weighted average of the no adverse selection prices.

Investors sell claims to their ongoing projects the lower is their current income, the larger is the scale of their ongoing projects, and the lower is the quality of those projects. Thus, we can use standard portfolio intuition to understand the selling decision. Loosely, there are two reasons investors issue claims to their ongoing projects: (1) the adverse selection reason and (2) a rebalancing reason. First, investors issue more claims to projects if they are low quality. Second, investors issue claims even to high quality projects if there is an imbalance in the scale of their ongoing project relative to their current income realization. Investors want both to smooth consumption and to smooth and diversify investment across vintages. Issuing claims to their

high time-rate of return if the asset is held rather than so realized."

ongoing projects supplements their current income for use in consumption and new investment, and reduces their exposure to risky project income.

The fact that ongoing projects are illiquid implies that the policy function will be characterized by two regions, an inaction region and a selling region. I will characterize the decision to sell first by characterizing the boundary to the selling/not selling regions, and then by showing that conditional on selling claims, the number of claims sold y_s is decreasing in income w and quality q and increasing in project scale y_o. In the Appendix, I characterize the boundary to the selling region for the finite horizon analogue in terms of the scale of the ongoing project that an investor owns analytically. This boundary is given by a linear function of current income, with a slope and intercept which are increasing in project quality. Simple comparative statics show that in this case, conditional on selling claims, the number of claims sold y_s is indeed decreasing in current income and project quality and increasing in the scale of the ongoing project. The finite horizon eliminates the investment allocation decision between projects of different vintages which allows for analytical characterization, but has several drawbacks as discussed in Section II.C.

I present numerical results for the policy functions and selling regions for the infinite horizon problem here. Figure 2 graphs the selling regions for the infinite horizon problem with the base case parameterization. White cells denote states in which no claims are sold and cells are darker as more claims get sold; a black cell denotes the maximum amount of claims (.75) is sold, which can only occur when $y_o = 0.75$, and gray cells darken in increments of 0.05. The boundary of the selling region appears to be almost linear in current income with a slope and intercept which are increasing in project quality.[18] Moreover, conditional on being inside the selling region, the number of claims sold is decreasing in income and project quality and increasing in the scale of the ongoing project.

[INSERT FIGURE 2 ABOUT HERE]

The policy functions at the upper tail of income for investors who hold the maxi-

[18] Note also that for the case $e = 0$ the problem for a given q is homogenous in the pair (w, y_o) and the boundary is again exactly linear.

mum scale project are affected by the binding upper bound on the grid for storage.[19] However, we do not need to concern ourselves with this region as long as there are few investors in the affected region of the state space and their contribution to the equilibrium variables of interest is economically insignificant. This is the case in this economy, 99.6 percent of investors have incomes w less than or equal to two, and holds for all of the economies discussed in Section IV. Accordingly, Figure 3 shows that this upper bound does not affect the equilibrium per capita quantities of high and low quality claims sold. Figure 3 shows the contribution of claims sold by investors in a specific state to the total amount of claims sold. White denotes states where no claims are sold or where there is no mass in the stationary equilibrium, the lightest shade of gray corresponds to a fraction of 0.1 percent of claims sold, and black denotes states in which at least one percent of sales occur. Essentially, only sales by low income investors get positive weight in the stationary equilibrium; investors with current income realizations smaller than two contribute 96.2 percent of high quality claims sold.

[INSERT FIGURE 3 ABOUT HERE]

D. *Comments on the Equilibrium*

In constructing the examples in this paper, I looked for economies with equilibria in which some high quality sales occur, but not all high quality projects are sold, and for economies which seemed to have a unique interior equilibrium for a broad range of parameters. This facilitates the numerical comparative statics which are the focus of this paper. Such examples require parameterizations which rule out the $\kappa = 0$ (no high quality sales) and the $\kappa = q_0$ (all high quality projects are sold in their entirety) equilibria.

[19] The policy functions for investors with very high wealth and the maximum scale project will be affected by the binding upper bound on storage no matter how high this upper bound is, but as long as few investors are affected this does not matter. Investors sell projects here because they have reached the upper bound on the amount of storage and new projects they can initiate and expect to be constrained in the next period as well.

To illustrate, consider the mapping which describes for any given initial value of κ and implied price of claims the resulting fraction of high quality claims on the market for a single iteration of the solution algorithm described in Section II.C. Figure 4 graphs this mapping $f : \kappa \to \kappa^+$ for the baseline economy as well as the economies studied in Section IV.A. The equilibrium claims price satisfies $f(\kappa^*) = \kappa^*$. Economies in which some high quality sales occur, but not all high quality projects are sold have mappings with $f(0) > 0$ and $f(q_0) < q_0$, where $q_0 = 0.5$ in the examples presented. For $f(0) > 0$, it must be the case that some high quality projects are sold even when the claims price equals the price of low quality claims. This can be achieved, for example, by narrowing the spread between high and low quality claims, by making agents more risk averse, or by decreasing agents' endowment. For $f(q_0) < q_0$, it must be the case that even if the price of claims is $q_0 p_H + (1 - q_0) p_L$, not all high quality projects are sold off in their entirety. This can be achieved, for example, by lowering ϕ, or by increasing q_H, both of which make selling more costly relative to holding on to a high quality project.

[INSERT FIGURE 4 ABOUT HERE]

The parameter ϕ turns out to allow a useful degree of freedom in parameterizing the expected return on risky projects from initiation to the intermediate stage relative to the expected return from the intermediate stage to completion. In particular, ϕ directly decreases the portion of the expected return which accrues over the first period of investment since it lowers the maximum possible equilibrium claims price for a given q_0, q_H, and q_L. This makes selling more costly, which in turn makes the $f(q_0) = q_0$ equilibrium where all high quality projects are sold in their entirety unlikely. It allows for numerical examples which are robust over a larger parameter range because it does this without affecting either the amount of information asymmetry at the intermediate stage, or the two period return on risky projects held to completion. For the examples presented in the paper, I chose values for q_H and q_L symmetric around 0.5, let q_0 be 0.5, and set $\phi = 0.9$ to ensure that $f(0.5) < 0.5$.[20]

[20] Qualitatively very similar results arise from choosing, for example, $\phi = 1$, $q_L = 0.4$, $q_H = 0.7$, and $e = 0.05$, with all other parameters unchanged.

The example economies constructed in this way typically have unique equilibria; they are characterized by $\kappa \to \kappa^+$ mappings which cross the 45° line once from above. However, as is common in economies with adverse selection, there are certain parameterizations of this economy which display multiple equilibria (at least numerically).[21] If adverse selection is present there are externalities from selling– when other high quality sellers participate in the pooled market each investor faces a smaller discount for selling high quality assets and may choose to sell, and vice versa if there are few other high quality sellers.

To see this, consider again the mapping in Figure 4. For each of the economies studied here there is a unique fixed point where $f(\kappa^*) = \kappa^*$; this is the equilibrium discussed above. For any initial κ below this fixed point, e.g. $\kappa = 0$, the solution algorithm converges to the equilibrium κ^* from below and vice versa for initial values above the fixed point, e.g. $\kappa = q_0$, without finding any other equilibrium points. It is possible, however, to construct economies with mappings with multiple fixed points, for example by constructing economies with mappings exhibiting a slope which exceeds one in the relevant range. Notice, though, that in all of the examples I computed, up to numerical precision, the mapping f shifted up as productivity increased. This suggests that even in economies with multiple equilibria, given an appropriate equilibrium selection mechanism, the comparative statics results from the examples presented here would carry over.

IV. Comparative Statics

For liquidity to be increasing in the state of the economy, high quality claims must constitute a larger fraction of the claims market (κ should increase) the more productive the risky technology is. The intuition for why liquidity increases with productivity is as follows: As the technology gets more productive, risky investment opportunities improve and investors optimally initiate larger scale projects at all income levels each period. Investors also store less and hence carry less insurance

[21]See, for example, Azariadis and Smith (1998). I thank Beatrix Paal for comments on this section.

against low payoffs to their risky projects.[22] This implies that the risky payoff has a larger impact on investors' current income; pre-trade income streams are more risky.[23] Moreover, when productivity increases it becomes more costly for investors to have an imbalance between ongoing and newly initiated risky projects. Risk-averse investors want to smooth both their consumption and their level of investment in the risky technology across vintages and when the risky investment is more productive the optimal investment level is higher. Thus, more claims are sold to high quality projects to supplement current income for use in consumption and new investment when the risky technology is more productive. The claims price, and hence market liquidity, increases. Furthermore, at the higher claims price investors with higher current income realizations and smaller scale ongoing projects issue claims to high quality projects. The claims price in the stationary equilibrium is the "fixed point" of these effects, where the quality distribution of claims sold is consistent with the claims price.

More claims issued for non-adverse selection reasons leads to a more liquid market when the risky asset is more productive. This seems intuitive–investors infer the reason assets are on the market from the state of the economy. In good times, it is likely assets are sold because of temporarily low cash flows or to raise money for high productivity new projects. Conversely, in bad times more claims are sold due to projects turning sour rather than to these "course of business" shocks.[24] Since investors with low quality projects sell claims mainly due to the low quality realization, the selling decision for these investors is less sensitive to changes in productivity and the state sensitive volume is high quality.

[22]Policy functions for storage (shown in the working paper version of this paper, Eisfeldt (2000a)), reveal that investors store a smaller fraction of their income when the risky asset is more productive.

[23]It is precisely the increased pre-trade riskiness of income that induces better post-trade risk sharing through the more liquid claims market. Interestingly, post-trade income can actually be less risky (i.e., we can observe counter cyclical income variation) even as risky investment increases since liquidity enables better income smoothing.

[24]Accordingly, Pilotte (1992) finds that the stock price response to new financing is less adverse when growth opportunities are better.

A. *Varying Productivity*

The productivity of the risky linear technology is given by the slope of the expected payoff $E[Y]$ relative to the initial investment cost I and can be summarized by the two period expected net return $E[Y/I] - 1 \equiv E[r_2]$. The simplest way to model increases in productivity without affecting conditional variances and while holding the low payoff constant at zero is by decreasing the initial investment cost. Comparative statics are over changes in I implied by two percent changes in the two period expected net return holding the expected payoff constant at 0.5. The initial investment cost I is such that the implied two period expected return $E[r_2]$ is 40 percent in the most productive economy and 12 percent in the least productive economy.

Figure 5 graphs the increase in liquidity as a function of increases in productivity. The top left panel shows how the quality distribution of claims sold shifts toward high quality projects as the initial investment cost decreases relative to the expected output. The top right panel plots the resulting claims price. In the least productive economy the fraction of claims which are high quality κ is small and hence the market claims price $P = \kappa p_H + (1-\kappa)p_L$ is very close to the true value of low quality claims p_L. As productivity increases, the claims price increases to a value close to the no-adverse selection price, $P = q_0 p_H + (1 - q_0)p_L$.

[INSERT FIGURE 5 ABOUT HERE]

Between $E[r_2] = 20\%$ and $E[r_2] = 22\%$, the claims price P overtakes the initial investment cost I. At this point, the endogenous one period return from projects to which claims are sold $P/I - 1 \equiv r_1$ becomes positive and it becomes better to invest in risky projects to which one can sell claims after one period than to invest in storage. Accordingly, the model economy is very sensitive to changes in productivity in this region.

The bottom panels of Figure 5 show that the tradeoff between the timing of income and expected return improves and that the cost of supplementing current income by selling claims to future output decreases. The left panel shows that the endogenous one period return increases faster with productivity than the exogenous two period return. The right panel graphs the discount from "true value" at which high quality claims sell. This discount decreases from about 18 percent to just over 10 percent.

Three effects on the high quality selling region drive the changes in liquidity. First, since risky investment increases, more investors realize individual states with large scale ongoing projects and low current income; states in the selling region. This effect alone drives the claims price up. Second, at the higher claims price agents within the selling region sell weakly more. Finally, the selling region grows; investors with smaller scale ongoing projects or higher current income realizations become willing to sell claims to high quality projects at the higher claims price. Figure 6 verifies that the selling region for high quality projects indeed increases with productivity. From top to bottom, the figure displays the selling region for investors with high quality projects when $E[r_2] = 18\%$, 20%, and 22%.[25] The selling region for low quality projects also increases, however the amount of high quality claims sold is much more sensitive to changes in productivity.

[INSERT FIGURE 6 ABOUT HERE]

Liquidity magnifies the effects of changes in productivity on investment and volume. Figure 7 graphs the decomposition of the effects of increases in productivity and liquidity compared to increases in productivity alone. The top line shows investment and volume as a function of the two period expected return when liquidity effects are accounted for. The bottom line nets out liquidity effects by holding the claims price for all economies constant at the equilibrium price of the least productive economy. Since κ is bounded between zero and one, magnification is necessarily local. For intermediate levels of productivity, liquidity is very sensitive to changes in productivity and in this region liquidity magnifies the effect of higher productivity on investment and volume. Moreover, investment and volume are always strictly greater when liquidity effects are included. Note also that in this model market breakdown works through investment. At low levels of productivity and hence liquidity (e.g.,

[25] Again, there are almost no investors in the upper tail of the income distribution (with incomes larger than 1.8). For clarity I do not graph this region here.

when $E[r_2]$=12%), investment which would occur if there were no adverse selection is not undertaken. Investment (and volume) is nearly zero.

[INSERT FIGURE 7 ABOUT HERE]

We can further measure the quantitative effect of liquidity by determining the amount of consumption an investor would be willing to forego to obtain an increase in liquidity alone, netting out the effect of productivity increases.

To compute the welfare cost of illiquidity, let $v(w, y_o, q)$ solve the problem in (4) and (5) for economy $\mathcal{E}$. Define the average value for an investor in this economy as $\hat{v}$. This average value satisfies:

$$\hat{v} \equiv \sum_{(W \times Y_o \times Q)} \lambda(w, y_o, q) v(w, y_o, q) = \sum_{t=0}^{\infty} \beta^t (1-\delta)^t \frac{\hat{c}^{(1-\sigma)}}{1-\sigma} \tag{10}$$

where $\lambda(w, y_o, q)$ is the measure of investors in state (w, y_o, q) in the stationary equilibrium. The constant consumption stream $\hat{c}$ associated with this value can be found by inverting equation (10).

The fraction of consumption an investor with the average value of utility would be willing to forego to move from an economy with low productivity to an economy with high productivity is

$$\theta_{(r_2^L, P^L) \to (r_2^H, P^H)} \equiv \frac{\hat{c}_{(r_2^H, P^H)} - \hat{c}_{(r_2^L, P^L)}}{\hat{c}_{(r_2^L, P^L)}} \tag{11}$$

where the subscripts on $\hat{c}$ give the productivity and liquidity of the associated economies. Part of this fraction is due to the change in productivity, and part is due to the higher liquidity of the economy with high productivity. To decompose this fraction, let $\hat{c}_{(r_2^L, P^H)}$ be the constant consumption stream associated with the average value of an investor who lives in an economy with low productivity but high liquidity. Define $\theta_{(r_2^L, P^L) \to (r_2^L, P^H)}$ analogously to equation (11), replacing $\hat{c}_{(r_2^H, P^H)}$ with $\hat{c}_{(r_2^L, P^H)}$. The fraction $\theta_{(r_2^L, P^L) \to (r_2^L, P^H)}$determines how much consumption an investor in the low productivity low liquidity economy would forego in order to move to the economy with higher liquidity but the same low productivity. The contribution of liquidity to the fraction of consumption an investor would give up to move from the low productivity low liquidity to the high productivity high liquidity economy is given by $\theta_{(r_2^L,P^L)\rightarrow(r_2^L,P^H)}/\theta_{(r_2^L,P^L)\rightarrow(r_2^H,P^H)}$.

For example, we can compare the economies from Figure 6. Investors would forego about 3.7 percent of consumption to move from the economy with $E[r_2] = 18\%$ to the economy with $E[r_2] = 20\%$, about 43 percent of which is due to the change in liquidity. The effect of liquidity varies. When liquidity has a larger effect, the difference in welfare across economies is magnified. Investors would forego about 2.2 percent of their consumption to move from the economy with $E[r_2] = 20\%$ to the economy with $E[r_2] = 22\%$, about 27% of which is due to the change in liquidity.

B. *Varying the Ex Ante Quality Distribution*

The intuition from the comparative statics in Section IV.A. applies to changes in other model parameters which essentially affect the relative productivity of the risky asset. This is because the key to higher liquidity in this model is more investment in the risky asset. For example, increasing productivity by increasing the exogenous fraction of high quality projects, q_0, also leads to higher liquidity. The endogenous fraction of high quality claims sold moves in the same direction as and by more than changes in the exogenous fraction of high quality projects initiated. The results of comparative statics over q_0 mimic those of Section IV.A. almost exactly.[26] Notice that in this experiment all changes in the return on projects to which claims are sold are due to changes in the claims price alone since the initial investment cost is held constant.

V. Cycles

This section extends the comparative statics to aggregate fluctuations of a single economy over time. An explicit extension is important since some of the results for comparisons across stationary equilibria discussed in Section IV may not carry over to an economy with fluctuations since investors can smooth cyclical shocks over time. The aggregate fluctuations I study here are deterministic cycles. In a model

[26] The complete results appear in the working paper version of this paper, Eisfeldt (2000a).

with stochastic aggregate fluctuations, distributions emerge as state variables which makes solving the model intractable due to the "curse of dimensionality". Thus, to illustrate, I study the effect of deterministic cycles.

The length of the cycles plays an important role in the effect of productivity on liquidity. As discussed above, the fact that investors initiate larger scale risky projects at all income levels when productivity increases drives the rebalancing sales which increase liquidity. If cycles are only two periods long, the high productivity period is also the period when investors have few ongoing projects (since the low productivity period always precedes the high productivity period). As a result, even though the investment opportunity for risky projects is good in periods of high productivity (as measured by the two period return) and the income from risky projects (initiated two periods prior, in the last high productivity period) has a large impact on current income, investors have few ongoing projects and thus sell few claims (and even fewer high quality ones). Thus, I study an economy with four period cycles consisting of two periods of low productivity followed by two periods of high productivity.

Clearly, the exact specification of the productivity process is important for the economy to exhibit procyclical liquidity. As argued above, the "persistence" of the deterministic productivity changes is a crucial determinant of the results. A quantitative stochastic business cycle study would require calibrating this persistence and the magnitude of the productivity shocks, as well as the timing of project payoffs. Similarly, as in the comparison across stationary equilibria, we would expect different effects from positive shocks to income vs. productivity. Positive income shocks might reduce contemporaneous high quality sales, whereas a positive productivity shock would increase risky investment and hence lead to more rebalancing sales in the future. In light of these comments, the results of the example with deterministic cycles must be viewed as illustrative.

Figure 8 plots productivity, the claims price (which determines the level of liquidity), investment, and volume over a four period cycle. The cycle consists of two periods where the two period return to risky projects is 16 percent followed by two periods where the two period return to risky projects is 26 percent. The pattern of investment closely follows that of the productivity level; when productivity is high investors invest more in the risky asset. Liquidity and volume lag productivity and investment by one period. As discussed in Section IV, investors issue claims when they have large scale ongoing projects relative to their current income from storage and past investments. In periods of high productivity investors of all income levels invest more in the risky asset and insure less through storage. The higher the investment in the risky asset relative to storage, the more likely it is that in the following period investors realize a state in which their exposure to the risky asset is too large relative to their current income. Thus liquidity and volume follow the cyclical pattern of productivity and investment, lagged by one period. Intuitively, participants in the claims market expect to see more sales of high quality claims and a higher claims price when investment was high in the previous period, since a high investment level leads to rebalancing sales.

[INSERT FIGURE 8 ABOUT HERE]

I conclude this section with a conjecture about the results of an extension to stochastic productivity shocks. Since the productivity changes in this model are deterministic, investors do not need to rebalance to take advantage of investment opportunities; the only shocks are to their current income. If productivity was stochastic, more rebalancing might occur in order to take advantage of productivity increases. In this case, liquidity would still be procyclical, but might not lag productivity as pronouncedly because the productivity shock itself would lead investors to rebalance their holdings between old (ongoing) and new (highly productive) projects.

VI. Conclusions

This paper studies a dynamic economy in which high productivity leads to higher investment in risky assets and hence more rebalancing trades, mitigating the adverse selection problem and improving liquidity. State dependent liquidity is important for understanding the following two features of asset market data: First, liquidity spreads vary both cross sectionally and in time series in a way which suggests that markets are more liquid in good times. Second, the unconditional level of illiquidity in most long-term asset markets (as measured, for example, by transactions costs) is too low to explain the observed demand for liquid assets.

Two aspects of the model are worth noting here. First, there is more heterogeneity than just across project quality in the economy studied here. The idea is to model the reasons why investors sell claims explicitly, and then to determine how many investors sell claims for each reason. It turns out that more investors sell claims for reasons other than a low quality realization when productivity is high. Second, the level of liquidity depends on the interaction between investment at different vintages. Increased risky investment at higher productivity levels leads to riskier pre-trade income streams from past investments. Moreover, when productivity is high the optimal new project scale is larger thus more current income is necessary to start it. As a result, more rebalancing sales occur and the volume of high quality sales is procyclical.

Appendix

The regions where investors sell or do not sell claims to their ongoing projects can be characterized analytically in a simplified two period analogue to the infinite horizon problem. The crucial simplification is that this finite horizon problem eliminates the interaction between projects of different vintages. Consider the following finite horizon version of the model with two dates 0 and 1. At time 0, an investor is born with income w_0, ongoing projects $y_{o,0}$, and a signal for ongoing projects q. The investor chooses how much storage x_1 to carry over to date 1 and the number of claims to ongoing projects to sell $y_{s,0}$. At time 1 the investor simply consumes the endowment and any income from risky projects and storage. Due to the finite horizon the investor does not initiate any new projects, which simplifies the analysis.

Formally, at dates 0 and 1 the investor solves the following problems, respectively:

$$
\begin{aligned}
[t=0] \qquad & v_0(w_0, y_{o,0}, q) = \max_{\{c_0, x_1\} \in \Re^2_+, y_{s,0} \in [0, y_{o,0}]} \{u(c_0) + \beta E\left[v_1(w_1)|q\right]\} \\
& \text{s.t. } c_0 \leq w_0 + y_{s,0}P - x_1 \\
& w_1 \leq x_1 + (y_{o,0} - y_{s,0})Y_1 \\
[t=1] \qquad & v_1(w_1) = \max_{c_1} u(c_1) \\
& \text{s.t. } c_1 \leq e + w_1
\end{aligned}
$$

given w_0, $y_{o,0}$, and q, where $u(c) = \frac{c^{(1-\sigma)}}{1-\sigma}$.

The first order conditions for this problem are:

$$0 = -u'(w_0 + y_{s,0}P - x_1) + \beta\left[qu'(e + y_{o,0} - y_{s,0} + x_1) + (1-q)u'(e + x_1)\right] \quad \text{(A1)}$$

$$0 = Pu'(w_0 + y_{s,0}P - x_1) - \beta qu'(e + y_{o,0} - y_{s,0} + x_1) + \nu_s^0 \quad \text{(A2)}$$

where ν_s^0 is the multiplier on the constraint that ongoing projects cannot be created. Here, I ignore the constraint that investors cannot sell unbacked claims since I am focusing on the boundary of the selling region, where $y_{s,0} = 0$.

Comparative statics for claims sold $y_{s,0}$ with respect to income w_0, number of ongoing projects $y_{o,0}$, and project quality q show that the number of claims investors issue is decreasing in w_0, increasing in $y_{o,0}$, and decreasing in q. Investors sell more ongoing projects the lower their income, the more ongoing projects they have, and the lower the quality of those projects.

Moreover, we can explicitly characterize the regions in which claims to projects are sold by characterizing the boundary to the regions (where $y_{s,0} = 0$) as follows. Assume that $x_1 > 0$. At $x_1 = 0$ a similar argument, using only equation (A2), applies. If the investor is indifferent between selling and not selling, at $y_{s,0}$=0, equations (A1) and (A2) reduce to

$$0 = -u'(w_0 - x_1) + \beta \left[qu'(e + y_{o,0} + x_1) + (1 - q)u'(e + x_1) \right] \tag{A3}$$

$$0 = Pu'(w_0 - x_1) - \beta qu'(e + y_{o,0} + x_1). \tag{A4}$$

It is straightforward to solve equations (A3) and (A4) for $y^*_{o,0}$, the critical level of ongoing projects at which investors begin to sell claims to their projects, in terms of parameters. One can easily verify that $y^*_{o,0}$ is an increasing linear function of w_0, with an intercept and slope which are increasing in q. Figure 9 describes the selling regions as a function of w_0, $y_{o,0}$, and q.

[INSERT FIGURE 9 ABOUT HERE]

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Tables

Table I
Parameterization of the Baseline Economy

This table presents the values for the risk aversion parameter σ, discount factor β, death rate δ, constant endowment e, initial investment cost for risky projects I, probability of receiving the high and low signals q_H and q_L, prices at which claims to high and low projects could be issued in an economy with no adverse selection p_H and p_L, and the discretized state space for storage x, ongoing risky projects y_o, and claims sold y_s used in the baseline economy.

Preferences		Endowment		Technology		Grid	
σ	2	e	0.2	I	0.4167	ystep	0.05
β	0.99			q_0	0.5	xstep	0.1
δ	0.03			q_L	0.45	x^{UB}	1.5
				q_H	0.55	y_o^{UB}	0.75
				p_L	0.405	y_s^{UB}	0.75
				p_H	0.495		

Table II
Results for the Baseline Economy

This table presents the price and quantity results for the baseline economy.

Prices: Two period expected returns $E[r_2]$, one period return on projects to which claims are sold r_1, fraction of claims which are high quality κ, claims price P, discount from true value at which claims are sold, $\frac{p_H - P}{p_H}$.

Quantities: Per capita quantities of storage $\bar{x}$, ongoing projects $\bar{y}_o$, projects to which claims are sold $\bar{y}_s$, projects to which claims are sold as a fraction of total projects $\frac{\bar{y}_s}{\bar{y}_o}$, and storage as a fraction of total investment $\frac{\bar{x}}{\bar{x}+\bar{y}_o}$.

Prices		Quantities	
$E[r_2]$	20%	$\bar{x}$	0.0623
r_1	-1.5%	$\bar{y}_o$	0.2552
κ	0.0605	$\bar{y}_s$	0.0787
P	0.4104	$\bar{y}_s/\bar{y}_o$	30.82%
$(p_H - P)/p_H$	17%	$\bar{x}/(\bar{x}+\bar{y}_o)$	19.61%

Figures

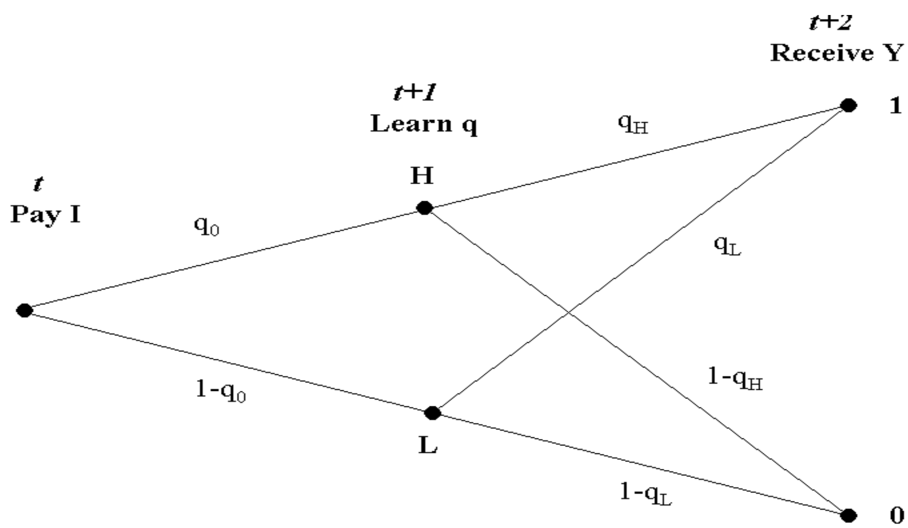

Figure 1. Timeline of a risky project.

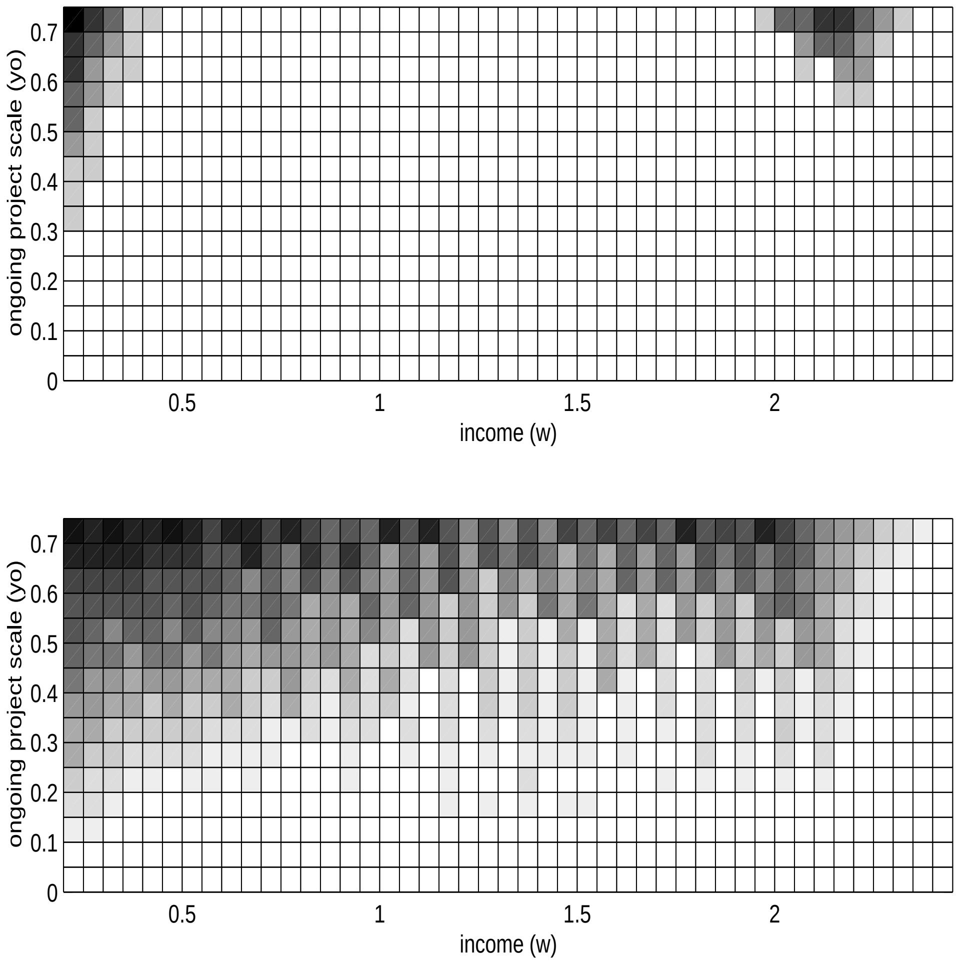

Figure 2. Selling regions. Claims sold y_s as a function of income w and ongoing project scale y_o for investors with high (top) and low (bottom) quality signal. White is 0, black is .75, and gray cells darken in increments of 0.05 claims.

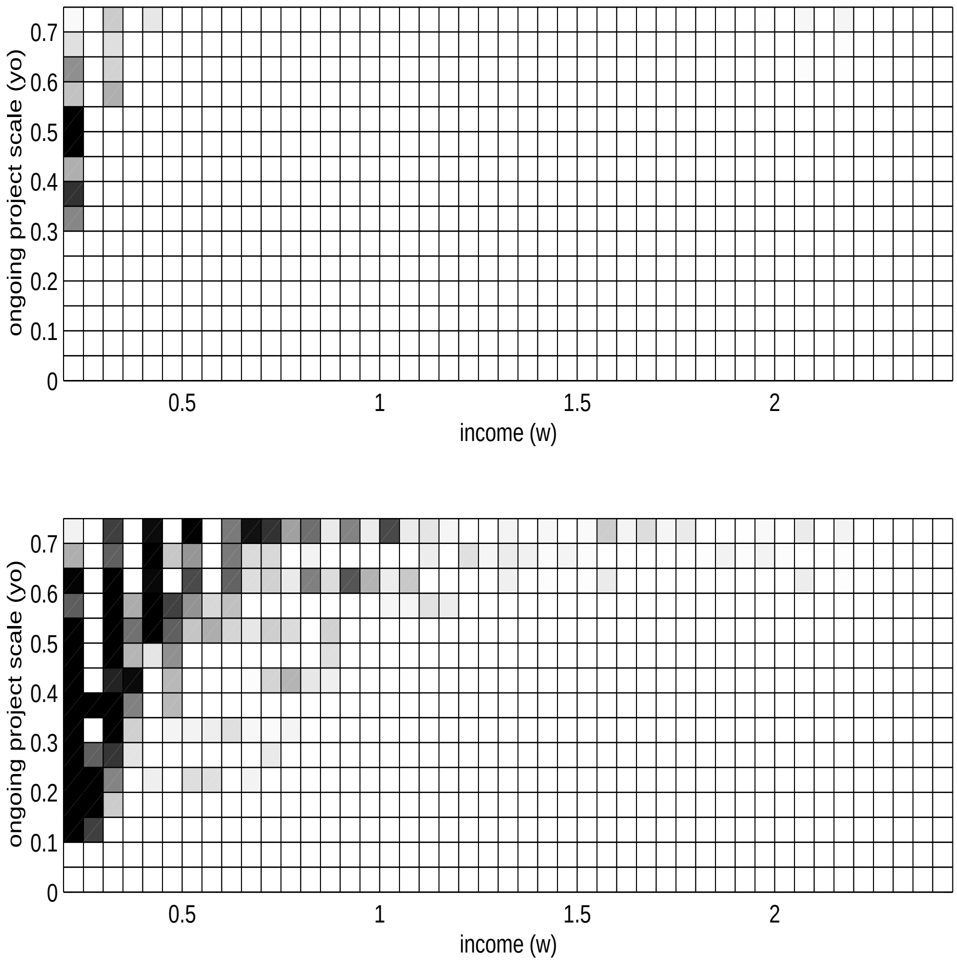

Figure 3. Weighted sales. Claims sold by state weighted by the weight on that state in the stationary equilibrium as a fraction of total claims sold by investors with high (top) and low (bottom) quality projects $\frac{y_s|_{(w,y_o,q)}\lambda(w,y_o,q)}{\bar{y}_s}$. White is 0, black is $\geq$ 1% and gray cells darken in increments of 0.1%.

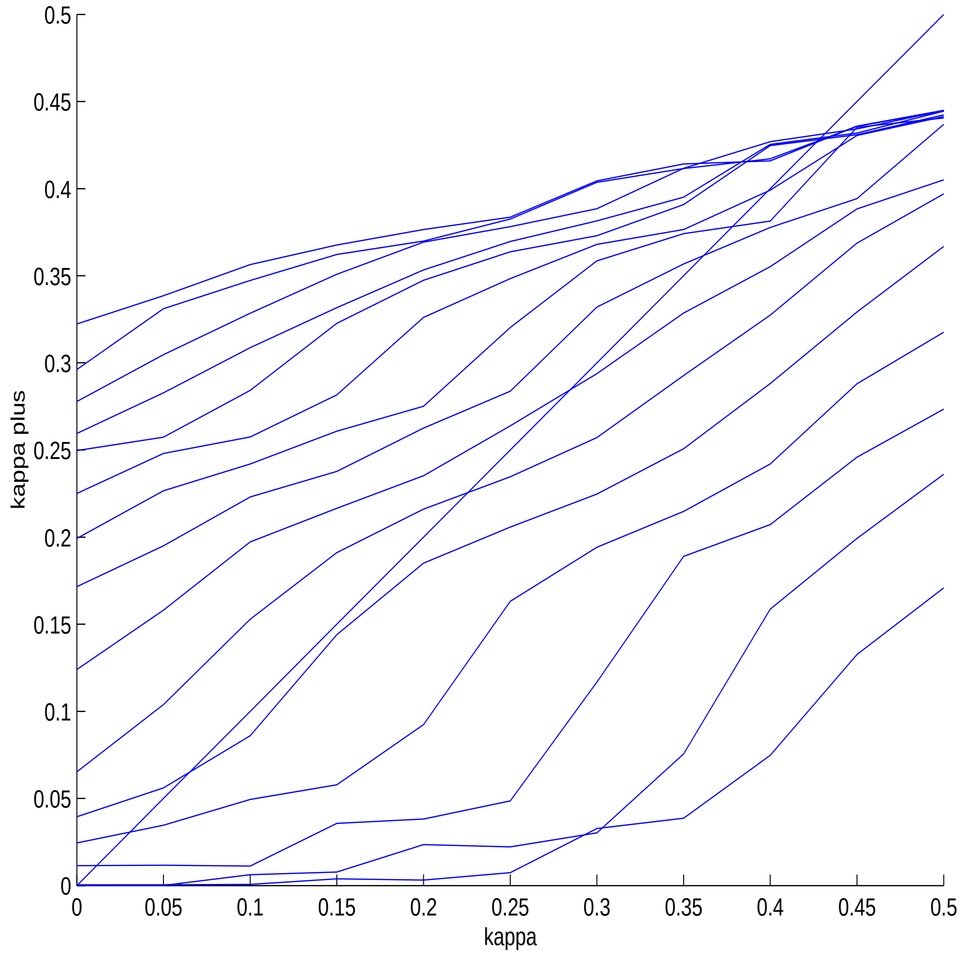

Figure 4. $\boldsymbol{f} : \boldsymbol{\kappa} \longrightarrow \boldsymbol{\kappa}^{+}$. Each line shows the mapping from κ into κ^{+} for a single economy from Section IV.A. For each economy, the mapping is plotted for $\kappa \in \{0, 0.05, 0.10, ..., 0.5\}$. The intersection of the mapping with the 45° line is the equilibrium fraction of high quality claims for that particular economy. The bottom mapping is for the economy with the lowest productivity, where $E[r_2] = 12\%$, the one above it is for the $E[r_2] = 14\%$ economy, and so on.

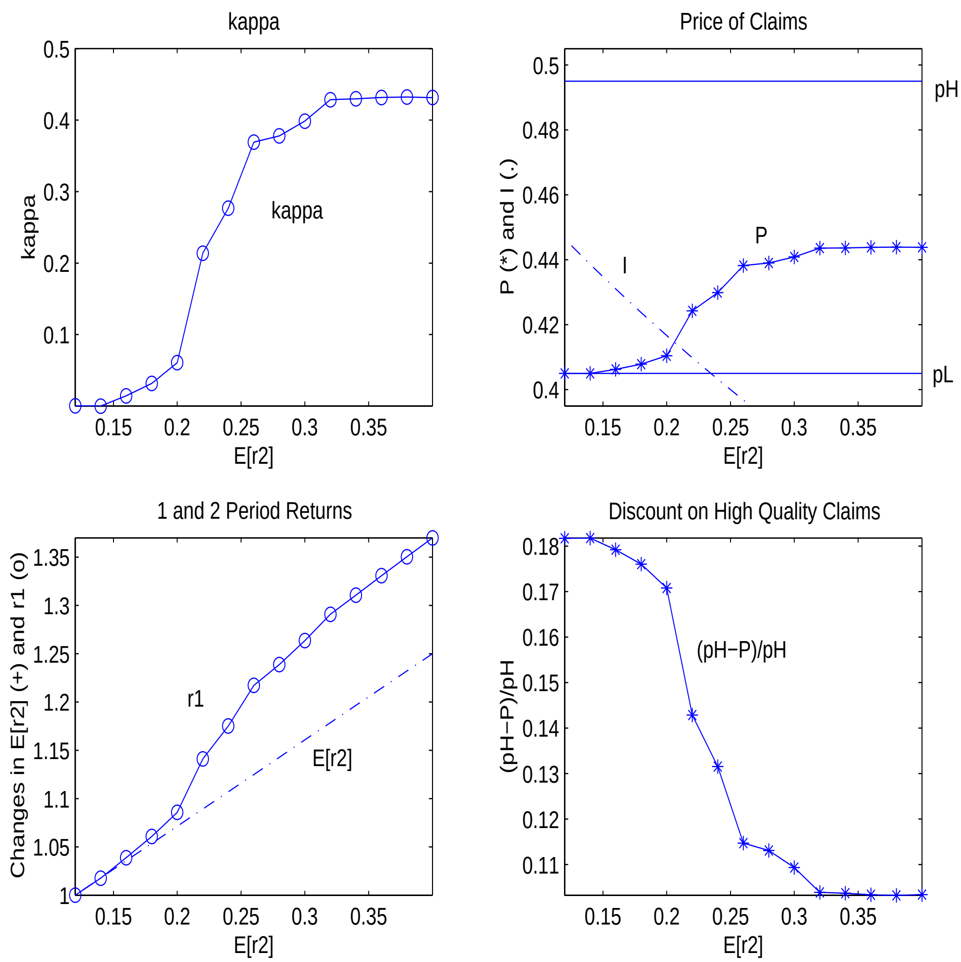

Figure 5. Effects of changing the initial investment cost I on liquidity.

Top Left Panel: Fraction of Claims Which are High Quality, κ, as a Function of the Expected Two Period Return, $E[r_2]$.

Top Right Panel: Claims Price P $(*)$ and Initial Investment Cost I $(\cdot)$ as a Function of the Expected Two Period Return, $E[r_2]$. True values of claims p_H and p_L are plotted as horizontal lines at 0.405 and 0.495, respectively.

Bottom Left Panel: Tradeoff Between Maturity and Expected Return as a Function of the Expected Two Period Return, $E[r_2]$: Endogenous One Period Return r_1 and Exogenous Two Period Expected Return Normalized by their $E[r_2] = 12\%$ values.

Bottom Right Panel: Cost of Transferring Future into Current Income as a Function of the Expected Two Period Return $E[r_2]$: Equilibrium Discount from True Value at which Claims to High Quality Projects Sell, $\frac{p_H - P}{p_H}$, as a Function of the Expected Two Period Return $E[r_2]$.

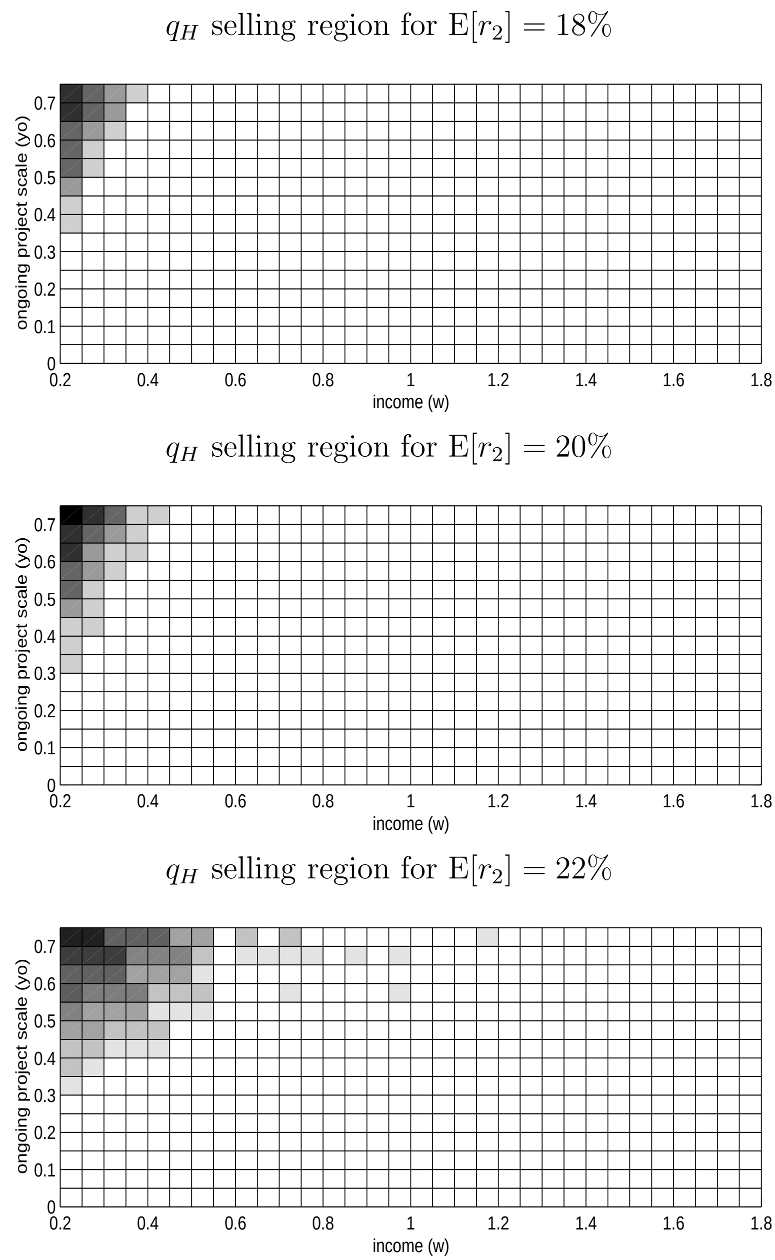

Figure 6. Selling regions increase as productivity increases. Claims sold y_s as a function of income w and ongoing project scale y_o for investors with the high quality signal. White is 0, black is .75 and gray cells darken in increments of 0.05 claims. The expected two period return $E[r_2]$ is 18% (top), 20% (center), 22% (bottom). For clarity I do not graph incomes greater than 1.8 where upper bounds on the state space may be binding. Investors with incomes over 1.8 contribute less to high quality claims volume as productivity increases and constitute less than 5% of high quality claims sold in all cases.

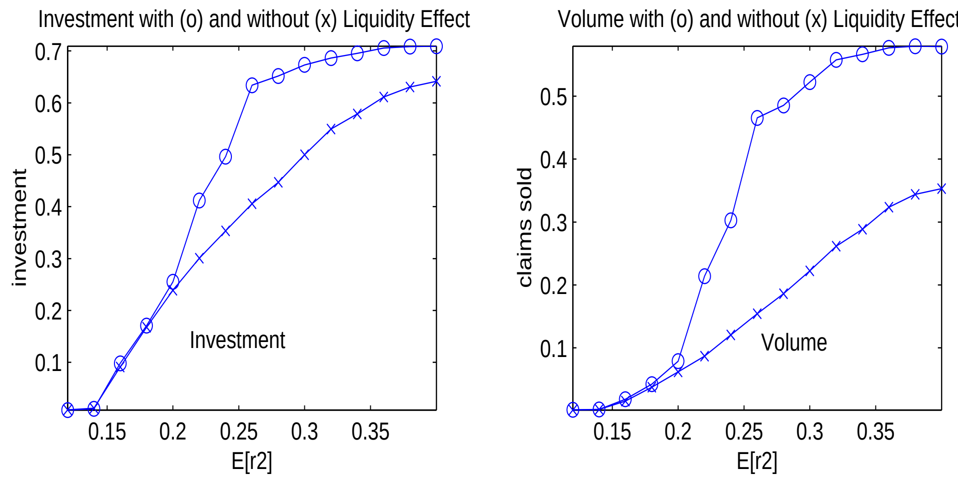

Figure 7. Liquidity magnifies the effects of changing the initial investment cost, I, on investment and volume.

Top Left Panel: Investment with (o) and without (x) Liquidity Effects. Liquidity is held constant by holding the claims price P constant at the level of the lowest productivity ($E[r_2] = 12\%$) economy.

Top Right Panel: Volume (Size of Claims Market) with (o) and without (x) Liquidity Effects. Liquidity is held constant by holding the claims price P constant at the level of the lowest productivity ($E[r_2] = 12\%$) economy.

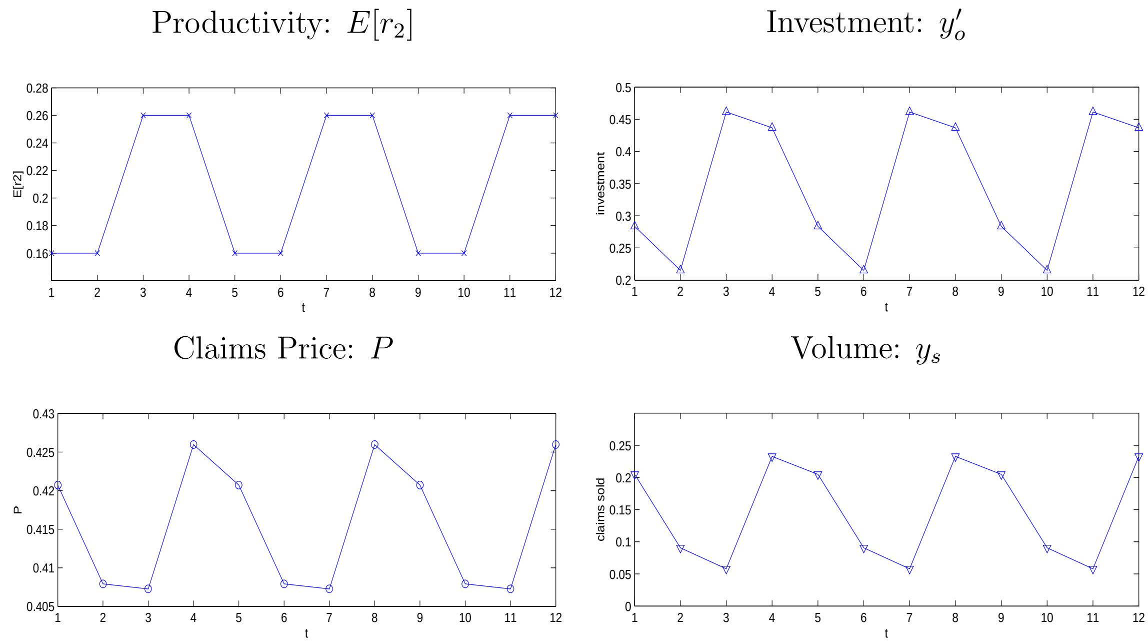

Figure 8. Aggregate fluctuations: Productivity, claims price, investment and volume over the cycle.

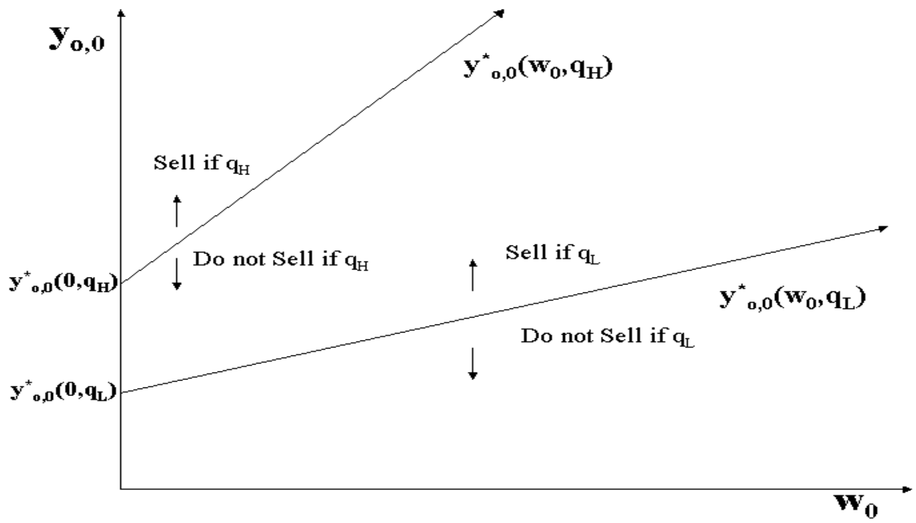

Figure A1. Selling regions: Finite horizon problem.