THE BREADTH OF CURRENCY CRISES

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Abstract. This appendix to the paper provides an explanation for potential asymptotic dependence of foreign exchange returns across currencies.

Appendix A. Data description and discussion (see paper)

Appendix B. Extreme currency linkages - classification and theoretical explanation

B.1. **Introduction.** The linkages in terms of returns and volatility between forex markets during periods of crisis is one of two types. The classification derives from the two ways in which multivariate random variables can behave far from the center. Random variables are either asymptotically independent or dependent, regardless their correlation. If asymptotically independent, the dependency when present, eventually dies out completely as the rate of depreciation increases. For example, if exchange rate returns are multivariate normal, the returns would be asymptotically independent. This does not meet the empirical evidence regarding the interdependencies between different foreign exchange markets. The empirical literature on foreign exchange markets frequently speaks of 'increased correlation' during periods of turbulence, suggesting forex markets are asymptotically dependent instead, see e.g. Bodart and Reding (1999), for this market speak. Moreover, it is a stylized fact that the marginal distributions exhibit fatter tails than the normal distribution. In Forbes and Chinn (2003) a linear factor model is used to describe the global linkages empirically. The monetary model of foreign exchange rate determination explains the

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exchange rate returns from the changes in the linear differences of domestic and foreign variables. We show that if the changes in the benchmark country fundamentals have a marginally fat tailed distribution, then the forex returns of the different countries are necessarily asymptotically dependent. This implies that linked currency crises have a much higher likelihood than if these fundamentals were driven by a normal distribution.

B.1.1. *Forex Model.* Consider the standard Monetary model of the logarithm of the foreign exchange rate derived from the quantity equation

$$\begin{aligned} s_{ij} &= (m_i - m_j) - \phi(y_i - y_j) + \lambda(R_i - R_j) \\ &= g_i - g_j \end{aligned}$$

say, where g is the composite of the logarithmic money measure m, the negative of the income elasticity times log real income $-\phi y$ and the semi interest rate elasticity times the interest rate R. In first differences the model reads

$$\Delta s_{ij} = \Delta g_i - \Delta g_j. \tag{B.1}$$

Regarding this specification we like to note two things which are important for the discussion that follows. The first point to note is that this model is linear in the first differences of the composite fundamental g and hence the individual fundamentals as well. Second, given multiple exchange rates quoted against the i-th country currency, it follows that all forex returns Δs_{ij}, $j = 1, ..., n$ given $n + 1$ different currencies, are partly driven by the same set of fundamentals Δg_i of the $i-$benchmark country. Thus all j-exchange rates have a similar exposure to the $i-$th currency fundamentals. For example, Aghion, Bachetta, Banerjee (2001) present a graph with the ratio of dollar denominated liabilities to claims with respect to foreign banks in 1997 right before the start of the Asian crisis. The ten highest exposures are by Thailand, Indonesia, Russia, Korea, Malaysia, South Africa, Philippines, Columbia, Mexico and Brazil respectively. Given the high content of dollar denominated debt, all these countries were therefore highly exposed to the same US interest rates moves. The linear specification conforms the linear factor model used in Forbes and Chinn (2003) who find that trade linkages are important transmitters of shocks between countries. Note that the monetary model captures the mirror image of the trade account through movements in the capital account.

B.2. **Measures of dependency.**

B.2.1. *The correlation measure.* A standard measure of dependency is the coefficient of correlation ρ. This measure is intimately connected with the normal distribution. One can ask how well ρ captures the dependency, if it is not known whether the data are normal or not. Moreover, Boyer, Gibson, and Loretan (1997) have noted that even if the normal model applies, verifying the market speak of increased correlation is beset with difficulties. The empirical literature such as Bodart and Reding (1999) and Engle, Ito and Lin (1990) finds little support for normality in stress situations. One of the problems associated with the concept of correlation is that the data may be dependent, while the correlation coefficient is zero. Consider e.g. the discrete uniform distribution on the 8 points $(\pm 1, \pm 1)$, $(\pm 2, \pm 2)$. Due to the symmetry it is immediate that $\rho = 0$, though the data are not independent. If $x = -1$, y cannot be equal to 2, and $P\{Y > 1|X > 1\} = 1/2$, while unconditionally $P\{Y > 1\} = 1/4$ only. Thus ρ does not capture the dependency that is in the data. Lastly, as economists we are not so much interested in the correlation measure itself, rather we have an interest in the trade-offs between risk measured as a probability and the gains or losses, which are the quantiles of the distributions. As such the correlation is only an intermediate step in the calculation of this trade-off between quantile and probability. Why not directly compute this trade-off?

B.2.2. *Crash probabilities.* The correlation measure is not an end in itself. Presumably it is computed because it suggests how one market is interrelated with another market. In economics one is interested in the losses and gains and the real consumption that an investment buys. Thus what worries supervisors and industry representatives is that a heavy loss in one market goes hand in hand with a heavy loss in another market, destroying the real value of a diversified investment portfolio. More specifically, one asks given that $Y > t$, what is the probability that $X > s$, where t, s are high loss levels. Since we are interested in the extreme linkage probabilities, we will try to directly evaluate the linkage probabilities, bypassing the correlation concept.

If two random variables X, Y are not independent, having some information about one variable, say X, implies that one has some information about the other variable Y as well. This can be readily expressed as a conditional probability. We adopt the conditional crash probability measure of section 2, which conditions on the fact that one market has crashed without indicating which market this is.[1] Let κ denote the number of markets that crash, and write the expected number of market crashes as $E\{\kappa|\kappa \geq 1\}$. From elementary probability theory we have that

$$E\{\kappa|\kappa \geq 1\} =$$

$$1\frac{P\{X > t, Y \leq t\} + P\{X \leq t, Y > t\}}{1 - P\{X \leq t, Y \leq t\}} + 2\frac{P\{X > t, Y > t\}}{1 - P\{X \leq t, Y \leq t\}} =$$

$$\frac{P\{X > t\} + P\{Y > t\}}{1 - P\{X \leq t, Y \leq t\}}.$$

The conditional expectation measure $E\{\kappa|\kappa \geq 1\}$ has the advantage that it can easily be extended beyond the bivariate setting (see also section 2).

To develop some intuition for the measure

$$E\{\kappa|\kappa \geq 1\} = \frac{P\{X > t\} + P\{Y > t\}}{1 - P\{X \leq t, Y \leq t\}}, \tag{B.2}$$

we calculate the measure for two extreme cases.

Case 1. *If X and Y are i.i.d. and writing $p = P\{X > t\}$, then*

$$E\{\kappa|\kappa \geq 1\} = \frac{2p}{1 - (1-p)^2} = \frac{2}{2-p}.$$

In the limit as $t \to \infty$, $p \to 0$ and hence $E\{\kappa|\kappa \geq 1\} \to 1$.

Case 2. *If $X = Y$ and writing $p = P\{X > t\}$, then*

$$E\{\kappa|\kappa \geq 1\} = \frac{2p}{1 - (1-p)} = 2.$$

Clearly, even as $p \to 0$, still $E\{\kappa|\kappa \geq 1\} = 2$.

These two cases show that $1 \leq E\{\kappa|\kappa \geq 1\} \leq 2$. In case the data are independent, then in the limit $E\{\kappa|\kappa \geq 1\}$ reaches its lower bound, while if the data are dependent, $E\{\kappa|\kappa \geq 1\}$ can at most be equal to the upper bound 2. In the first case the X and Y are asymptotically independent, while in the latter case they are asymptotically dependent. Also note that even though in the first case the data are independent, the dependency measure $E\{\kappa|\kappa \geq 1\}$ is higher than 1 at all finite levels

[1]For presentational purposes of this appendix, as in Hartmann, Straetmans and de Vries (2003), we fix quantiles and leave the exceedance probabilities open. We also take the two quantiles on which we condition equal to t, but this is by no means necessary.

of p since even when independent, there is a positive probability that 'two markets will crash, given that one market crashes'.

B.2.3. *The question.* We are after the conditional expectation $E\{\kappa|\kappa \geq 1\}$ for t large. Theoretically speaking two possibilities for dependent data arise. In the limit, either $E\{\kappa|\kappa \geq 1\} = 1$, or $1 < E\{\kappa|\kappa \geq 1\} \leq 2$. The empirical literature has concluded that distributions which exhibit asymptotic independence do not reflect the nature of the typical dependency that one sees in asset data. Therefore empirical research has started modelling the asymptotic dependency structure. It is, however, an open question as to why the return data do exhibit asymptotic dependence. This is the topic of the next section.

B.3. **Asymptotic dependency.** In this section we provide an economic explanation for the observed limiting dependency in the forex data. To do this, we capitalize on the linear economic structure of the forex model (B.1) laid out above. This linear linkage structure is similar to the linkage between financial institutions, see e.g. Allen and Gale (2000). This linkage is often just a linear relation between the returns on the projects financed by the different financial institutions. The linear linkage is due to the structure of e.g. the interbank deposit market or the syndicated loans market.

Assume that each composite fundamental g is independent and has a marginal distribution which either exhibits light or heavy tails. It is more or less a stylized fact that asset returns are heavy tailed. We show that this necessarily leads to asymptotic dependence. Conversely, we also show that if the fundamentals exhibit light tails, such as in the case of the normal distribution, then the forex returns are asymptotically independent.

For the results it is sufficient to consider a three currency system with composite fundamentals $\Delta g_0 = X$, $\Delta g_1 = -Y$, and $\Delta g_2 = -S$. Thus $\Delta s_{01} = X + Y$ and $\Delta s_{02} = X + S$. Suppose that X, Y, and S are independent and identically distributed. It is immediate that the two exchange rates Δs_{01} and Δs_{02} are dependent, since (with bounded second moments) the correlation coefficient is 1/2.

B.3.1. *Fundamentals with light tails.* In this subsection we consider a specific light tailed distribution the bivariate the normal distribution. Suppose the three fundamentals X , Y and S are i.i.d. standard normally distributed. We will repeatedly use the following asymptotic approximation: as $t \to \infty$

$$\Pr\{\theta X > t\} \sim \frac{1}{\sqrt{2\pi}} \frac{\theta}{t} \exp\left(-\frac{1}{2}(\frac{t}{\theta})^2\right).$$

To show asymptotic independence we use Sibuya's (1960) approach. Note that

$$\begin{aligned} E\left\{\kappa|\kappa\geq1\right\} &= \frac{\Pr\{\Delta s_{01}>t\}+P\{\Delta s_{02}>t\}}{1-\Pr\{\Delta s_{01}\leq t,\Delta s_{02}\leq t\}} \\ &= \frac{1}{1-\frac{\Pr\{\Delta s_{01}>t,\Delta s_{02}>t\}}{\Pr\{\Delta s_{01}>t\}+P\{\Delta s_{02}>t\}}}. \end{aligned}$$

To indicate convergence in distribution, we will use the symbol $\Longrightarrow$. Evidently

$$\begin{aligned} \Pr\{\Delta s_{01} &> t\}=\Pr\{X+Y>t\} \\ &\Longrightarrow \Pr\{\sqrt{2}X>t\} \\ &\approx \frac{2}{\sqrt{\pi}}\frac{1}{t}e^{-t^2/4} \end{aligned}$$

by the above asymptotic approximation. We bound the probability $\Pr\{\Delta s_{01}>t,\Delta s_{02}>t\}$ in the numerator

$$\begin{aligned} \Pr\{\Delta s_{01} &> t,\Delta s_{02}>t\}\leq\Pr\{\Delta s_{01}+\Delta s_{02}>2t\} \\ &= \Pr\{2X+Y+S>2t\} \\ &\Longrightarrow \Pr\{\sqrt{6}X>2t\}=\Pr\{X>\frac{2}{\sqrt{6}}t\} \\ &\approx \sqrt{\frac{3}{\pi}}\frac{1}{2t}e^{-t^2/3} \end{aligned}$$

Thus under normality

$$\frac{\Pr\{\Delta s_{01}>t,\Delta s_{02}>t\}}{\Pr\{\Delta s_{01}>t\}+P\{\Delta s_{02}>t\}}\leq\frac{\sqrt{3}}{8}\exp\left(-\frac{t^2}{3}+\frac{t^2}{4}\right)\to0\text{ as }t\to\infty.$$

Hence

$$\lim_{t\to\infty}E\left\{\kappa|\kappa\geq1\right\}=1$$

It follows that the two forex returns Δs_{01} and Δs_{02} are asymptotically independent, even thought the correlation coefficient is as much as

$$\rho=1/2.$$

B.3.2. *Fundamentals with heavy tails.* Before we can give the positive result we need a precise definition of what we mean by heavy tails. In particular we will assume that the distribution $F(x)$ far into the tails has as a first order term identical to the Pareto distribution, i.e.

$$F(x)=1-x^{-\alpha}L(x)\quad\text{as }x\to\infty, \tag{B.3}$$

where $L(x)$ is a slowly varying function (see below). In other words the distribution varies regularly at infinity, i.e.

$$\text{(B.4)} \qquad \lim_{t\to\infty} \frac{1-F(tx)}{1-F(t)} = x^{-\alpha}, \quad \alpha > 0,\ x > 0.$$

and

$$\lim_{t\to\infty} \frac{L(tx)}{L(t)} = 1, \quad x > 0.$$

Regular variation implies that the first order term in (B.3) has the Pareto form. Generally, the tail index α corresponds to the number of bounded moments. And because not all moments are bounded, we speak of heavy tails. Distributions like the Student-t, F-distribution, Burr distribution, sum-stable distributions with unbounded variance all fall into this class. One shows that the unconditional distribution of the ARCH and GARCH processes also belongs to this class, see De Haan et al. (1989). Note that the Student distribution is often used in the empirical modelling of the unconditional return of exchange rates, see e.g. Boothe and Glassmann (1987), while ARCH process are popular conditional models, see Baillie and McMahon (1989).

We need the following famous convolution Lemma from Feller, (1971, VIII.8).

Theorem 1. *Let the X_i be independent identically distributed random variables with regularly varying symmetric tails, i.e. as $x \to \infty$*

$$\Pr\{X_i \le -x\} = \Pr\{X_i > x\} = x^{-\alpha}L(x).$$

Then as $s \to \infty$

$$\Pr\{\sum_{i=1}^{n} X_i \le s\} = 1 - ns^{-\alpha}L(s).$$

In three dimensions note that this theorem implies by the independency of the X_i that

$$\begin{aligned}\Pr\{\sum_{i=1}^{3} X_i &\le s\} = 1 - \sum_{i=1}^{3}\Pr\{X_i > s\} \\ &= \Pr\{X_1 \le s, X_2 \le s, X_3 \le s\}\end{aligned}$$

In other words, the probability on the area below the plane $\sum_{i=1}^{3} X_i = s$ equals the probability on the lower bar. The first step is the Lemma 1. The second step is a consequence of the independence which implies that the joint probabilities like $\Pr\{X_1 > x, X_2 > x\} = \Pr\{X_1 >$

$x\}\Pr\{X_2 > x\}$ are of smaller order than the marginal $\Pr\{X_2 > x\}$. Thus for large quantiles s all mass concentrates along the axes, so that hyperplanes and bars which cut the three axes at the same points separate the same probability mass. A-two dimensional graphical explanation is added at the end of this appendix. This implies the following:

Claim 1. *Let X , Y and S be independent identically distributed random variables with regularly varying tails , i.e. as $x \to \infty$*

$$\Pr\{X \leq -x\} = \Pr\{Y \leq -x\} = \Pr\{S \leq -x\} = x^{-\alpha}L(x),$$

$$\Pr\{X > x\} = \Pr\{Y > x\} = \Pr\{S > x\} = x^{-\alpha}L(x).$$

Then for

$$\begin{aligned} &\lim_{s\to\infty} E\{\kappa|\kappa \geq 1\} = \\ = &\lim_{s\to\infty} \frac{\Pr\{\Delta s_{01} > s\} + P\{\Delta s_{02} > s\}}{1 - \Pr\{\Delta s_{01} \leq s, \Delta s_{02} \leq s\}} \\ = &\lim_{s\to\infty} \frac{\Pr\{X + Y > s\} + \Pr\{X + S > s\}}{1 - \Pr\{X + Y \leq s, X + S \leq s\}} \\ = &\lim_{s\to\infty} \frac{2s^{-\alpha}L(s) + 2s^{-\alpha}L(s)}{3s^{-\alpha}L(s)} = \frac{4}{3}. \end{aligned}$$

Proof. The reason that

$$\begin{aligned} 1 - \Pr\{X + Y &\leq s, X + S \leq s\} \\ &= 3s^{-\alpha}L(s) \end{aligned}$$

is that the lines $X + Y = s$ and $X + S = s$ are two of the three edges of the triangular plane $\sum_{i=1}^{3} X_i = s$ in the positive quadrant. Since for large s all mass is along the three axes

$$\begin{aligned} 1 - \Pr\{X + Y &\leq s, \ X + S \leq s\} \\ &= 1 - \Pr\{X \leq s, Y \leq s, S \leq s\} \\ &= 1 - 3s^{-\alpha}L(s). \end{aligned}$$

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Note that in this case the two exchange rates returns Δs_{01} and Δs_{02} are asymptotically dependent, since $\lim_{s\to\infty} E\{\kappa|\kappa \geq 1\} = 4/3 > 1$.

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B.4. **Intuition in two dimensions for the proof of claim 1.** For large s all mass is concentrated along the two axes. That is why the probability to be above the line in the first figure is equal to the probability to be outside the lower square in the second figure.

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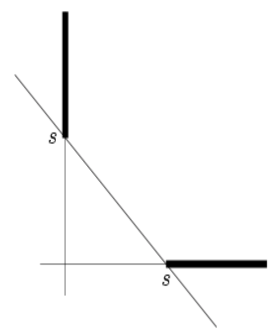
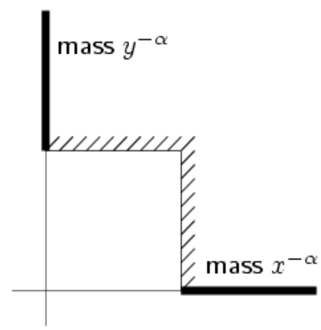
Figure 17: Convolution 2



$$\Pr[X + Y \leq s] = \Pr[X \leq s, Y \leq s] \sim 1 - 2s^{-\alpha}$$

FIGURE 1

Figure 16: Convolution 1



$\Pr[X \leq x, Y \leq y] = 1 - x^{-\alpha} - y^{-\alpha}$

FIGURE 2