The Costs of Credit Boons

Guido Lorenzoni*

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Abstract

This paper presents a simple model of a credit expansion driven by an expected increase in the productivity of the entrepreneurial sector. We study how the presence of financial constraints affects the welfare properties of the equilibrium. In particular we show under what circumstances firms' financing decisions make the economy overly sensitive to aggregate shocks. We show that this type of excessive fragility can be associated with overborrowing and overinvestment.

We characterize the equilibrium and the constrained efficient allocations in terms of a private and a social coefficient of risk aversion. When the private coefficient of risk aversion is lower than the social coefficient firms undervalue the costs of volatility and excess volatility arises.

We discuss some policy implications of our framework. A contractionary monetary policy during the boom is a blunt instrument to attack the inefficiency described as it reduces investment by reducing both sources of inside and outside finance. Capital requirements, by affecting the composition of finance, can achieve the same level of financial stability with a smaller sacrifice in terms of investment ex ante.

Over the last two decades both industrialized and emerging economies have experienced "credit booms", that is periods of high investment associated to an expansion in credit, high asset prices and fast growth. In some cases these booms have been followed by a bust associated with low investment,

*Princeton University, Department of Economics and Woodrow Wilson School of Public and International Affairs.

bankruptcies, a credit contraction and a contraction in output.[1] An extensive literature has showed how aggregate shocks can be propagated and amplified by the presence of credit market frictions. In this paper we use a simple model with credit market frictions to analyze the welfare properties of a credit boom. The main objective of the paper is to show in what circumstances a boom can display excessive fragility, i.e. an equilibrium financial structure that is excessively sensitive to aggregate shocks, and to show that excessive fragility can be associated with over-borrowing and over-investment.

We define broadly a "credit boom" as a period of high investment that is financed by recourse to outside finance. We study economies where a credit boom is driven by high expected productivity but there is uncertainty regarding productivity in the short run. The high level of expected productivity warrants high levels of investment. However, the entrepreneurs who undertake the investment have limited wealth so investment needs to be financed mostly with outside funds. With some probability there is a negative aggregate productivity shock. In this case entrepreneurial wealth shrinks and investment has to be cut back. Entrepreneurs can invest more during the boom phase by increasing their leverage, however this jeopardizes their financial position if the negative shock hits. Therefore, the crucial trade-off arising during a credit boom is between investment *ex ante* and financial stability *ex post*.

In this paper we study a simple model in which this trade off is present and we show that the presence of credit market frictions generates a wedge between the private and social benefits of financial stability. As a consequence there can be excessive volatility and overborrowing. The model shows that during the early phase of the credit cycle entrepreneurs under-estimate the social damage coming from a negative shock to their net worth. The inefficiency arises solely from the presence of the borrowing constraint and it is not due to irrational or speculative pricing of financial assets nor to the lack of state-contingent clauses in financial contracts. *Ex post* the economy is faced with a debt-overhang problem. Even though entrepreneurs correctly forecast the probability of the debt-overhang and have access to contingent contracts they tend to use contingent contracts less than optimally and leave their balance sheets over-exposed to the aggregate shock.

[1] The main stylized facts on credit booms and on boom-bust cycles are described in Gourinchas et al. (2001), Borio and Lowe (2001), Bordo and Jeanne (2002) and Tornell and Westermann (2002).

In models with collateral constraints little attention has been devoted to the costs associated to the boom side of a credit cycle. From a first best perspective models of credit constraints always display under-borrowing. It is almost tautological that in a model with borrowing constraints, agents borrow less than they would in a frictionless world. Policy makers may be concerned with credit crunches but it is not clear why they should be concerned with credit booms that boost the net worth of firms, reduce the outside-finance premium and increase investment in high return projects. In a monetary economy an investment boom may generate an expansion in aggregate demand, but as long as monetary policy adjusts the interest rate to its natural level, and keeps inflationary pressures under control, there seem to be no additional reason to increase interest rates to contain the credit boom.

This paper shows that if one moves to a second best perspective the costs of credit booms can be analyzed in a standard fashion. In particular, we compare the competitive equilibrium with a constrained efficient allocation that is subject to the same financial constraints present in the competitive economy. The financial constraints limit both the level of investment and financial stability that can be achieved in equilibrium. A firm can buy insurance against a bad productivity shock by reducing the contingent payments it has to make if that shock materializes. This protects the firm net worth and allows the firm to invest more in bad states of the world but reduces total borrowing and thus reduces investment *ex ante*. We study a simple pecuniary externality associated with the stabilization of entrepreneurial net worth. An increase in entrepreneurial net worth in bad states of the world has a positive effect on input prices (wages) that firms do not take into account. Essentially, workers would be willing to pay the entrepreneurs *ex ante* to increase their net worth in bad states. A higher net worth in the bad states increases the demand for labor and, in equilibrium, increases the wage rate. This increases the amount of resources that the entrepreneurs can commit to pay to the rest of the economy. Competitive financial contracts do not take into account these pecuniary externality and thus can generate excessive borrowing *ex ante* and excessive volatility *ex post*.

We use this framework to analyze in what circumstances excess volatility arise and to compare the effect of different policies. In particular, we show that the costs of the boom depend on the uncertainty regarding future productivity growth and on the relative importance of temporary and permanent productivity shocks. In terms of policy, we show that policies oriented at reducing fragility are more effective than policies oriented at reducing the level of investment during the boom. In particular a contractionary monetary policy during the boom is a blunt instrument. The problem with monetary policy is that it reduces borrowing by increasing interest rates and by reducing profits. The substitution effect associated to the interest rate increase goes in the right direction, as it induces firms to reduce borrowing, however the wealth effects associated to the same interest rate increase and to the reduction in profits have detrimental effects as they reduce firms' net-worth and make firms more reliant on outside funds in all states of the world. Regulatory capital requirements instead attack the problem of excess fragility at its roots and tend to reduce excess fragility while having a smaller effect on investment. Therefore a tightening of capital requirements during a credit boom may be more effective at reducing the social costs of financial instability.

This paper is related to the large literature on the effects of financial frictions on macroeconomic volatility.[2] The existing papers have compared the volatility arising in a model with financial constraint with a first best benchmark in which no financial constraints are present. The contribution of this paper is to conduct a second best analysis of a model with financial frictions and to study the welfare costs associated to the "boom side" of the credit cycle. Most of the literature imposes restrictions on the liabilities used by the entrepreneurs to finance their projects, and rules out state contingent liabilities that depend on *aggregate shocks*. However, the corporate finance arguments that have been used to explain the use of rigid liabilities (debt) cannot be invoked to justify this type of rigidity, given that economy-wide aggregate shocks are usually outside the control of the single manager or entrepreneur and are easily observable it is not . In this paper we show that even in presence of state contingent liabilities collateral constraints tend to make investment and output fluctuations larger and more persistent, both compared to a frictionless first best and to a second best benchmark. Moreover in equilibrium financial contracts tend to be *too rigid* with respect to aggregate shocks. Adding a cost of writing state contingent clauses in financial contracts our model could be used to show that in equilibrium private contracts may fail to include clauses contingent on the aggregate shock that would have first-order effects on *ex ante* welfare.

[2]Bernanke and Gertler (2001), Kiotaki and Moore (1997), Holmstrom and Tirole (1996), Aghion, Banerjee and Bachetta (2000), .

This paper is also related to the large and growing literature on the optimal policy response to an expansion in private credit and to asset price booms associated with it. Most of the recent literature has focused on the use of monetary policy. Bernanke and Gertler (2001a, 2001b) have used a macro model with financial frictions to study wether monetary policy should respond to non-fundamental asset prices movements. They have shown the potential disruptive effects of an accommodating monetary policy that leads to an amplification in aggregate fluctuations because of a feed-back effect between net worth, investment and aggregate demand. The authors argue that once the central bank has chosen an aggressive anti-inflationary policy that mutes this amplification effect, it should pay no further attention to asset price movements.

Cecchetti et al. (2000), Blanchard (2001) and Dupor (2002) have argued on the contrary, that when an asset price boom is associated to overinvestment then monetary policy has a dual objective: to maintain price stability *and* to correct the overinvestment problem. Therefore, optimal monetary policy during an asset price boom may need to be more contractionary than what would be required by a simple inflation targeting rule.

This conclusion however rests on two assumptions: first that the overinvestment is driven by asset prices that reflect irrational optimism, secondly that central bank holds correct expectations and can easily identify an asset price boom that is not driven by fundamentals. These two assumptions are clearly subject to debate, and the policy conclusions clearly depend on the stance taken on the rationality of asset prices and on the ability of central banks to spot irrational fads.

A recent paper by Bordo and Jeanne (2003) attacks the problem from a different perspective. Asset price movements, wether rational or irrational, can be disruptive if some agents in the economy hold highly leveraged positions. Our paper takes a similar perspective. As we show in this paper, in a world of imperfect capital markets even in the absence of irrational mispricing it is possible to have overborrowing associated to excessive fluctuations in investment and output. In this framework overborrowing is simply driven by the presence of a pecuniary externality and the effects of different policies can be studied in a relatively standard framework.

Bordo and Jeanne take as given the objective function of the central bank, and assume that a certain degree of output stabilization is desirable from a social point of view. In the present paper we define a social welfare objective starting from agents preferences, derive the social preference for stabilization and show in what circumstances the social and private margins that determine financial stability differ. Another difference between our approach and Bordo and Jeanne's is that they use a reduced form keynesian monetary model, while we use a real model. By using a monetary model they focus on the volatility of investment for its effects on aggregate demand, while by using a real model we focus on the volatility of investment for the impact it has on capital accumulation in the entrepreneurial sector. Notice that in a microfounded neo-keynesian macro model the volatility of investment can have effects on output volatility *only if* the monetary authority decides to accommodate the investment shock. A policy of inflation targeting that keeps output near its natural level would eliminate any inefficient output volatility driven by investment volatility. When we discuss monetary policy we think of our model as an economy in which the central bank is already successfully targeting the natural level of output, where by "natural level" we mean the level of output that would arise in an economy with fully flexible prices. Therefore, all our monetary policy recommendations will be in terms of deviations from this ideal inflation targeting benchmark. A final difference between our approach and Bordo and Jeanne is that our model considers a broader array of policy tools and shows that given the nature of the inefficiency, monetary policy appears a blunt instrument compared with regulatory requirements that directly attack the overborrowing problem.

From a methodological standpoint this paper is related to the literature on limited enforcement of credit contracts and on the welfare properties of competitive equilibrium with limited enforcement. Kehoe and Levine (1993) have shown that in this type of economies a first welfare theorem holds when there is one good, and have shown that in economies with more than one good the equilibrium can be constrained inefficient. The model presented here differs from theirs because it includes investment and assumes a weaker enforcement technology, however we obtain similar results: when the technology is linear our economy is essentially a one good economy and the equilibrium is constrained efficient, when the technology is not linear the equilibrium is not in general efficient. In the latter case factor prices are endogenous and these prices affect the financial constraints, the inefficiency arises because optimal private contracts fail to take into account this effect (pecuniary externality). We discuss this connection more in detail in section 2. Caballero and Krishnamurty (2001, 2002) have used a similar notion of constrained efficiency in a "dual model of liquidity" to study overinvestment in a small open economy. Even though the environments we study are quite different we share their emphasis on general equilibrium effects and on pecuniary externalities. The mechanism at work in this paper is different, as we do not have a dual model of financial frictions. In their model financial frictions make transactions within some group of agents (entrepreneurs) less costly than with outside investors and the pecuniary externality arise in the transactions between agents in the "local" group. The main structural difference between our model and theirs is that they concentrate on the effects that equilibrium prices have on the reallocation of wealth among entrepreneurs, while here we concentrate on the effect they have in reallocating wealth between entrepreneurs and outside investors.

1 A simple model with debt constraints

Consider an economy lasting three periods. There is one good that can be used for consumption or investment, and there are two groups of agents: consumers/workers and entrepreneurs. There is a large number of agents of each type, we normalize the population of each type to 1.

Consumers are risk neutral with preferences represented by the utility function

$$\sum_{t=0}^{2} c_t$$

where c_t is consumption. Consumers are endowed with 1 unit of leisure in each period which they supply inelastically on the labor market. We will assume that parameters are such that in equilibrium c_t for is positive in all periods, so that the gross real interest rate in this economy is given and equal to 1. In periods 0 and 1 we assume that workers are employed in the traditional sector which employs only labor and has a linear technology, so the wage rate is given and equal to $\overline{w}$ in these two periods.

Entrepreneurs have preferences represented by the utility function

$$\sum_{t=0}^{2} c_t^E$$

and they have an initial wealth of n_0 date zero. Entrepreneurs have access to the following technology: by investing k_1 at date 0 they obtain $a_1 k_1$ units of capital at date 1, where a_1 is stochastic; by investing k_2 at date 1 they produce consumption goods at time 2 according to the constant returns to scale production function $f(k_2, l_2)$. Assume that f satisfies standard Inada conditions. We also assume full depreciation. Given that the price of capital is constant and equal to one we can as well assume that the production functions just described include the undepreciated part of the capital stock. With these assumptions on technology we interpret the three periods in the following way. Period 0 is the period in which a new technology is available to entrepreneurs with limited wealth and they start to invest in it. Period 1 is an intermediate period in which the capital invested can appreciate or depreciate according to the shock a_1. The value of the assets invested is $a_1 k_1$, so entrepreneurs face a shocks to the asset side of their balance sheet. Given the state of their balance sheet entrepreneurs determine their level of investment for period 2. Finally, period 2 represents the medium/long run in which the new technology is fully developed.

The level of productivity of the entrepreneurial technology in period 1, a_1, is the only source of uncertainty in the economy, and all uncertainty is resolved in period 1. The productivity a_1 is the same for all entrepreneurs, so there is only aggregate uncertainty. There is a discrete set S of states of the world. At date 1 the state of the world $s \in S$ is realized with probability π_s and productivity takes the value a_{1s}. We assume that

$$E[a_1] > 1$$

All variables dated $t = 1, 2$, with the exception of k_1, will be random variables that are function of a_{1s}, even though we will often omit the subscript s. Accordingly, all expressions involving these variables should be intended "for all s".

Goods and factor markets are competitive at all dates. The real wage rate is denoted by w_t. Entrepreneurs profits at date 2 can be written as $r_2 k_2$ where r_2 is the shadow rental rate on capital r_2 defined by

$$r_2 \equiv \max_l \{f(1, l) - w_2 l\}$$

The wage rate, and hence the rate of return r_2, are taken as given by the individual entrepreneur. This will give us a linear problem for the entrepreneurs' optimal financial contract.

1.1 Enforcement technology

Entrepreneurs at date 0 issue financial contracts, a financial contract specifies a payment B from consumers to entrepreneurs at date 0 and state contingent payments P_{1s} and P_{2s} in periods 1 and 2. We assume that financial contracts are subject to limited enforcement. Let θ be a fixed parameter that satisfies $0 < \theta < 1$ and $\theta < a_{1s}$ in all states. At dates 1 and 2 the entrepreneur can default on his financial obligations. Let the value of the firm at time t be v_t. Upon default the maximum repayment that outsiders can enforce is θk_t and the entrepreneur escapes with the residual $v_t - \theta k_t$[3]. If the entrepreneur defaults at date 1 he can use the residual wealth to start anew and issue new financial contracts due in period 2. This weak form of punishment is assumed for simplicity, the substance of the results is not changed by making the punishment on a defaulting entrepreneur at date 1 stronger.

The presence of imperfect enforcement limits the amount of resources that entrepreneurs can credibly commit to deliver to outside financiers at date 1 and 2. This makes the investment levels at date 1 and 2 positively related to entrepreneurial wealth. There are a number of alternative models of financing constraints that can generate a limited financial commitment in this sense and a positive relation between entrepreneurial wealth and investment[4]. A model with limited enforcement has an advantage in terms of simplicity given that the only constraints on the equilibrium allocation take the form of participation constraints.

It is useful to characterize the financial contracts in terms of the present value of the entrepreneur financial liabilities at dates 1 and 2. Given that we will focus on equilibria with zero interest rate we can define:

$$\begin{aligned} d_{1s} &= \left(P_{1s} + P_{2s}\right)/k_1 \\ d_{2s} &= P_{2s}/k_2 \end{aligned} \tag{1}$$

with this notation the participation constraint, or no default constraint, for

[3] Given that there is full depreciation the value of the firm is equal to the current profits and is equal to $a_{1s}k_1$ in period 1 and to $r_{2s}k_{2s}$ in period 2.

[4] For example Bernanke and Gertler (1986) derive such a relationship in a model with costly state verification, Holmstrom and Tirole (1996) derive it from a moral hazard problem, and Kiyotaki and Moore (1997) derive it in an environment with incomplete contracting and renegotiation.

the entrepreneur can also be written as

$$\begin{aligned} d_{1s} &\leq \theta \\ d_{2s} &\leq \theta \end{aligned}$$

We can interpret these as "collateral constraints", the value of an entrepreneur outstanding liabilities cannot exceed at any point in time a given fraction of the capital stock. All promises of payment made by entrepreneurs must be backed by this form of collateral. A similar notion of financial contracts limited by collateral is used in Geanakoplos (1996) and in Lustig (2001).

1.2 Optimal financial contracts

We will focus our attention on competitive equilibria in which $c_t > 0$ for all t, so that the interest rate is given and equal to 1. In this case the problem determining the optimal financial contract for an entrepreneur is:

$$\max_{\{c_t^E\},\{k_t\},\{B,P_1,P_2\}} \quad E[\sum_{t=0}^{2} c_t^E] \tag{P}$$

$$\text{s.t.} \quad c_0^E + k_1 \leq n_0 + B \tag{2}$$

$$c_1^E + k_2 \leq a_1 k_1 - P_1 \tag{3}$$

$$c_2^E \leq r_2 k_2 - P_2 \tag{4}$$

$$c_1^E + c_2^E \geq z\left(a_1 k_1 - \theta k_1\right) \tag{5}$$

$$c_2^E \geq \left(r_2 - \theta\right) k_2 \tag{6}$$

$$B \leq E\left[P_1 + P_2\right] \tag{7}$$

$$c_t^E \geq 0 \tag{8}$$

$$r_2, w_2 \text{ given} \tag{9}$$

Constraints (2) to (4) are standard budget constraints. Constraints (5) and

(6) are the participation constraint for an entrepreneur and require that at all dates the entrepreneur continuation utility is larger than the utility he would achieve after default. The variable z represents the rate of return on entrepreneurial wealth after default and it will be described in more detail. Notice that we did not introduce participation constraints for the consumers. It is possible to introduce default by consumers and assume that wages cannot

be used as collateral. In that case one would add the constraints:

$$\begin{aligned} P_1 + P_2 &\geq 0 \\ P_2 &\geq 0 \end{aligned}$$

to ensure that consumers do not default financial contracts can never involve a positive transfer to entrepreneurs in net present value. The analysis in Holmstrom and Tirole (1998) centers on the effect of this type of constraints. They show that public liquidity can be introduced to relax these constraints. In the following analysis we abstract from these constraints by assuming that consumers can fully commit to financial contracts. In the examples we will describe these constraints are not binding, that is entrepreneurs liabilities are always positive.

To complete the description of the problem (2) we need to specify the rate of return on entrepreneurial wealth, z. Consider an entrepreneur with a given wealth level n_1 at date 1. The collateral constraint imposes an upper bound on the amount of capital that he can invest:

$$k_2 \leq n_1 + \theta k_2$$

if $r_2 > 1$ this constraint is binding and entrepreneurs' utility is $\frac{r_2-\theta}{1-\theta} n_1$, if $r_2 \leq 1$ the constraint is not binding and the utility is simply n_1. Therefore we can define

$$z = \max \left\{ \frac{r_2 - \theta}{1 - \theta}, 1 \right\} \tag{10}$$

as the rate of return on entrepreneurial wealth at date 1. An advantage of considering a weak punishment for entrepreneurs is that z is the rate of return on entrepreneurial wealth for both defaulting and non-defaulting agents.

A competitive equilibrium is defined by a financial contract $\{B, P_1, P_2\}$ and factor prices $\{r_2, w_2\}$ such that the financial contract solves (2) and goods and financial markets clear. We will focus our attention on competitive equilibria in which $w_2 > \overline{w}$ so that all workers are employed in the entrepreneurial sector in period 2. Given that the traditional technology is linear this assumption ensures that the level of investment in the entrepreneurial sector has a positive effect on equilibrium wages. In this case a competitive equilibrium is given by a financial contract that is optimal for the entrepreneur and factor prices that satisfy

$$\begin{aligned} r_2 &= f_K(k_2, 1) \\ w_2 &= f_L(k_2, 1) \end{aligned}$$

To characterize the optimal financial contract we can define entrepreneurial wealth at date 1 as

$$n_1 = a_1 k_1 - P_1 - P_2$$

the quantities $a_1 k_1$ and $P_1 + P_2$ represent the assets and liabilities of the entrepreneur at date 1. Using the definition of total liabilities at date 1 (1) we can rewrite the entrepreneur problem at date 0 in terms of a constrained maximization of total surplus:

$$\begin{aligned}
\max_{k_1,\{d\}} \quad & E\left[a_1 k_1 - k_1 + f\left(k_2, 1\right) - k_2\right] && (11) \\
s.t. \quad & k_1 \leq n_0 + E\left[d_1\right] k_1 && (12) \\
& k_2 \leq \theta k_2 + \left(a_1 - d_1\right) k_1 && (13) \\
& d_1 \leq \theta
\end{aligned}$$

By choosing the state contingent liabilities $\{d_{1s}\}$ an entrepreneur affects the level of investment at date 0, through (12), and affects the distribution of investment at date 1 across states of the world at date 1, through (13). Entrepreneurs would like to increase their wealth levels and their investment in states of nature where the return z is high. To increase wealth in those states entrepreneurs have to reduce d_1, this reduces investment at date 0, k_1, and therefore it reduces the wealth level in all other states.

It is useful to define the expected gross rate of return on entrepreneurial wealth at date 0 which is equal to

$$z_0 = E\left[z \frac{a_1 - d_1}{1 - E\left[d_1\right]}\right]$$

notice that $z_0 - 1$ corresponds to the Lagrange multiplier on the financing constraint (12) in problem (11) and $z_0 - 1 > 0$ reflects the presence of an outside finance premium. With this notation the optimal financial contract is characterized by the following inequalities:

$$\begin{aligned}
z_s &\leq z_0 && (14) \\
d_{1s} &\leq \theta
\end{aligned}$$

with at least one strict equality in each state s. Notice that these conditions require $z_s \leq z_0$ in all states of the world. If in any state $z_s > z_0$ then the optimum financial contract would involve $k_1 = 0$, and this is incompatible with equilibrium, so we can restrict our attention to the case $z_s \leq z_0$.

To interpret condition (14) consider the following marginal choice between two financial strategies. An entrepreneur has a dollar of inside funds n_0 to allocate, he can use it to increase investment and buy capital at date 0. Investment in physical capital can be leveraged so by investing 1 dollar of inside funds at date 0 he can invest $\frac{1}{1-E[d]}$ units of capital and earn the random return $a_1 - d_1$. Reinvesting this return at date 1 at the rate of return z gives a total return of z_0. Alternatively he can use the dollar to reduce his financial liabilities in state s, this is identical to buy an Arrow security that pays $1/\pi_s$ in state s. In this way with probability π_s he will have an extra dollar and invest it at the rate of return z_s. The expected return of this strategy is z_s.

If the return on investment is higher than the return on reducing debt in state s then the entrepreneur finds it optimal to use up all his borrowing capacity and the constraint $d_{1s} \leq \theta$ is binding. If instead $z_s = z_0$ then the entrepreneur is indifferent between investment at date 0 and investment in state s and the constraint can be slack. In these states the entrepreneur is willing to leave some unused borrowing capacity in order to protect his net worth in states in which the contingent return is high.

The rate of return z governs the incentives to save entrepreneurial net worth. Entrepreneurs are willing to reduce investment at date 0 to buy securities that pay off in states where the rate of return z is high. The distribution of z induces a hedging motive for entrepreneurs. Given that $E[a_1] > 1$ if z is constant across states then it is always the case that

$$z_0 = z \frac{E[a_1 - d_1]}{1 - E[d_1]} > z$$

and entrepreneurs want to borrow up to the maximum at date 0. If instead z is volatile and it is negatively correlated with a_1 then entrepreneurs may prefer to save net worth in high z states, and to sacrifice the return on capital from period 0 to period 1 in order to have extra funds in these states. In general equilibrium the volatility of z is determined by the volatility of entrepreneurial net worth. We will see that in equilibrium z_s is high in states of the world where n_{1s} is low, so we will interpret the hedging behavior of entrepreneurs in terms of net worth stabilization.

In this simple framework we assume that entrepreneurs use state contingent claims to insure their net worth against aggregate shocks. In a more realistic framework the type of contracts that can be used to stabilize the wealth of entrepreneurs can involve a variety of financial contracts. A simple way of obtaining some state contingency is a combination of equity financing and non state contingent debt. If the insider issues shares against a fraction S of the firm value and D in non state contingent debt we would have the following expression for the total liabilities of the insider:

$$P_1 + P_2 = S\left(a_1 k_1 - D\right) + D$$

Other possible ways to stabilize the net worth of the insider are the purchase of financial assets with countercyclical payoffs and the use of committed lines of credit from financial institutions. In this paper we abstract from the specific instruments used to stabilize the wealth of insiders and we allow for fully state contingent debt.

1.3 Equilibrium

The features of a competitive equilibrium depend on the parameters of the model, and in particular on entrepreneurs' initial net worth n_0 and on the fraction of collateralizable assets θ. When n_0 is large enough the competitive equilibrium corresponds to the first best allocation and financial constraints are not binding. When n_0 is smaller than some cutoff level $\overline{n}$ the entrepreneur is financially constrained at date 0. In period 1 the entrepreneur financial constraint may be binding or not depending on the realization of the aggregate shock a_1. The next proposition gives a characterization of the equilibria at date 1.

Proposition 1 *A competitive equilibrium exists, is unique, and is characterized by two cutoff levels a', a'' with $a' \leq a''$, such that:*

1. *If $a_{1s} \geq a''$ then $z_s = 1$;*
2. *If $a' \leq a_{1s} < a''$ then $z_s > 1$, z_s is increasing in a_{1s} and $d_s = \theta$ (borrowing capacity is exhausted);*
3. *If $a_{1s} < a'$ then $z_s = z_0$ and $d_s = \theta - (a' - a_{1s}) < \theta$ (borrowing capacity is slack).*

The characterization above is driven by the equilibrium relation between the level of entrepreneurial net worth and the equilibrium rate of return z. When the value of assets at date 1 is high entrepreneurial net worth is large.

Entrepreneurs can finance the first best level of investment at date 1 and the rate of return z_s is equal to the gross interest rate. The outside finance premium $z_s - 1$ is zero. If productivity is in an intermediate range entrepreneurs are credit constrained at date 1, entrepreneurial capital is scarcer and has higher marginal productivity, and the return z is higher. When productivity and profits at date 1 are lower investment in the entrepreneurial sector is reduced. As entrepreneurial capital is scarcer the rate of return r_2 increases and so does z. However as long as z is smaller than z_0 entrepreneurs borrow up to the maximum and save no funds. When productivity falls below the level a' entrepreneurial capital become so scarce and z so large that entrepreneurs prefer to borrow less than the maximum amount in this states.

Everyone in this economy is risk-neutral, however the concavity of f and the presence of collateral constraints induce a motive for insuring the net-worth of entrepreneurs. In the absence of financial frictions investment would be independent of entrepreneurial net-worth and the return to entrepreneurial net worth would be equal to 1 for any net worth level. When financial frictions are present, though, investment depends on entrepreneurial net worth and the rate return on capital is negatively related to the total net worth of the entrepreneurial sector. This generates a motive for financial stabilization. The economy faces a trade-off between financial stability *ex post* and investment *ex ante*. Given the limited collateral available in good states, in order to raise additional funds at date 0 entrepreneurs must promise part of the collateral available in bad states. By doing so, they reduce their net worth in bad states and this pushes up the return z_s.

The degree of financial stabilization obtained in a competitive equilibrium depends on the slope of the relation between z and entrepreneurial net worth. We can write this relation as $z = h(n_1)$ and think of it as a function that gives us the marginal return of entrepreneurial wealth[5]. The form of the function h depends only on the shape of the production function f. We can capture the effective "risk aversion" of the entrepreneurial sector, i.e. the desire for net worth stabilization in the economy using the coefficient of absolute risk

[5] The function h is given by

$$h(n) = \max \left\{ \frac{f_K\left(\frac{n}{1-\theta}, 1\right) - \theta}{1-\theta}, 1 \right\}.$$

aversion

$$\sigma(n) = \left| \frac{h'(n)}{h(n)} \right|$$

With this definition it is easy to compare economies with different production functions and obtain the following result.

Proposition 2 *Consider two economies that are identical except for the production function which is f in the first economy and $\hat{f}$ in the second. Let d and $\hat{d}$ be the equilibrium financial contracts. If the first economy has a higher degree of risk aversion, —i.e. $\hat{\sigma}(n) > \sigma(n)$ for all n— then*

$$\begin{aligned} \hat{d}_s &\leq d_s \text{ for all } s \\ \hat{k}_1 &< k_1 \\ Var\left(\frac{\hat{k}_2}{\hat{k}_1}\right) &< Var\left(\frac{k_2}{k_1}\right). \end{aligned}$$

A steeper function h induces a more cautious behavior on the part of entrepreneurs, and results in a greater degree of financial stability and smaller investment *ex ante*. The risk aversion captured by the function h determines the equilibrium level of investment and financial stability.

2 Efficiency, excess volatility and overinvestment

We turn now to the efficiency properties of a competitive equilibrium. As a first step, let us briefly characterize the allocation arising in an economy with no collateral constraints. This will be our first best benchmark. Since $E[a_1] > 1$ all the endowment available at date 0 will be invested so

$$k_1^* = \overline{w} + n_0.$$

In period 1 r_2 and k_2 are constant across states and satisfy:

$$\begin{aligned} r_{2s} &= r_2^* = 1 \\ k_{2s} &= k_2^* \\ f_K(k_2^*, 1) &= 1 \end{aligned}$$

Investment at date 1 and output at date 2 are independent of the productivity shock at time 1, given that this shock carries no information on future productivity. This means that the dependence of k_2 on the productivity shock in our economy is only due to the presence of financial frictions. The following proposition summarizes the comparison between the first best and the competitive economy.

Proposition 3 *There is $\hat{n}$ such that if $n_0 \geq \hat{n}$ the competitive equilibrium coincides with the first best allocation. If $n_0 < \hat{n}$ we have*

$$\begin{aligned} k_1 &< k_1^* \\ k_{2s} &\leq k_2^* \\ Var\left(\frac{k_2}{k_1}\right) &> Var\left(\frac{k_2^*}{k_1^*}\right) = 0. \end{aligned}$$

Proposition 3 makes it clear that from a first best point of view the model can only display under-investment and under-borrowing. If the financial constraint is binding the amount of outside funds that can be raised by entrepreneurs is limited and the rate of return on investment, z_0, is higher than the equilibrium interest rate, 1, and larger than the interest rate that would arise in the frictionless economy, $E[a_1]$. That is, there is a non-negative outside finance premium. If the financial constraint is not binding the rate of return on investment is equal to the interest rate $E[a_1]$ the outside-finance premium is zero and the economy achieves the first best level of investment. In the first best economy there is no trade off between investment and financial stability. Since entrepreneurs have unlimited access to outside funds the economy can achieve maximum investment at date 0 and maximum stability at date 1.

Not surprisingly, in presence of financial constraints borrowing at date 0 is lower than at a first best point of view the model can only display under-borrowing and under-investment. If the financial constraint is binding the amount of outside funds that can be raised by entrepreneurs is limited and the rate of return on investment, z_0, is higher than the equilibrium interest rate, 1, and larger than the interest rate that would arise in the frictionless economy, $E[a_1]$. That is there is a non-negative outside finance premium. If the financial constraint is not binding the rate of return on investment is equal to the interest rate $E[a_1]$ the outside-finance premium is zero and the economy achieves the first best level of investment. In the first best economy there is no trade off between investment and financial stability. Since entrepreneurs have unlimited access to outside funds the economy can achieve maximum investment at date 0 and maximum stability at date 1.

Let us now turn to second best analysis. We consider a social planner that intervenes by setting the financial contract $\{B, P_1, P_2\}$ at date 0. The rest of the allocation is determined by competitive markets as in the previous section. In particular the planner does not intervene in the financial markets at date 1, in the factor markets at date 2 and cannot affect the punishment for defaulting agents. With these assumptions the no-default constraints are the same for the planner as for the competitive economy. The point of this exercise is to understand the inefficiencies associated with the choice of the financial contracts at date 0 taking as given the contractual frictions in the economy. The assumption that the planner does not intervene in the financial markets at date 1 can be relaxed if we assume that the planner cannot monitor the investment of the firm, that is if we assume that the planner cannot observe wether funds at date 1 are invested in physical capital or in risk free bonds.

At date 1 entrepreneurs have net worth equal to

$$n_1 = a_1 k_1 - (P_1 + P_2)$$

Optimal investment by the entrepreneurs at date 1 and equilibrium in the factor market at date 2 imply that capital will be determined by

$$k_2 = \min\left\{k_2^*, \frac{1}{1-\theta} n_1\right\}$$

and the entrepreneurs equilibrium utility in period 1 will be equal to

$$c_1^E + c_2^E = z n_1$$

where z is defined in (10). The no default condition at date 1 can be written as

$$P_1 + P_2 \leq \theta k_1.$$

Finally, consumers expected utility is given by

$$E\left[\overline{w} - B + (\overline{w} - P_1) + (w_2 - P_2)\right]$$

Therefore, the social planner problem can be written as:

$$\begin{aligned}
\max_{c_0,k_1\{B,P_1,P_2\}} \quad & c_0 + E\left[\frac{r_2-\theta}{1-\theta}n_1\right] & (P') \\
s.t. \quad & c_0 + k_1 \leq n_0 + B & \\
& n_1 = a_1 k_1 - P_1 - P_2 & \\
& B \leq E[P_1+P_2] + \left(2\overline{w} + E\left[w_2\right] - U^C\right) & (15) \\
& r_2 = f_K\left(k_2, 1\right) & (16) \\
& w_2 = f_L(k_2, 1) & (17)
\end{aligned}$$

where (15) is the participation constraint for the consumers and (16) and (17) are the equilibrium factor prices. Again, we are implicitly assuming that at an efficient allocation is characterized by $w_2 > \overline{w}$. A financial contract $\{B, P_1, P_2\}$ that solves P' and the corresponding equilibrium allocation are defined "constrained efficient".

The crucial difference between problems P and P' is the fact that factor prices are taken as given in the former and are endogenous in the latter. If one modifies problem P' by substituting conditions (16) and (17) with given r_2 and w_2 one can define allocations that are, using Kehoe and Levine (1993) terminology, "conditional constrained efficient". Using that definition it is possible to prove a first welfare theorem for our economy and show that the allocation corresponding to the competitive equilibrium in section 1.3 is conditional constrained efficient. This means that, conditional on given factor prices, the financial contract chosen by the entrepreneurs in a competitive setting is socially optimal. Conditional constrained efficiency does not corresponds to constrained efficiency because factor prices in general do depend on the financial contract chosen by entrepreneurs. The fundamental reason why factor prices have welfare consequences is because they appear in the participation constraints for entrepreneurs, that is changes in factor prices can relax or tighten the financial constraints in the economy and in this way change the set of feasible allocations.[6] To see how this effect works observe that the no default constraints (5) and (6) can be rewritten in terms

[6] Kehoe and Levine (1993) contain a general discussion of the notions of constrained efficiency and conditional constrained efficiency in the presence of participation constraints and endogenous prices.

of workers' consumption as:

$$\begin{aligned} c_1 + c_2 &\leq 1 + w_2 + \theta k_1 \\ c_2 &\leq w_2 + \theta k_2 \end{aligned}$$

this means that the maximum amount that outside investors can collect from entrepreneurs consists of the collateral θk_t *and* of the wages. In private financial contracts entrepreneurs commit directly to make payments that are backed by the collateral θk_t. However, on top of that entrepreneurs have an indirect way of committing to make future transfers. If they increase the demand for labor at date 2 by accumulating more capital they indirectly commit to pay higher wages. The planner takes into account this second effect and that explains the difference between the planner problem P' and the individual problem P.

The social planner problem can be restated in terms of the total liabilities

$$d_1 = (P_1 + P_2) / k_1$$

If we focus on financial efficient allocations that display $c_t > 0$ for all t then problem P' can be restated in terms of a constrained maximization of total surplus analogous to (11)

$$\max_{k_1, \{d\}} \quad E\left[a_1 k_1 - k_1 + f(k_2, 1) - k_2\right] \tag{18}$$

$$\begin{aligned} s.t. \quad & k_1 \leq n_0 + E[d_1] k_1 + \left(2\overline{w} + E[w_2(k_2)] - U^C\right) \qquad (19) \\ & k_2 \leq \theta k_2 + (a_1 - d_1) k_1 \\ & k_2 \leq k_2^* \\ & d_1 \leq \theta \end{aligned}$$

A comparison of (11) and (18) shows that the term $\left(2\overline{w} + E[w_2(k_2)] - U^C\right)$ in (19) looks like an additional source of finance for the entrepreneur. In particular, if the function w_2 is concave a reduction in the volatility of k_2 has the effect relaxing the constraint (19). This is the reason why the social planner may value financial stability more than the private economy. Now we turn to the characterization of the constrained efficient allocation and describe the social planner problem in terms of a trade-off between investment and financial stability analogous to the one faced by the private economy. Let $\tilde{d}_1$ be entrepreneurs liabilities in a constrained efficient contract and $\tilde{n}_1$ be the corresponding net worth. Then the following proposition characterizes the constrained efficient contract.

Proposition 4 *Suppose a financial efficient contract has $c_t > 0$ at all t and $w_{2s} > \overline{w}$ for all s. Then there is a constant $\tilde{z}_0$ and a random variable $\tilde{z}$ such that:*

$$\begin{aligned} \tilde{d}_{1s} &\leq \theta \\ \tilde{z}_s &\leq \tilde{z}_0 \end{aligned} \qquad (20)$$

with at least an equality in each state. The $\tilde{z}_0$ and $\tilde{z}$ satisfy

$$\begin{array}{ll} \tilde{z}_s = z_s + \frac{\tilde{z}_0 - 1}{1-\theta}\frac{\partial w_{2s}}{\partial k_{2s}} & \text{if } \tilde{n}_{1s} < (1-\theta)\, k_2^* \\ \tilde{z}_s \in \left[z_s, z_s + \frac{\tilde{z}_0 - 1}{1-\theta}\frac{\partial w_{2s}}{\partial k_{2s}}\right] & \text{if } \tilde{n}_{1s} = (1-\theta)\, k_2^* \\ \tilde{z}_s = z_s & \text{if } \tilde{n}_{1s} > (1-\theta)\, k_2^* \end{array} \qquad (21)$$

where $\frac{\partial w_{2s}}{\partial k_{2s}} = f_{LK}(k_{2s}, 1)$.

Notice the similarity between the first order conditions (20) and the first order conditions (14) characterizing the equilibrium financial contract. The expression in (21) can be interpreted similarly to the expression (10): it gives the social rate of return on entrepreneurial wealth at date 1. The difference between z and $\tilde{z}$ is that the first only captures the private return on an extra dollar of net worth n_1, while the second captures the additional effect that an extra dollar of net worth has on wages. When entrepreneurs are constrained at date 1 an extra dollar of net worth increases investment by $\frac{1}{1-\theta}$, thus it increases labor demand at date 2 and it increases wages by $\frac{1}{1-\theta}\frac{\partial w_{2s}}{\partial k_{2s}}$. An increase in entrepreneurial wealth generates a pecuniary transfer from entrepreneurs to workers, if this pecuniary transfer was incorporated in private contracts it would increase the resources available to entrepreneurs at date 0 and reduce their resources at date 2. Since entrepreneurs marginal value of wealth at date 0 is $\tilde{z}_0$ while their marginal value of wealth at date 2 is 1 the total welfare effect of this transfer corresponds to $(\tilde{z}_0 - 1)\frac{1}{1-\theta}\frac{\partial w_{2s}}{\partial k_{2s}}$. In a frictionless economy an agent marginal value of wealth is constant across dates and across states and pecuniary externalities have zero welfare effects. In this model if n_0 is large enough and the economy achieves the first best allocation then the term $(\tilde{z}_0 - 1)$ would be zero and the externality would disappear.

In order to discuss excess fragility and overborrowing let us introduce a function $\tilde{h}$ analogous to the function h introduced in section 1.3, that associates to the net worth n_1 the corresponding level of the total return $\tilde{z}$[7]. As the slope of h captures the risk aversion of the private sector, the slope of $\tilde{h}$ captures the social degree of risk aversion. The possibility of overborrowing and excess volatility is related to the relative slopes of these two functions. In particular when $\tilde{h}$ displays a higher absolute risk aversion than h the entrepreneurs underestimate the cost of a negative shock to net worth.[8]

We can now compare the competitive allocation with the constrained efficient frontier. Let $\{d_{1s}\}$ be entrepreneurs liabilities in a competitive equilibrium. Take any constrained efficient allocation that weakly dominates the competitive allocation. Let $\left\{\tilde{d}_{1s}\right\}$ be the value of entrepreneur liabilities in this allocation.

Proposition 5 *Suppose $\tilde{h}$ is more risk averse than h. Suppose $c_0 > 0$ in the constrained efficient allocation. Then the following inequalities hold*

$$\begin{aligned} d_{1s} &\geq \tilde{d}_{1s} \\ Var\left(\frac{k_2}{k_1}\right) &\geq Var\left(\frac{\tilde{k}_2}{\tilde{k}_1}\right) \end{aligned}$$

Therefore, either the equilibrium is constrained efficient or the equilibrium displays excess volatility.

There is a crucial effect that tends to make $\tilde{h}$ more risk averse than h. In states in which entrepreneurial wealth n_{1s} is high the economy achieves the first best level of investment and $z_s = \tilde{z}_s = 1$. Therefore, for high levels of wealth the pecuniary externality captured by $\partial w_2 / \partial k_2$ is muted and the

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[7] The function $\tilde{h}$ is, more precisely, a correspondence and is defined by

$$\tilde{h}(n) = \begin{cases} \frac{f_K - \theta + (\tilde{z}_0 - 1) f_{KL}}{1-\theta} & \frac{n}{1-\theta} < k_2^* \\ \left[\frac{f_K - \theta}{1-\theta}, \frac{f_K - \theta + (\tilde{z}_0 - 1) f_{KL}}{1-\theta}\right] & \frac{n}{1-\theta} = k_2^* \\ 1 & \frac{n}{1-\theta} > k_2^* \end{cases}.$$

where $k = \frac{n}{1-\theta}$.

[8] Given that $\tilde{h}$ is non differentiable we will use the following definition: $\tilde{h}$ is more risk averse than h iff

$$\frac{\tilde{h}(n')}{\tilde{h}(n)} \geq \frac{h(n')}{h(n)}$$

for $n' < n$.

social and private return on entrepreneurial wealth are identical. For lower levels of wealth, however, the pecuniary effect is positive and $\tilde{z}_s > z_s$. This effect tends to make $\tilde{h}$ more risk averse than h. However, with a general production function the curvature of f can undo this effect for low values of n_2 and it is possible to construct examples in which there is under-borrowing and too little volatility in equilibrium.[9]

The proposition above gives a result of over-borrowing in terms of a borrowing ratio: in equilibrium the ratio of borrowed funds to total assets $E[d_1]$ is too large. The absolute level of borrowing $E[d_1]k_1$ and the level of investment k_1, however depends on the parameters of the economy and on the way in which the surplus is divided between consumers and entrepreneurs. That is, the presence of over-borrowing depends on the point on the constrained efficient frontier that is used to make the comparison. We now turn to an example to illustrate the results above and to show the possibility of overborrowing and overinvestment.

2.1 An example

We will now study an example in which the excess fragility result in proposition 5 applies and in which over-borrowing and over-investment arise.

Suppose the production function f takes the Cobb-Douglas form

$$f(k,l) = Ak^{\alpha}l^{1-\alpha} + (1-\delta)k$$

with $1-\delta \geq \theta$.

We want to compare the competitive equilibrium and the financial efficient allocation that delivers the same utility to entrepreneurs[10]. With this production function proposition (5) applies, moreover it is possible to show that with this production function the following additional inequality holds:

$$k_1 \geq \tilde{k}_1$$

[9] A sufficient condition for overborrowing is

$$\frac{f_{KK}}{f_K - \theta} \leq \frac{f_{LKK}}{f_{LK}}$$

This condition is satisfied in the case of a Cobb-Douglas production function, where $f = Ak^{\alpha}l^{1-\alpha} + (1-\delta)k$ and $1-\delta \geq \theta$.

[10] Given that the utility possibility frontier is hump-shaped for this economy, such an allocation may fail to exist. Therefore we consider competitive equilibria where U_E^{CE} is large enough that there is an efficient allocation with $U_E = U_E^{CE}$.

so there is both excess fragility and over-investment.

Let us consider a production function with parameters $\alpha = 0.3$, $\delta = 0.5$ and $\alpha A = \delta$. There are two shocks for a_1, $\{0.75, 1.5\}$ with probabilities $\{0.2, 0.8\}$. The parameter θ is equal to 0.5 and the initial wealth of entrepreneurs is $n_0 = 0.5$. Table I compares the equilibrium with the constrained efficient allocation that keeps the entrepreneurs indifferent.

	Competitive equilibrium	Financial efficient allocation
d_1	$\begin{cases} 0.50 \\ 0.50 \end{cases}$	$\begin{cases} 0.33 \\ 0.50 \end{cases}$
n_1	$\begin{cases} 0.25 \\ 1.00 \end{cases}$	$\begin{cases} 0.41 \\ 0.98 \end{cases}$
k_1	1.00	0.98
k_2	$\begin{cases} 0.50 \\ 1.00 \end{cases}$	$\begin{cases} 0.82 \\ 1.00 \end{cases}$
$E[d_1]\,k_1$	0.50	0.46
w_2	$\begin{cases} 0.94 \\ 1.17 \end{cases}$	$\begin{cases} 1.10 \\ 1.17 \end{cases}$

Table I

The economy faces an expected gross return on capital of $E[a_1] = 1.35$ in the first period. This induces entrepreneurs to maximize leverage and to choose a very rigid financial structure with d_1 constant and equal to θ. This corresponds to using exclusively (non-state contingent) debt. This rigid financial structure determines a high volatility of net worth, that in turns generate a high volatility of investment at date 0. If the good shock is realized the entrepreneurs are able to finance the first best level of investment $k_2^* = 1$. However, if the bad shock realizes, due to their high leverage entrepreneurs have to cut back investment of 50%. This determines a loss in wages of 0.24. Workers would be willing to pay 0.2×0.24ex ante to reduce wage volatility and this would more than compensate the entrepreneurs for the reduction in investment *ex ante*.

In the financial efficient allocation the volatility in capital and in wages is reduced, workers pay 0.025 to the entrepreneurs and in exchange entrepreneurs reduce investment at date 0 and reduce their financial obligations in the future periods. The optimal combination of financial stability and investment is achieved when wage volatility is reduced by 2/3. The wage gain for the workers is equal to 0.16 and the net gain for the workers is equal to $0.2 \times 0.16 - 0.025 = 0.007$. The *ex ante* gain is clearly small, which is not surprising given workers risk neutrality. However, the difference in volatility in the two allocations is striking. Moreover, it is useful to observe that, since workers are compensating entrepreneurs for the reduction in volatility, the cost in terms of reduced investment in period 0 is small. The biggest difference between the efficient allocation and the equilibrium, is in terms of financial structure. If we use the notation introduced at the end of section 1.2 we can see that in equilibrium entrepreneurs use 100% debt financing, while in the efficient allocation entrepreneurs uses 60% equity financing[11].

2.2 Volatility and temporary shocks

We can identify two properties of the economy that are essential in generating excess fragility. First, not surprisingly, when the volatility of a_1 is low enough the equilibrium is always constrained efficient. In the extreme case of non-random a_1 the following lemma establishes constrained efficiency.

Lemma 6 *If there is no uncertainty regarding a_1 then the competitive equilibrium is constrained efficient.*

When there is no uncertainty regarding a_1 the social return on investment at date 0 $\tilde{z}_0$ is equal to $\tilde{z}\frac{a_1-d}{1-d}$ and it is always the case that $\tilde{z}_0 > \tilde{z}$. In this case delaying investment is never beneficial neither from a private nor from a social point of view.

Second, it is crucial that shocks to entrepreneurs balance sheet are temporary. In our model the shock a_1 reduces entrepreneurial wealth but does not affect the long run productivity of the new technology (i.e. f is unaffected). It is easy to modify the model to allow for permanent shocks. One can modify the model above and allow for a multiplicative shock to the production function at time 2:

$$a_2 f(k_2, l_2)$$

[11] Solving for

$$\begin{aligned} d^H &= s a_1^H + (1-s) d \\ d^L &= s a_1^L + (1-s) d \end{aligned}$$

one finds $s = .22$ and $d = .20$. So the entrepreneur sells 22% of the total equity, raises debt for $.20 * k_1$ and the raises funds against equity for $E[sa_1] * k_1 = 0.30 * k_1$.

When a_1 and a_2 are positively correlated both the private and the social motive for financial stability are weaker. In presence of permanent shocks entrepreneurs net worth tends to be low precisely in those states in which expected productivity is low. The following lemma describes a case in which the correlation between a_1 and a_2 completely eliminates any incentive for financial stabilization, both at the private and at the social level.

Lemma 7 *Consider a Cobb-Douglas economy with a random a_2 that is perfectly correlated with a_1 according to:*

$$\ln(a_{2s}) = \kappa_0 + (1 - \alpha)\ln(a_{1s} - \theta)$$

then there is a competitive equilibrium with $d_s = \theta$ for all s and the equilibrium is constrained efficient.

These two result highlight the fact that an inefficient credit boom is more likely to arise when the balance sheet of entrepreneurs is subject to large shocks and when these shocks are weakly correlated with the long run productivity of the entrepreneurial projects. Notice that in a richer environment asset price movements may affect the entrepreneurs' balance sheet directly or indirectly, by affecting the balance sheet of banks that provide financial insurance to entrepreneurs. In this sense, the possibility of speculative movements in asset prices may be crucial in affecting the costs of a credit boom.

3 Some policy implications

The inefficiency identified in the previous section can be addressed with a number of possible policy instrument. Here, we will focus on the model implications for prudential regulation and for monetary policy.

3.1 Capital requirements

It is straightforward to interpret the results above in terms of capital requirements. Regulatory interventions that impose minimum capitalization on financial firms are widespread in industrialized economies, and often their introduction is justified based on the idea that competition in the financial sector may bring about excessive volatility at the macroeconomic level. The model presented gives a simple framework that rationalizes this idea.

Consider a capital requirement at date 0 of the type

$$\frac{n_0}{k_1} \geq \nu$$

this imposes a lower bound on the ratio of inside funds to total assets. The presence of this constraints effectively reduces the rate of return on investment at date 0, z_0, by increasing the shadow cost of outside funds. This tilts the trade-off in favor of financial stabilization, expands the set of states of the world in which entrepreneurs acquire net worth insurance and increases net worth in these states. The next proposition shows that a capital requirement can implement a constrained efficient allocation.

Proposition 8 *Suppose $\tilde{h}$ is more risk averse than h. Consider a constrained efficient allocation that dominates the competitive equilibrium. Let*

$$\begin{aligned} \tau &= \left(1 - E\tilde{d}_1\right)\tilde{k}_1 - n_0 \\ \nu &= \left(1 - E\tilde{d}_1\right) \end{aligned}$$

Then a capital requirement ν and a transfer τ to entrepreneurs at date 0 implement the constrained efficient allocation.

In practice capital requirements are imposed on a specific class of firms, namely on financial firms, especially on commercial banks. To have a fully fledged theory of capital requirements the model above should be integrated into a model of financial intermediation. If financial intermediaries specialize in the provision of contingent credit lines and other forms of net-worth insurance then the stabilization of their net-worth will be instrumental in providing net-worth insurance to the non-financial corporate sector. Moreover, if entrepreneurial firms which rely more on outside funding are more dependent on bank credit, capital requirements on banks can help indirectly to stabilize the balance sheet of these firms. The analysis of capital requirements in a explicit framework with intermediation remains a topic for future research.

The model can be used to study the relation between macroeconomic volatility and optimal capital requirements. Lemma 6 implies that for low levels of volatility the optimal level of capital requirements will be zero. It is more difficult to obtain general comparative statics result for positive levels of volatility. Figure 1 plots the optimal capital requirement ν for the economy in example 2.1 where we fix $Ea_1 = 1.35$ and we let $a_1^H - a_1^L = \Delta a$ vary. The dotted line represents the ratio n_0/k_1 in the competitive equilibrium. In this example when volatility increases the benefits of financial stability are larger. For low levels of Δa there is no need of capital requirements, while for $\Delta a > .55$ the optimal capital requirement is positive and increasing in Δa.

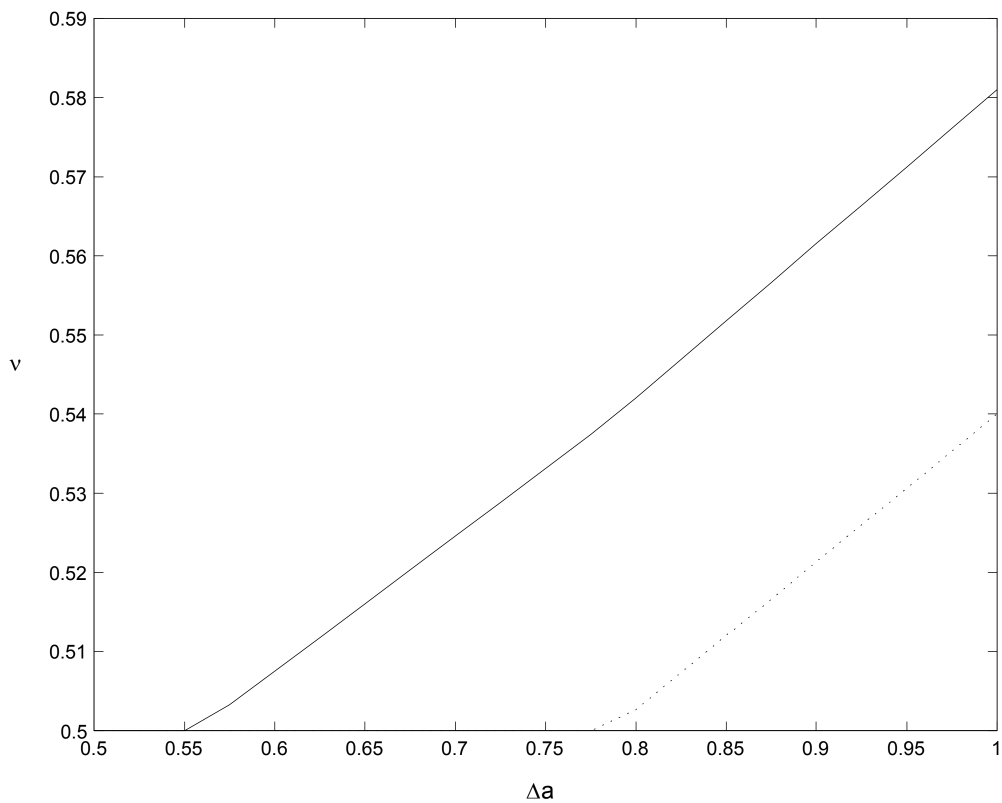

Figure 1. Volatility and capital requirements

A consequence of our approach is that if different capital requirements can be imposed on different sectors (or on different categories of investment) then they should take into account not only the different volatility but also the correlation of that sector with the rest of the economy. It is possible to extend the model above to the case of more sectors. In that case the sectors that are positively correlated with the aggregate economy are sectors that generate the larger external costs in case of under-capitalization, because the correlation reduces the input demand in periods when total demand is low. Therefore optimal capital requirements will be larger for these sectors.

3.2 Bail outs

The second type of intervention that we consider is a bail-out policy, i.e. an intervention that transfers resources to distressed firms at date 1. This type of intervention has neutral effects in our framework. If the private incentives to save collateral are unchanged private contracts exactly undo whatever government transfers do. More precisely suppose the government can induce a non-distortionary transfer of resources $t_s k_1$ from investors to entrepreneurs at date 1, and suppose that these transfers are compensated by an equivalent transfer $E[t]k_1$ from entrepreneurs to investors at date 0. Proposition 9 shows that the competitive equilibrium is unchanged.

Proposition 9 *Suppose the government implements a scheme of state contingent transfers from investors to entrepreneurs* $\{t_s k_1\}_s$ *associated to an ex ante compensating transfer* $-E[t]k_1$. *Then the equilibrium prices and quantities are unchanged and the equilibrium financial contract at date 0 is*

$$d = d + t$$

3.3 Monetary Policy

Let us first consider the effects of a monetary contraction in period 0, oriented at reducing borrowing. This is the policy that Bordo and Jeanne dub "pro-active" monetary policy. To introduce monetary policy in the model here we use a simple sticky price version of the model above. The model is a simple variant of the real model described where the consumers utility function is modified as follows

$$u(c_0) - v(l_0) + c_1 + c_2$$

and where we specify that the net worth at date 0 depends on current activity according to

$$n_0 = f(k_0, l_0) - wl_0 - d_0 k_0$$

where the initial level of debt d_0 and of the capital stock k_0 are given. The sticky price model is described in detail in Appendix II. There we show that a monetary contraction at date 0 affects entrepreneurs behavior in two ways: it increases the interest rate ρ and it reduces the net worth n_0 of entrepreneurs. The first channel is a standard interest rate channel, the second channel is a type of balance sheet channel that we will call the profit channel. If we abstract from the distortionary consequences of this policy we can describe the effects of monetary policy as changing the budget constraint of entrepreneurs at date zero according to

$$k_1 = n_0 - \tau(\rho) + \mu(\rho) E[p] k_1$$

where ρ is the real interest rate $\tau(\rho)$ is an increasing function that captures the profit channel and $\mu(\rho) = 1/(1+\rho)$ captures the interest rate channel.

A contractionary monetary policy decreases investment at date 0 through an increase in both τ and μ. The interest rate channel has a beneficial effect because it makes outside finance more costly. It reduces the return on investment at date 0 financed with outside funds, reduces the shadow value z_0 and induces entrepreneurs to increase the degree of financial stability. However, the two channels also have a wealth effect. Let ρ^* be the natural interest rate, corresponding to the equilibrium of the economy without monetary frictions. The total wealth effect of a given monetary policy is

$$\tau(\rho) + (\mu(\rho^*) - \mu(\rho)) E[p] k$$

Therefore, monetary policy reduces investment not only by reducing the leverage ratio k_1/n_0 but also by reducing the level of inside funds.

Monetary policy can achieve the efficient financial contracts described in 8 only if it is accompanied by a transfer to entrepreneurs. However, the size of the transfer is *larger* than in the case of capital requirements, because the transfer not only has to redistribute the gains from financial stability but it also has to undo the reduction of n_0 brought about by the monetary contraction itself.

It is useful to compare the effects of monetary policy and capital requirements in the case where transfers *ex ante* are not feasible. When transfers *ex ante* are not feasible neither capital requirements nor monetary policy will be able to achieve a Pareto improvement over the competitive allocation. Still, by reducing financial volatility they can move the economy towards the constrained efficient frontier. However, to achieve a given level of financial stability monetary policy generates a larger reduction in investment *ex ante*. Going back to the trade-off between investment and financial stability the advantage of capital requirements over monetary policy is that they require a smaller sacrifice in terms of investment in order to achieve the same level of financial stability. Capital requirements virtue is that they increase the cost of outside funds without generating a negative effect on entrepreneurs' wealth. On top of that, in a sticky price framework, a contractionary monetary policy induces a distortion on the labor supply decision and on the relative prices.

A very different type of monetary intervention that can affect equilibrium financial contracts is a state contingent monetary policy at date 2. In particular an expansionary policy in bad states of the world that increases r_2 gives a reward to firms that have maintained a sufficient level of capitalization. This policy increases z in the bad states and encourages net-worth stabilization *ex ante*. Abstracting from the distortions associated to the monetary intervention let us look at a policy that offers a state contingent subsidy to the rental rate on capital r_2 and suppose this policy can be financed with lump-sum taxation.

In this case an appropriate system of subsidies to capital income in bad states of the world, together with appropriate transfers at date 0, can implement the second best efficient allocation. Suppose the government implements a scheme of state contingent subsidies to capital income $\{\sigma\}$ that satisfy

$$z\,(1+\sigma) = \tilde{z}.$$

Then the competitive equilibrium supports a constrained efficient allocation. The subsidies needed to support a constrained efficient allocation are countercyclical, i.e. σ_s is increasing in a_s.

A model similar to the one in the appendix can be used to show that a state contingent monetary policy that stabilizes output in bad states of the world at date 2 provides a subsidy of this type. However, this monetary policy has also distortionary effects on output and relative prices, so the positive incentives effects just discussed would have to be balanced against these distortionary effects.

The message one gets from the model is that a monetary policy that tries to intervene during the credit boom and reduces investment *ex ante* is not necessarily effective at restoring efficiency, while a monetary policy that tends to stabilize output *ex post* may also have favorable incentive effects *ex ante*.

Even though our model displays over-investment, over-investment is just a symptom of an overly fragile financial structure. Any policy that reduces investment *ex ante* but has no effect on the composition of investment finance tends to reduce the level of entrepreneurial net-worth in all future states with small stabilizing effects. In order to attack the inefficiency in our model the policy maker can resort to regulatory interventions or to state contingent monetary policy that is announced and understood by the public. The latter will favor the firms that have protected their balance sheets by increasing the *ex post* rate of return on capital and in this way induce less aggressive borrowing *ex ante*.

4 Concluding remarks

The model and the examples presented in this paper show how excessive financial instability and overborrowing can arise due to the presence of financial constraints and endogenous input prices. Clearly, there are other general equilibrium effects of financial instability aside from the effect on input prices. In particular asset prices can be endogenized. Asset prices will be a function of entrepreneurs' wealth and at the same time affect the value of their capital that in the present model was taken as exogenous and equal to $a_1 k_1$. This may generate feed-back effects that amplify the effects studied here and introduce pecuniary externalities among entrepreneurs. Secondly, the risk premia may be endogenous. In this paper we have assumed risk-neutral investors to show in the simplest way how a form of "risk aversion" arises solely because of financial constraints. In models with risk-averse investors the risk premia used at date 0 to evaluate financial contracts will depend on the future distribution of wages and therefore a higher volatility of wages at date 1 will reduce the ability of entrepreneurs to attract funds at date 0. In turns, if entrepreneurs can raise funds more cheaply at date 0 their liabilities at date 1 will be smaller and the level of investment at date 1 will be less sensitive to aggregate shocks. If these effects are large multiple, pareto-ranked equilibria can arise[12]. We leave the analysis of models with endogenous asset prices and endogenous risk premia to future research.

Another limitation of the model presented is that the effects of monetary policy have been studied considering simple temporary deviations from a

[12] An example in this direction is presented in section 6.1 of Holmstrom and Tirole (2001).

simple inflation targeting regime. A full analysis of the effects of monetary policy on the choice of financial structure should take into account monetary policy inertial character. We believe that when the analysis is extended in this direction, it will still be possible to decompose the effects of a interest rate increase on investment into a 'substitution effect' and a 'wealth effect', with the former having a positive effect on financial stability and the second having only a negative effect on investment. However, this conclusion and a quantitative assessment of these effects should await further research.

Finally, as observed in section 3.1, it will be useful to explicitly introduce financial intermediaries in order to analyze more realistic forms of capital requirements.

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5 Appendix I

Proof of Proposition 1

It is useful to first prove the characterization part and then prove existence and uniqueness.

Notice that the Inada condition implies that in equilibrium $z_s \geq 1$. It follows that $z_0 > 1$. Consider the states $S_0 = \{s : z_s < z_0\}$. We want to show that if $s \in S_0$ and $a_{s'} > a_s$ then $s' \in S_0$. Notice that if $z_0 > z_s$ then $d_{1s} = \theta$, $d_{1s'}$ satisfies $d_{s'} \leq \theta$ and we have $n_{1s'} = (a_{1s'} - d_{1s'})\, k_1 > (a_{1s} - \theta)\, k_1 = n_{1s}$ which implies $k_{2s'} \geq k_{2s}$. Concavity of f implies $z_{s'} \leq z_s < z_0$.

Consider now the states $S_1 = \{s : z_s = 1\}$. Since $z_0 > 1$ it follows that $S_1 \subset S_0$. If $s \in S_1$ and $a_{s'} > a_s$ then $d_{s'} = d_s = \theta$ and therefore $k_{2s'} \geq k_{2s}$ and $z_{s'} = 1$.

In equilibrium if $d_s < \theta$, then $z_s = z_0$. This implies that n_{1s} is constant in S/S_0. This implies $a_{1s} - d_{1s} = a_1' - \theta$. Denote the cutoff as a. Therefore, the optimal financial contract d_1 can be expressed in terms of the cutoff a as

$$d_1 = \theta + \min\{a_1 - a, 0\}.$$

Therefore an optimal financial contract is fully characterized by a. Let the CDF of a_1 be F and the support of a_1 be $[\underline{a}, \overline{a}]$.

Let $H(a) = E[xh(n_1)]$ where

$$x_s = \iota_s - \frac{a_{1s} - \theta}{1 - \theta} \tag{22}$$

where $\iota_s = 1/F(a)$ if $d_{1s} < \theta$, and zero otherwise. Notice that $H(\overline{a})$ is proportional to $\left(1 - E\left[\frac{a_{1s}-\theta}{1-\theta}\right]\right)$ and so $H(\overline{a}) < 0$.

Differentiating H one obtains

$$H'(a) = E\left[x_s h'(n_{1s}) \frac{dn_{1s}}{da}\right]$$

where

$$\frac{dn_{1s}}{da} = \left(1-\pi_s\frac{a_{1s}-d_{1s}}{1-Ed_1}\right)k_1 \text{ if } d_s<\theta$$
$$\frac{dn_{1s}}{da} = -\pi_s\frac{a_{1s}-d_{1s}}{1-Ed_1}k_1 \text{ if } d_s>\theta$$

Consider an a^* such that $H(a^*)=0$. Then in all states with $d_s<\theta$ we have $1-\pi_s\frac{a_{1s}-d_{1s}}{1-Ed_1}>0$ (otherwise $z_s\le\pi_s\frac{a_{1s}-d_{1s}}{1-Ed_1}z_s<z_0$) and $x_s>0$ (otherwise $H(a^*)>0$). In the remaining states we have $\frac{dn_{1s}}{da}<0$ and $x_s<0$. Therefore $H'(a^*)<0$. So H is quasiconcave, either $H(\underline{a})<0$ and there is an equilibrium with $d_s=\theta$ for all s, or $H(a^*)=0$ for $\underline{a}<a^*<\overline{a}$. In both cases the equilibrium is unique.

Proof of proposition 2

From proposition **1** we know that the equilibrium is fully characterized by the cutoff a_1', so we only need to prove that $\hat{a}_1'>a_1'$.

Let n_1 be the equilibrium net worth in the first economy. Let $\underline{n}=\min\{n_{1s}\}$. The condition $h(\underline{n})=E\left[\frac{a_{1s}-\theta}{1-\theta}h(n_1)\right]$ implies $\tilde{h}(\underline{n})>E\left[\frac{a_{1s}-\theta}{1-\theta}\tilde{h}(n_1)\right]$ because $\tilde{h}$ is more risk averse than h. Let $\tilde{H}$ be defined as in proposition **1** for the second economy. The last inequality implies $\tilde{H}(a_1')>0$ and, by the quasiconcavity of $\tilde{H}$, we have $a_1'<\hat{a}_1$.

Proof of proposition 5

First we can show that the when $\tilde{h}$ is more risk averse than h then $f_L(k,1)$ is concave. By contradiction: if $f_{LKK}>0$ for some $k<k^*$ then the coefficient of absolute risk of h and $\tilde{h}$ are

$$\left|\frac{f_{KK}+\kappa f_{LKK}}{f_{KL}-\theta+\kappa f_{KL}}\right|<\left|\frac{f_{KK}}{f_{KL}-\theta}\right|.$$

Then program P' is concave. As in the proof of **1** we can show that an efficient allocation is fully characterized by a cutoff $\tilde{a}$ such that $\tilde{d}_1=\theta+\min\{a_1-\tilde{a},0\}$.

We will prove the proposition considering the constrained efficient allocation corresponding to $U^C=U_c^C$, the proof can be exteneded to any efficient allocation that weakly dominates the equilibrium.

Let n_1 be the equilibrium wealth levels and let x be defined as in (22). Since x and n_1 are monotonically related and $E[xh(n)]=0$ it follows that

$E\left[x\tilde{h}(n)\right] \geq 0$. The concavity of the program can then be used to show that the constrained efficient cutoff satisfies $\tilde{a} \geq a_1'$.

Proof of Proposition 8 The entrepreneur problem subject to capital requirements can be written as

$$\begin{aligned} \max_{k_1,\{d\}} \quad & E\left[z\left(a_1 - d\right)k_1\right] \\ s.t. \quad & k_1 \leq n_0 + \tau + E\left[d\right]k_1 \\ & k_1 \leq \nu n_0 \\ & d_s \leq \theta \end{aligned}$$

let μ be the Lagrange multiplier on the capital requirement constraint then the first order conditions are

$$\begin{aligned} (1 - Ed)\,z_0 + \mu &= E\left[z\left(a_1 - d\right)\right] \\ z_0 - z &\geq 0 \\ d &\leq \theta \\ (z_0 - z)\left(d - \theta\right) &= 0 \end{aligned}$$

Consider a constrained efficient allocation and let $\{z_s\}$ be the rate of return on inside funds at the constrained efficient allocation. Let $\hat{n} = \min\{n_{2s}\}$ and let $x = \frac{a_1 - d}{1 - Ed}$. The constrained efficient allocation satisfies

$$\tilde{h}(\hat{n}) = E\left[x\tilde{h}(n_1)\right]$$

This implies that

$$h(\hat{n}) > E\left[xh(n_1)\right]$$

and we can let

$$\begin{aligned} z_0^* &= h(\hat{n}) \\ \mu^* &= \frac{E\left[z\left(a_1 - d\right)\right]}{(1 - Ed)} - z_0^* \geq 0 \end{aligned}$$

The Lagrange multipliers (z_0^*, μ^*) and the constrained efficient levels of d^* satisfy the first order conditions of the entrepreneur.

6 Appendix II (monetary model)

Here is a sketch of the monetary model used to discuss the effects of monetary policy in section 3. There is a continuum of consumption goods which enter consumers' utility according to the CES aggregator:

$$x = \left(\int_0^1 x_j^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}$$

From consumer optimization we have $x = \frac{y}{p}$, where y is nominal income and $p = \left(\int p_j^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$. There is a continuum of monopolistic competitive firms that rent capital from entrepreneurs and produce consumption goods. These firms are owned by consumers. Firms' technology at date 0 is described by the production function

$$f(k,l) = Ak^{\alpha} l^{1-\alpha}$$

Assume prices are fixed at date -1 and then at date 0 a fraction κ of firms adjust their prices. At date 1 and 2 assume prices are flexible. The real factor prices w and r are assumed to be fully flexible. We have the inverse demand faced by firm j

$$\frac{p_j}{p} = \left(\frac{x_j}{x} \right)^{-\frac{1}{\sigma}}$$

Profits are given by

$$\max_{x_j} (1+s) \left(\frac{x_j}{x} \right)^{-\frac{1}{\sigma}} x_j - c(r,w) x_j$$

where the linear cost function $c(r,w)x_j$ is derived by standard cost minimization under constant returns to scale.

For flex price firms we have:

$$(1+s) \left(1 - \frac{1}{\sigma} \right) \frac{p_j}{p} = c(r,w)$$

Because of constant returns to scale for all firms we have the same optimal factor proportion for all firms

$$\frac{k}{l} = h(\frac{r}{w}).$$

The subsidy s is set to eliminate the distortion due to monopoly pricing

$$(1+s)\left(1-\frac{1}{\sigma}\right)=1$$

Since $x_j = f(k_j, l_j) = f(k,l)\frac{l_j}{l}$ and $\frac{l_j}{l} = \frac{x_j}{x} = \left(\frac{p_j}{p}\right)^{-\sigma}$ we get total final goods output

$$\begin{aligned} x &= \left[\left[\int \left(\frac{p_j}{p}\right)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}\right]^{-\sigma} f(k,l) = \\ &= vAk^{\alpha}l^{1-\alpha} \end{aligned}$$

In this setup inflation has two costs:

(1) it distorts the efficiency condition

$$c(r,w)=1$$

(2) it reduces output in terms of consumption goods because of the variance term v above, which is $v<1$ whenever p_j is not constant across sectors.

In this environment monetary policy can be set so as to achieve a given employment level l. The real wage is then determined by

$$w = v'(l)$$

and then for a given capital stock k the rental rate for capital r is determined by

$$r = wh^{-1}\left(\frac{k}{l}\right)$$

The associated inflation is then determined by the condition

$$\frac{p_{flex}}{(1-\kappa)\,p_{-1}+\kappa p_{flex}} = c(r,w)$$

which gives us the price level for flex price firms.

In this framework, monetary policy is essentially a policy that distorts the labor supply margin l at date 0 and in this way it distorts r and affects capital accumulation in the entrepreneurial sector. The "natural" benchmark is characterized by

$$f_L\left(l^*,k\right)=v'(l^*)$$

When monetary policy targets an $l > l^*$, output is higher and so are w and r, we have $c(r, w) > 1$ and there is inflation. When it targets $l < l^*$ the opposite happens.

We can think of monetary policy as targeting a certain level of l at date 0. For a given level of l we can derive the equilibrium of the corresponding real economy and derive the corresponding real interest rate ρ and the level of entrepreneurial wealth $n_0 = f_K(l, k_0)$.