Ownership Dynamics with Multiple Insiders: The case of REITs

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ABSTRACT. We study ownership dynamics of multiple strategic risk-averse insiders facing a moral hazard problem. We show that, when insiders cannot commit, ex-ante, to an ownership policy, the aggregate insider stake gradually declines towards the competitive allocation. Moreover, both the speed of decline and the long-term equilibrium aggregate insider ownership level are greater for companies with a higher number of insiders, *ceteris paribus*. We, then, test the model on data from the U.S. Real Estate Investment Trusts (REITs) industry and find that the predictions of the model are supported by the data.

Classification Codes: G14, G32, D43.

Key Words: Corporate Insiders, Moral Hazard, Ownership Dynamics, REITs

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Introduction

In this paper we study the dynamics of corporate ownership for companies with multiple risk-averse insiders facing a moral hazard problem. This problem arises naturally, among others, within the context of the U.S. REIT industry. The industry has been undergoing substantial structural changes since the 1990s (see, e.g., Edelstein, Urošević and Wonder (2005) (EUW)). As a result of these changes, a typical REIT today has multiple individuals with significant ownership stakes. We study, both theoretically and empirically, dynamics of the aggregate ownership stake of significant REIT insiders. On the theoretical side, our model builds upon the dynamic moral hazard model of DeMarzo and Urošević (2001) (DU) and EUW. These models study an optimal ownership policy of a risk averse large shareholder/insider facing a moral hazard problem, and determine the corresponding equilibrium share price.[1] In this paper we extend these models to incorporate strategic interactions among multiple insiders. This allows us to explore how the make-up of the aggregate insider stake influences the nature and the speed of its adjustment. On the empirical side, this is the first paper that explores links between the ownership dynamics and the composition of the aggregate insider stake, i.e. the number of insiders, in a company. In addition, our paper fills an important void in the empirical literature on REITs. Namely, while for C-corporations Mikkelson et al (1997), Urošević (2002), and Harjoto and Garen (2005) (for the U.S.) and Franks et al. (2005) (for the U.K.), document a significant and steady decline of the aggregate insider ownership stake after the initial public offering (IPO); to the best of our knowledge no similar studies have been performed for REITs.

Let us now describe the model. In an economy consisting of one risky firm and a riskless bond, there are *N* equally risk-averse agents whom we label insiders[2] and a continuum of small outside risk-averse investors, all with CARA preferences. We consider the case in which each insider can commit to an optimal ownership policy, and the case in which such commitment is impossible. As in DU and EUW, the commitment policy is time-inconsistent in the sense that after a planned sale, any insider with marginal valuation below that of the aggregate investor's will be tempted to trade again. The problem is that the insider will no longer internalize the capital loss on the shares just sold. Put in another way, insiders' risk-aversion creates a wedge between their valuation of company shares and the value placed on the company shares by the outside investors (the market). The optimal time-consistent policy, therefore, is for insiders to gradually adjust their stakes in the company until the perfect risk sharing allocation is achieved (in contrast to EUW, for reasons of tractability, in this paper we do not consider private benefits of control by

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[1] These models build upon one-period models of Admati et al. (1994) and two period models of Kihlstrom (1998) and Stoughton and Zechner (1998). DeMarzo and Bizer (1993) establish the connection between the durable goods monopolist problem and securities markets and, thus, connection with the Coasian conjecture (see Coase (1972)).

[2] Here, and in the rest of the paper, we use the word "insider" to denote anyone who files SEC insider forms and not necessarily someone who "trades on information". In our model, insiders do not trade on private information, but to reduce the amount of idiosyncratic risk that they bear. Other authors have analyzed insider trading on private information in the presence of moral hazard: for example, in Yung (2005), moral hazard arises because a firm's returns are a function of the insider's costly effort.

corporate insiders. As a result, the competitive allocation coincides with the perfect risk sharing allocation[3]).

There are two competing forces driving the results of the model. On one hand, insiders are facing a moral hazard problem so that the expected firm cash flows and, therefore, the company value increases with insider holdings, *ceteris paribus*. On the other hand, high company stakes imply a significant risk exposure to insiders who, as a result, have the incentive to decrease their stakes over time. Importantly, the ownership policy of each insider influences the share price, and, therefore, every insider's future ownership decisions. The equilibrium in the economy simultaneously specifies the optimal ownership policies of each insider, as well as the corresponding share price process.

Since outside investors are assumed risk-averse, there is an additional strategic reason for a dynamic stake adjustment in this model vis-à-vis the one-agent case. Namely, a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among the insiders a "race to diversify". As a result, in the unique subgame-perfect equilibrium, the speed of adjustment towards the perfect risk sharing allocation increases with the number of company insiders N. Intuitively, as N increases, the asset price more quickly becomes competitive, though the adjustment towards the long-run equilibrium is gradual.

Our model has three main testable predictions. The first is that the aggregate insider's stake gradually declines towards the perfect risk sharing allocation (the long-term equilibrium). The second is that the long-term equilibrium allocation level increases with an increase in number of a REIT's insiders. Finally, the third is that the initial speed of adjustment of the aggregate insider stake towards the long-term equilibrium level increases with an increase in number of REIT's insiders. We test these three predictions on a sample of 137 U.S. REITs that went public between January 1986 and December 1999. Using t-tests and linear regressions, we find that all three predictions are clearly supported by the data. We observe a reduction in the aggregate insiders' ownership by a 18% during the first 5 years, and by a 42% during the first 10 years after the IPO; moreover, the aggregate insider ownership during the tenth year of a REIT's life and the speed of adjustment during the first five years of a REIT's live increase with the number of insiders.

Independently of this work, Pritsker (2005) develops a related model. He also constructs a model of multiple "large traders" in a CARA/normal setting and borrows, like this paper, some modeling techniques from DU. In many important ways, however, our models are quite different. The key difference is in the economic environment that the two models portray. While both of our papers incorporate diversification as a key motivation to trade, Pritsker (2005) studies market liquidity, shock transmission and market manipulation by large institutional traders in a multi-asset economy with no moral

[3] Benefits of control are likely substantial for REITs, but we see no obvious relation between the effect of those benefits and the number of corporate insiders, which is the focus of our empirical work in this paper.

hazard. In contrast, we study the evolution of insider ownership stakes in a single asset economy in which insiders face a moral hazard problem.[4]

This paper is organized as follows. In Section 1, we describe the model. In Section 2, we find the benchmark commitment and price-taking allocations. In Section 3, we solve for time-consistent equilibrium and derive key empirical predictions. In Section 4, we test the model on the post-IPO data on U.S. REITs. Section 5 contains conclusions and suggestions for future research. Section 6 contains the list of references. Proofs are relegated to the Appendix.

1. The Model

In this section we describe the model. By incorporating several strategic insiders instead of a single strategic agent, this model constitutes an important extension to the work of DU and EUW. In many other respects of the model setup, on the other hand, our models bear significant similarities. For that reason we omit derivations that closely parallel those in DU and EUW focusing, instead, on those aspects of the model that differ from its one-agent counterparts.

We consider a going-concern publicly traded REIT with a supply of shares that is normalized to one and with a cumulative free cash flow process described by the following diffusion process

$$dD = \hat{\mu}dt + \hat{\sigma}dz$$
$$D(0) = 0$$

where Z is standard Brownian motion (the expressions for $\hat{\mu}, \hat{\sigma}$ are specified below). Consequently, the cash flows in each period are normally distributed and independent across periods. Normality is important for tractability of the model and while unlimited liability is not desirable, it does not play an important role in the forces driving our results. Inter-temporal independence of cash flows means that there is no learning from the past.[5] Shares of the firm trade in the market at the price V, which needs to be determined in equilibrium. In addition to this firm there exists a riskless investment that pays a continuously compounded return of r, with a perfectly elastic supply. Since real life REITs pay out all of their cash flows as dividends, we assume the same in our model (an added benefit of this assumption is an improved tractability of the model).

All agents in the economy maximize CARA expected utility. In particular, there are $N>1$ agents that have an ability to monitor the REIT and affect decisions within the firm. We refer to them as insiders. For simplicity we assume that the insiders may have different initial company stakes but are otherwise identical. In particular, the coefficient of absolute risk aversion is assumed identical across insiders and is equal to γ. In addition to insiders, there exists a continuum of competitive outside investors that can be represented by an aggregate investor; the coefficient of absolute risk aversion of the aggregate investor is denoted as γ^I.

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[4] A paper related to Pritsker (2005) is Brunnermeier and Pedersen (2005) who study the issue of predatory trading in a multi-trader context.

[5] See DU, for example, for more detailed discussion of these points.

All trades occur in a competitive market.[6] Let $\alpha^l(t) \in [0,1]$ (l=1:N) be the fraction of the firm held by the insider l at time t. We restrict each $\alpha^l(t)$ to be right-continuous, and interpret $\alpha^l(t-) \equiv \lim_{\tau \uparrow t} \alpha^l(\tau)$ as the shares held at the "start" of period t by the insider l; thus, $\alpha^l(t) - \alpha^l(t-)$ is the discrete number of shares purchased by the insider l in period t or, since the two are interchangeable in this model, the change in the insider's l ownership stake at time t. The insider l has an initial endowment $\alpha^l(0-) = \alpha^{l-}$. Initial endowments are, generically, different from each other.[7] Let $A(t) \equiv \sum_l \alpha^l(t)$ be the aggregate insider stake at time t and $A^- \equiv \sum_l \alpha^{l-}$ be the initial aggregate insiders stakes.

By market clearing, in equilibrium the aggregate investor's holdings at time t are given by the expression $1 - A(t),\ t \geq 0$. Sometimes, it will be convenient to separate in this sum holdings of an insider l, α^l, from the aggregate holdings of all other insiders $\beta^l(t) \equiv A(t) - \alpha^l(t)$. Furthermore, whenever confusion cannot occur, we shall drop the index l when describing a generic insider.

Insiders face a simple moral hazard problem:[8] insider l's costly monitoring effort $e_l, l = 1,..,N$ affects the expected free cash flow of the REIT in a linear fashion, $\hat{\mu}(\vec{e}) = \sum_{l=1}^{N} e_l$. One way to interpret this is to think of each insider as working on an independent task within the firm. While insiders do not affect the effort of other insiders directly, they do so indirectly by recognizing their own and other insiders' impact on the process of share price formation (see below). We assume, further, that the variance of the REIT's cash flows cannot be altered by the actions or holdings of the insiders; we set $\hat{\sigma} = \sigma$. An insider's l cost of effort is independent of other insiders' effort choices and quadratic in effort, i.e., $f(e_l) = e_l^2/(2\mu)$, where parameter μ is identical for all insiders. None of the parameters of the model depend on time.[9]

Since the effort cannot be contracted on, each insider's effort choice must be incentive compatible. Because of the CARA/normal setting, each insider's problem can be expressed in terms of the certainty equivalent. In each instant, insiders simultaneously choose their efforts in order to maximize the instantaneous certainty equivalent of their payoff,[10]

$$z^l(\alpha^l(t), \beta^l(t)) \equiv \max_{e^l} \alpha^l(t)\hat{\mu}(\vec{e}) - f(e^l) - \tfrac{1}{2}\alpha^{l2}(t)\gamma r \sigma^2, l = 1,..., N \qquad (1)$$

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[6] That means, in particular, that the insiders in the economy trade with competitive outside investors. In contrast, Vayanos (1999) considers a model with N strategic traders in the absence of competitive investors.
[7] The initial stakes are exogenous to the model. Stoughton and Zechner (1998) and DU endogenize initial stakes in a one-agent moral hazard setting.
[8] Moral hazard plays an important role in ownership decisions of corporate insiders. In particular, Brav and Gompers (2003) find that the moral hazard problem facing corporate insiders is the main reason for the existence of lock up restrictions on share trading immediately following an Initial Public Offering (IPO).
[9] Private benefits of control are excluded from the model. For a discussion of the effects of private benefits of control in the case of one large shareholder, see EUW.
[10] The derivation of this expression follows along the same lines as in the one-agent case and is omitted for brevity (see DU, Section 3, for more details).

The expression (1) is quite intuitive. The instantaneous certainty equivalent is equal to the total expected dividends received by the insider, net of her cost of effort and adjusted for the insider's risk aversion. Here, $\alpha^{l2}(t)\sigma^2$ captures the variance of the dividends received by an insider, γ is the insider's risk aversion, and the scaling by the interest rate r appears since the insider can smooth shocks over time. Given $\hat{\mu}(\vec{e})$ and $f(e_l)$, the maximization problem in (1) is solved by $e^l(t) = \alpha^l(t)\mu, l = 1,..,N.$ Thus, the certainty equivalent payoff "flow" to each insider can be rewritten entirely in terms of her and other insiders' holdings:

$$z^l(\alpha^l(t), \beta^l(t)) = (\mu - \gamma r\sigma^2)\alpha^{l2}(t)/2 + \mu\alpha^l(t)\beta^l(t), l = 1,.., N$$

Whenever confusion does not arise we shall drop in the above expression the index l and write it, instead, for a generic insider, as

$$z(\alpha(t), \beta(t)) \equiv (\mu - \gamma r\sigma^2)\alpha^2(t)/2 + \mu\alpha(t)\beta(t) \tag{2}$$

The instantaneous certainty equivalent of an insider is a function of that insider's holdings, given the aggregate holdings of all other insiders. The quantity $\mu \geq 0$ measures the expected free cash flow sensitivity with respect to the change in corporate ownership and thus parameterizes the importance of the moral hazard problem in this model.

Equation (2) determines the total risk-adjusted payoff to an insider from holding a fraction α of the firm. It is useful to restate this in terms of the marginal value of ownership to the insider:

$$\frac{\partial z}{\partial \alpha}(\alpha(t), \beta(t)) = \mu A(t) - \alpha(t)\gamma r\sigma^2 \tag{3}$$

That is, the marginal value of a share to the insider is simply the expected dividend per share, $\mu A(t)$, adjusted by the insider's "risk premium" given holdings $\alpha(t)$. In fact, to find the total risk-adjusted payoff it is sufficient to know $\frac{\partial z}{\partial \alpha}$, since $z(\alpha, \beta) = \int_0^\alpha d\hat{\alpha} \frac{\partial z}{\partial \alpha}(\hat{\alpha}, \beta)$.

An analogous expression can be derived for the aggregate investor.[11] Namely, the marginal value of a REIT share from the aggregate investor's perspective is given by the expression

$$v(A(t)) = \mu A(t) - (1 - A(t))\gamma^I r\sigma^2 \tag{4}$$

Namely, the marginal value of a share to the aggregate investor is the expected dividend per share, $\mu A(t)$ but, in contrast to (3), adjusted by the aggregate investor's risk premium given holdings $(1 - A(t))$. Equations (3) and (4) summarize the primitives of the model. The moral hazard of the insiders is, therefore, reflected in the dependence of the dividend on their aggregate holdings A, whereas the motivation for trading is provided by the difference in marginal valuations of insiders and the aggregate investor.

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[11] The derivation of this expression practically coincides with the one-agent case of DU and is, thus, omitted for brevity.

In the model, insiders live infinitely but trade and actively monitor the company only for a finite period of time. In particular, while insiders may consume, make effort choices, and trade in the riskless security continuously, we assume that they are restricted to trade shares of the firm on a finite set of dates $T = \{t_1 = 0, t_2, ..., t_N = T\}$ *common to all insiders*. In this case, $\alpha(t) = \alpha(t_i)$ for all $t \in [t_i, t_{i+1})$, where we define $t_{N+1} = \infty$. Technically, a finite number of trading periods is necessary for the uniqueness of the sub-game perfect equilibrium. In practice, companies frequently impose "windows" within which company insiders can trade say, every quarter.[12] The timing in the model is as follows:

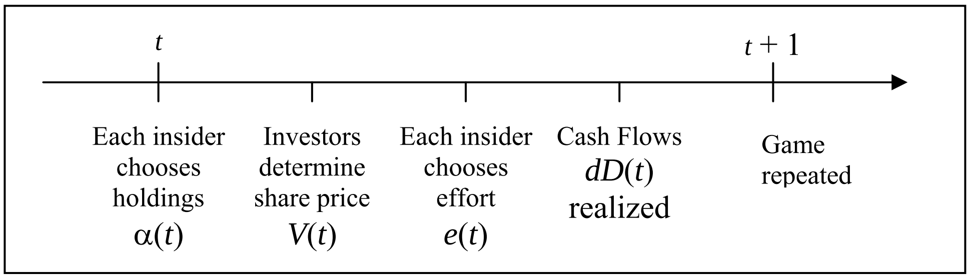

Figure 1: Timing in the model

Insiders' decisions are sequentially rational so that each agent is playing a multi-period simultaneous-move game with the other insiders in the company. In particular, each insider knows her own, as well as the other insiders', past trades. They also know that their trading decisions today affect the share price and, consequently, the future trading decisions of all insiders in the economy (including their own). Although the outside investors trade competitively as price-takers, they are aware of the strategic interaction among the insiders and the fact that the insiders' current trading decisions have an impact on the insiders' current and future trading decisions. Outside investors are rational and make their demands for shares after they observe the insiders' trading decisions for that time period. In particular, there is no asymmetric information about the dividend process or about the insiders' trading decisions. In other words, all information about the company and insider trades is revealed instantaneously to the investment community.[13] Finally, we do not explicitly restrict the aggregate investor to trade only on dates T since, in equilibrium, investors would only trade when the insiders trade.

[12] Urošević (2002) finds that the average interval between two successive trades by a corporate insider is 109 days, roughly corresponding to quarterly trading windows; see, also, Seyhun (1998).

[13] In contrast, Vayanos (2001) considers a model with one strategic trader, market makers, and noise traders in which the agent's trading decisions are his private information; Vayanos (1999), on the other hand, considers a model with N strategic players with private endowments and no noise traders or market makers. While these models are very different from each other (and from our model), the agents' allocations converge in both models towards the competitive risk sharing allocation.

2. Commitment and Price-Taking Strategies

The setup in this model is similar to DU and EUW with some important differences: a) they consider one and we consider multiple strategic insiders and their interactions; b) In contrast to their models, we restrict the analysis to the case of a stationary linear moral hazard problem where insiders cannot influence volatility and have no benefits of control. Preliminary results of DU (their Sections 3.1 to 3.3) can be adopted in our case with only minor changes. For example, the equilibrium share price in this model is given by

$$V\left(\vec{\alpha}(t)\right) = \int_t^{\infty} e^{-r(\tau-t)} \nu\left(A(\tau)\right) d\tau . \tag{5}$$

The expression in (5) requires some discussion. In one sense, it is straightforward – the equilibrium share price is simply equal to the discounted risk-adjusted dividend flow to investors. Less trivially, future dividend flows depend upon the insiders' anticipated trading strategies (through both the expected dividends and the future risk premium), which must be determined in equilibrium. Indeed, (5) states that the share price at time t is determined by the aggregate insider holdings at time t since, in a unique sub-game perfect equilibrium (see Section 3), an insider's holdings today influence each insider's trading strategy and, thus, their holdings in the future. Therefore, the share price is given by the discounted risk-adjusted dividends calculated *at the equilibrium trading strategy* $\vec{\alpha}(\tau)$. Due to the symmetry of the model, in the sub-game perfect equilibrium (see Section 3), the share price V at time t is, in fact, a linear function of the aggregate insider holdings $A(t)$.

In order to determine insiders' trading strategies, we must formulate their optimization problems. For an insider whose stake at time t is equal to $\alpha(t)$, her total certainty equivalent at time t is given by the following, rather intuitive, expression: [14]

$$k(\alpha(t),\beta(t)) = \int_{[t,\infty]} e^{-r(\tau-t)} \left[z\left(\alpha(\tau),\beta(\tau)\right) d\tau - d\alpha(\tau) V\left(\vec{\alpha}(\tau)\right) \right] \tag{6}$$

Due to the symmetry in the model the form of the expression in (6) is the same for each insider. The meaning of the right hand side in (6) is that each insider's certainty equivalent consists of her capitalized risk adjusted aggregate benefits from holding shares (the first term) and her capitalized trading gains from selling shares over time (the second term). These expressions depend on the insider's own future trading strategy as well as on the future strategies of all of the other insiders. These strategies need to be determined in equilibrium simultaneously with the share price process.

In general, each insider would trade at least once. Indeed, since relatively under-diversified insiders are typically more risk averse than the pool of outside investors, each insider would have an incentive to trade away from the initial allocation. The following proposition describes a commitment Nash equilibrium when each insider's strategy can

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[14] We adopt a set notation for the limits of integration to avoid ambiguity given discontinuities in α. Thus, $\int_{(t,T]} d\alpha(\tau) = \alpha(T) - \alpha(t)$ and $\int_{[t,T)} d\alpha(\tau) = \alpha(T-) - \alpha(t-)$, where $\alpha(t-) = \lim_{\tau \uparrow t} \alpha(\tau)$.

depend on time only; that means that, in particular, no insider is allowed to condition her trading decisions on the past insider trades.

PROPOSITION 1. Suppose that at time *t*, each agent announces a trading policy that depends only on time, $\alpha^l(\tau)$, $\tau \geq t$, and ***cannot be revised*** in the future. Then, each insider's strategy is given by the following Nash equilibrium strategy:

$$\alpha^c(\tau) \equiv \arg\max_{\alpha(\tau)} z\left(\alpha(\tau), \beta(\tau)\right) + \left(\alpha(t^-) - \alpha(\tau)\right) \nu\left(\alpha(\tau) + \beta(\tau)\right) \quad (7)$$

The aggregate insider equilibrium holdings in the commitment case are given by the following expression:

$$A^c = \frac{(\mu + \gamma^I r\sigma^2) A^- + N\gamma^I r\sigma^2}{\mu + (N+1)\gamma^I r\sigma^2 + \gamma r\sigma^2} \quad (8)$$

Since the parameters of the model do not depend on time, the commitment equilibrium allocation does not depend on time either. In addition, from (7) one can easily check that the first order conditions for the problem read:

$$\alpha^c = \frac{r\gamma^I \sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}(1-\beta) + \frac{\mu + r\gamma^I \sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}\alpha^- \quad (9)$$

In all of these expressions, the superscript *l* has been omitted. One has to bear in mind, however, that (7) and (9) stand for *N* different equations. In particular, since the initial insider holdings are generically different across the set of insiders, equilibrium insider holdings in the commitment case also differ across insiders. Moreover, an insider's commitment equilibrium stake increases with her initial holdings. That means that whenever insiders can commit to an optimal allocation, those who initially hold larger stakes would tend to optimally choose a higher level of ownership to commit to. In addition, an insider's commitment holdings *decrease* with an increase in other insiders' holdings. That means that by holding a smaller company stake each insider aims to raise the market risk premium, thus lowering the stock price, and consequently, the motivation of the other insiders to trade down in the future. This effect will play an important role in the time-consistent model (see below).

In the commitment equilibrium, each insider anticipates the impact that her trades will have, as well as those of other insiders, on the stock price. On the other hand, if insiders ignore such impact, i.e., if they take ν and, consequently, *V* as given, the following proposition holds:

PROPOSITION 2. Define $\alpha^p = \gamma^I / (\gamma + N\gamma^I)$. Then, if $\mu < r\gamma\sigma^2$, a Walrasian equilibrium exists in which each insider is a price-taker, the equilibrium trading strategy for each insider is given by α^p, and the aggregate price-taking equilibrium allocation is

$$A^p = N\alpha^p = \gamma^I / (\gamma / N + \gamma^I) \quad (10)$$

From **Proposition 2** it follows that the price-taking equilibrium exists when the (beneficial) incentive effect is smaller than the insiders' aversion towards risk. The aggregate perfect risk sharing allocation A^p can be seen as the competitive allocation of one insider with *N* times higher risk tolerance. Notice that (10) increases in the number of insiders *N*. That leads to an important testable prediction of our model. Note, also, that when N=1, expressions (8) and (10) coincide, respectively, with the commitment and the competitive allocations in DU, Section 3.4.

3. Time-Consistent Trading Strategy for Insiders

In the previous section, we solved for the optimal trading strategies assuming that all insiders could commit ex-ante to future trades. In that case, the current share price depends on all future trades that the insiders will make. Here, we no longer allow the insiders to commit to future trades. Thus, each insider's trading strategy must be time consistent. The previous results suggest that for a generic insider the commitment policy α^c is not time consistent. To see the intuition for this, note that from (9) it follows that α^c is increasing with the initial insider shareholdings. Therefore, once shares are initially sold, the insiders' decision-making process starts again, this time with smaller shareholdings but still above the competitive allocation. Thus, the insiders have an incentive to sell again. This second sale and the resulting change in effort impose a negative externality on the initial buyers of shares that the insiders do not consider when making a second sale. The same is true for each individual insider.

To solve for the equilibrium without commitment, note that the value of the shares at any time *t* must depend on the investors' expectations of the insiders' future trading decisions. Thus, investors must anticipate the insiders' ex-post incentives to trade. In addition, at each point in time, each insider recognizes that her trading decision today impacts not only her future trading decisions, but also those of all of the other insiders.

The problem is solved by backward induction. For that reason, consider, first, the insiders' decision-making process at time *T* (the last trading date). Recall that the insiders have the opportunity to trade only on the discrete dates $T = \{t_1=0,\ t_2,\ ...,\ t_N=T\}$. Implicitly, the insiders commit not to trade during the intervals $(t_i,\ t_{i+1})$. For simplicity, we assume that time intervals Δ between trades are constant (except for the last trading interval that is infinite) and introduce the capitalization factor δ_t where $\delta_t \equiv \delta \equiv (1-e^{-r\Delta})/r$, except in the case when $t_{N+1}=\infty$, when we set $\delta_T = 1/r$. At time *T* insiders, by assumption, commit not to trade again so one can use the technique developed in Section 2. Denote a generic insider's holdings at time *t* as α_t and the aggregate holdings of all of the other insiders by β_t. Let J_t be the optimal certainty equivalent of an insider at time *t*, i.e. the value function. Finally, let the share price at time *t* be denoted as V_t. Then, given a vector of the initial insiders' holdings, one can specify the recursive solution. Furthermore, under the assumptions of the model this recursive equilibrium is a unique sub-game perfect equilibrium. It is characterized by the following proposition:

PROPOSITION 3. For each $t \leq T$, the value function of the dynamic programming problem above is a quadratic form in variables (α_t, β_t):

$$J_{t+1} = J_{t+1}^{\alpha,\alpha}\alpha_t\alpha_t + J_{t+1}^{\alpha,\beta}\alpha_t\beta_t + J_{t+1}^{\beta,\beta}\beta_t\beta_t + J_{t+1}^{\alpha,0}\alpha_t + J_{t+1}^{\beta,0}\beta_t + J_{t+1}^{0} \quad (11)$$

while the share price is an affine function in $A_t = \alpha_t + \beta_t$

$$V_t = v_{0t} + v_t A_t = v_{0t} + v_t A_t \quad (12)$$

The optimal holdings of each insider at time t are determined as an affine transformation of her own and other insiders' holdings at time t-1:

$$\alpha_t = l_t^{\alpha,\alpha}\alpha_{t-1} + l_t^{\alpha,\beta}\beta_{t-1} + l_t^{\alpha,0}, \quad \beta_t = l_t^{\beta,\alpha}\alpha_{t-1} + l_t^{\beta,\beta}\beta_{t-1} + l_t^{\beta,0} \quad (13)$$

A complete set of recursive relations and the appropriate boundary conditions that determine the coefficients in (11)-(13) are given in the Appendix (Equations (A10) to (A13)). Under the assumptions of the model (in particular a $\mu \geq 0$ and a constant volatility), this equilibrium is the unique sub-game perfect equilibrium in the economy.

A straightforward but tedious calculation shows that, when $N \to 1$, the solution given by **Proposition 3** coincides with the solution in DU. Therefore, the equilibrium of **Proposition 3** generalizes to the multi-agent setting the time-consistent equilibrium in DU.

3.1. The Solution and Comparative Statics

Proposition 3 specifies the procedure for determining the optimal aggregate insider ownership policy when commitment is not possible. The solution is obtained in terms of a system of coupled recursive relations. Since the aggregate investor is assumed to be risk-averse, the time-consistent solution can, in general, only be analyzed numerically.[15] Now we will convey the results of such an analysis. In particular, we describe three important properties of the model:

Property 1. The aggregate stake gradually declines towards the perfect risk sharing allocation (the long-term equilibrium).

Property 2. The long-term equilibrium allocation level increases with an increase in number of REIT insiders, *ceteris `paribus*.

Property 3. The initial speed of adjustment of the aggregate insider stake towards the long-term equilibrium level increases with an increase in number of REIT insiders, *ceteris `paribus*.

Property 1 follows from the fact that insiders cannot credibly commit to a level of ownership above the competitive allocation. The second property (see (10)) follows

[15] In a limit when the outside investors are risk-neutral the problem effectively decouples into N single-agent problems, each of them equivalent to a problem discussed in DU and EUW, which can be solved exactly.

intuitively from the fact that with an increase in a number of insiders *N,* each insider's risk exposure diminishes. This allows insiders to absorb more risk in aggregate. In order to explain Property 3, note that a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among insiders a "race to diversify". As a result, in the unique sub-game perfect equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. Intuitively, as there are more strategic agents in the economy, prices more quickly become competitive, although the adjustment towards the long-run equilibrium is gradual.

The model confirms that intuition. In Figure 2, we present the dynamics of the aggregate insider stakes when the number of insiders varies from N=1 to N=5.

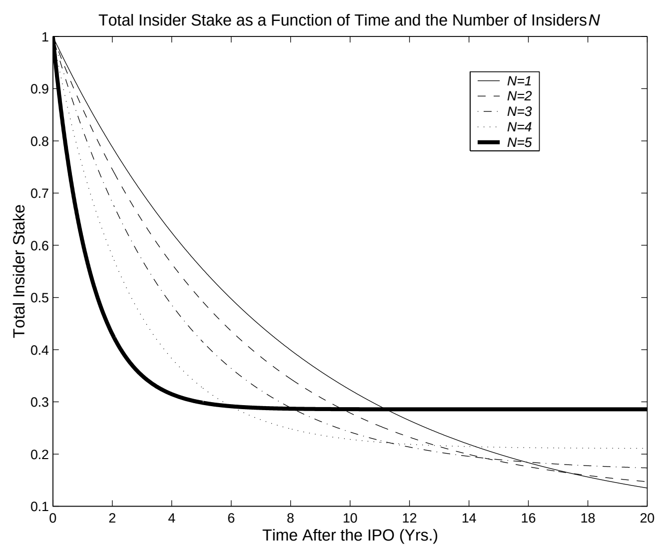

Figure 1: Aggregate Insider Ownership Policy Varies With the Number Of Insiders (Risk-Averse Investors). Here, $\gamma^I\sigma^2 r$=5, γ=15γ^I, μ=100, r=4%, and the number of insiders varies from N=1 to N=5. Notice that the speed of adjustment increases as the number of insiders in the company increases. At the same time, the long-term aggregate equilibrium allocation also increases. As a result, as the number of insiders increases, the aggregate insider stake adjusts relatively quickly to a relatively high long-term equilibrium level. Trading is quarterly.

While an increase in the number of insiders, *ceteris paribus*, raises the speed of adjustment towards the competitive allocation (10), it raises that level as well. This leads to an interesting empirical prediction. To wit, if outside investors are risk averse one would expect that companies with a relatively large number of (identical) insiders, *ceteris paribus*, should have a relatively short period of steep insider ownership adjustment towards a relatively high aggregate insider ownership level thereafter. In contrast, when the number of insiders in a company is relatively small, *ceteris paribus*, one would expect to observe slower adjustment towards a relatively low level of the aggregate insider ownership stake. Thus, if outside investors are risk-averse, the dynamics of the aggregate insider stake depends on the number of corporate insiders in the company.

4. Empirical Analysis

In this section we test the three key model predictions:

Prediction 1: The aggregate insider stake gradually declines towards a long-term equilibrium level.

Prediction 2: The long-term equilibrium level of the aggregate insiders' stake increases with the number of insiders.

Prediction 3: The initial speed of adjustment of the aggregate insiders' stake also increases with the number of insiders.

In order to test the predictions of the model empirically, we study the evolution of the aggregate insiders' ownership stake in 137 publicly traded U.S. REITs during the first ten years after with their IPO. We find support for **Predictions 1-3**: there is a significant reduction in the aggregate insiders' stake post-IPO. Moreover, the long-term aggregate ownership level of REIT insiders, as well as the initial speed of adjustment, is higher for REITs with a larger number of insiders than in REITs with a smaller number of insiders.

4.1. The Description of the Data

Our initial universe of companies consists of those publicly traded U.S. companies that appear in the *Center for Research in Security Prices (CRSP)* data file as REITs. All the market data fields come from the CRSP database.

We obtain insider ownership data using *Thomson Financial Insiders Filings (TFIF)* database on insider transactions between January 1986 and December 2004. According to Section 16(a) of the *Security and Exchange Act of 1934 (SEA),* large beneficial shareholders and managers of a publicly traded firm are required[16] to file their transactions in the company stock with the *Securities and Exchange Commission (SEC)*; these reports are collected in the TFIF dataset. One of the fields that insiders are supposed to fill is the resulting number of shares owned at the time of filing; using this information jointly with information on the number of shares acquired or disposed of at each transaction, we obtain estimates of insider positions in common shares of REITs[17]. After merging the estimated insider holdings at the end of each calendar month with the

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[16] The threshold for reporting is 5% ownership by non-managers, but some external shareholders choose to report their trades even when they hold less than 5% of a firm, as explained by Lakonishok and Lee (2001)

[17] The actual procedure that we follow to determine the total holdings for a particular insider for each document is the following:

1. To determine direct holdings we use those that are reported with the largest sequence number.
2. To determine indirect holdings (holdings of a trust, spouse, children) we aggregate all of the reported indirect holdings in the same document.

Note that the same document must contain both, direct and indirect holdings, if the later exist.

monthly data on shares outstanding from the CRSP file (correcting for stock splits), we compute the ownership stake of each insider at the end of each month.

For the purposes of our analysis of the first and second predictions we focus on companies with at least 10 years worth of post-IPO[18] data (a total of 59 REITs); for our analysis of the third prediction we focus on companies with 5 years worth of data post-IPO (a total of 130 REITs). Since we are interested in the behavior of insiders with significant company stakes, we eliminate from further consideration in the 5 years sample companies in which no insider owned more than 1% of company shares outstanding at any point in time during the first 5 years post-IPO, i.e. companies with no insiders with a significant stake post-IPO; and from further consideration in the 10 years sample companies in which no insider owned more than 1% of company shares outstanding at any point in time during the first 10 years post-IPO.[19]

In **Figure 2** we plot the distribution of IPOs in our sample. Whether measured by number of IPOs or by market capitalization, there is a concentration in the 1993-1994 and 1997-1998 periods. The largest IPO in our sample is that of *Equity Office Properties Trust* in 1997; the value of this REIT at the end of the first trading month was 4016.657 million dollars (expressed in real terms as Jan 1995 dollars). Although there were 4 IPOs in 1999, their size was quite small, only 110.4 million dollars (again expressed in Jan 1995 dollars).

In **Table 1** we show the distribution of companies in our sample by the initial number of significant insiders ($N(0)$). Most REITs in the sample have more than one significant insider; this is a motivation to construct a model like ours, which considers the strategic interaction between these multiple insiders. The number of large insiders is not large, on average; no more than ten percent of REITs have more than six significant insiders. Finally, there are three REITs in our sample that did not initially have a single significant insider at the IPO date.

The summary statistics on insider ownership are reported in **Table 2**. We report, on an annual basis, for the first five and ten years post IPO, the mean and the standard deviation (the latter quantities are given in cursive) of the number of insiders with significant stakes ($N(t)$), the aggregate insider stake ($A(t)$), the average insider stake ($A(t)/N(t)$), the largest stake owned by an insider ($\alpha^{\max}(t)$), and the stake of an insider with the largest holdings at the IPO date ($\alpha^{\text{ini}\max}(t)$). Here, t is the number of months that elapsed after the IPO. The average number of insiders with significant company stakes remains roughly constant (around 3 insiders per REIT), with a small increase over time. Just as the model predicts, the mean values of A, $\alpha^{\max}$, A/N, and $\alpha^{\text{ini}\max}$ decrease over time; on average A decreases by 18% during the first 5 years, and by 42% during the first 10

[18] We consider the IPO date to be the date of a company's first appearance in CRSP database.
[19] There are 7 REITs in which no insider owned more than 1% of company shares outstanding at any point in time during the first 5 years post-IPO, but they did at some point in time during the following 5 years. For this reason, we don't include all 137 REITs in our 5 years sample.

years after the IPO. The standard deviations of these quantities are large, which indicates a large cross-sectional variation across REITs.

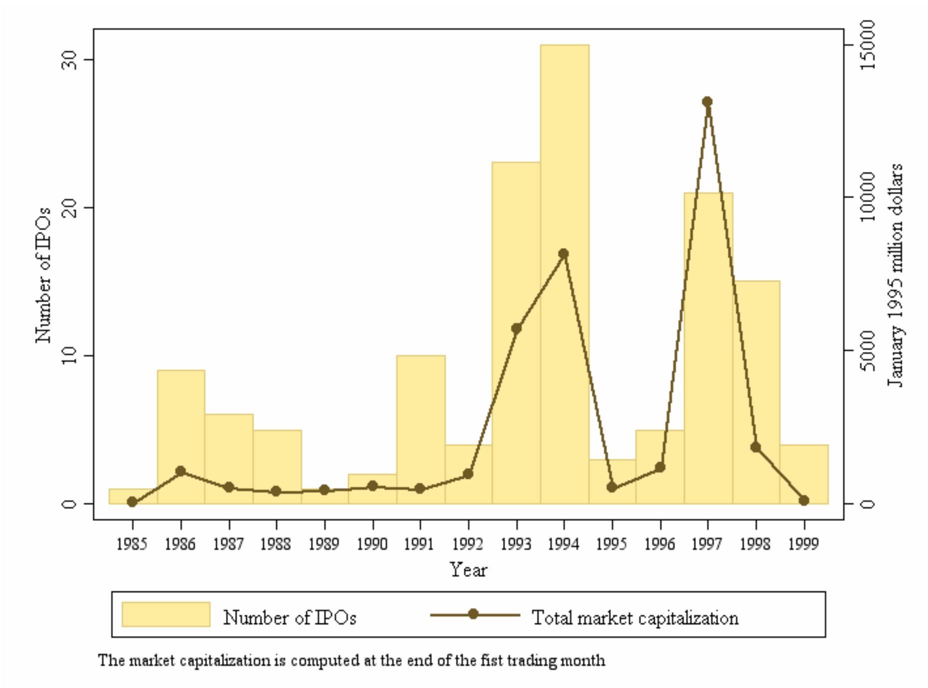

Figure 2: Distribution of the IPOs of the REITs in our sample.
We consider insiders that own at least 1% of shares outstanding at some point in time during the first 10 years post IPO. Our universe consists of 137 REITs that went public between January 1986 and December 1999, which appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership.

Table 1: Distribution of REITs by $N(0)$
The table shows the distribution of REITs by the number of insiders at the IPO date. We consider only those insiders that, at some point in time during the first 5 or 10 years post IPO, owned more than 1% of shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders' ownership, and for which at least one insider has a stake higher than 1% at some point in time in the first five or ten years after the IPO date.

	Subsample for which we have 5 years of data			Subsample for which we have 10 years of data		
$N(0)$	Number of REITs	Percentage	Cumulative	Number of REITs	Percentage	Cumulative
0	3	2.31	2.31	3	5.08	5.08
1	37	28.46	30.77	11	18.64	23.73
2	29	22.31	53.08	12	20.34	44.07
3	22	16.92	70	11	18.64	62.71
4	10	7.69	77.69	4	6.78	69.49
5	10	7.69	85.38	5	8.47	77.97
6	11	8.46	93.85	7	11.86	89.83
7	4	3.08	96.92	4	6.78	96.61
8	2	1.54	98.46	2	3.39	100
12	1	0.77	99.23			
Total	**130**	**100**		**59**	**100**	

Table 2: Descriptive statistics of insider ownership.

The mean and standard deviations (the latter quantity is in cursive) of the number of insiders (N), the aggregate insider stake (A), the average insider stake (A/N), the largest stake owned by an insider ($\alpha^{\max}$), and the stake of an insider with the largest holdings at the IPO date ($\alpha^{\text{ini max}}$). The first column (t) is time elapsed after the IPO (in months). The statistics are reported at the IPO date and every year up to the year five after the IPO. We consider only those insiders that, at some point in time during the first 5 or 10 years post IPO, owned more than 1% of shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders' ownership, and for which at least one insider has a stake higher than 1% at some point in time in the first five or ten years after the IPO date.

	Subsample for which we have 5 years of data					**Subsample for which we have 10 years of data**				
t	$N(t)$	$A(t)$	$A(t)/N(t)$	$\alpha^{\max}(t)$	$\alpha^{\text{ini max}}(t)$	$N(t)$	$A(t)$	$A(t)/N(t)$	$\alpha^{\max}(t)$	$\alpha^{\text{ini max}}(t)$
0	2.938	0.173	0.069	0.107	0.107	3.305	0.189	0.063	0.112	0.112
	2.127	*0.201*	*0.131*	*0.151*	*0.151*	*2.191*	*0.216*	*0.132*	*0.158*	*0.158*
12	3.115	0.163	0.06	0.098	0.095	3.492	0.174	0.05	0.097	0.091
	2.184	*0.184*	*0.108*	*0.131*	*0.131*	*2.216*	*0.189*	*0.079*	*0.12*	*0.12*
24	3.2	0.155	0.058	0.094	0.089	3.593	0.158	0.045	0.091	0.083
	2.187	*0.183*	*0.106*	*0.132*	*0.134*	*2.229*	*0.184*	*0.071*	*0.118*	*0.119*
36	3.215	0.151	0.061	0.092	0.081	3.627	0.14	0.041	0.077	0.067
	2.178	*0.17*	*0.109*	*0.122*	*0.122*	*2.274*	*0.157*	*0.063*	*0.085*	*0.086*
48	3.215	0.148	0.062	0.092	0.083	3.627	0.137	0.041	0.078	0.065
	2.178	*0.164*	*0.109*	*0.123*	*0.122*	*2.274*	*0.167*	*0.067*	*0.105*	*0.098*
60	3.223	0.142	0.059	0.088	0.078	3.644	0.13	0.038	0.07	0.06
	2.179	*0.165*	*0.108*	*0.123*	*0.122*	*2.288*	*0.159*	*0.056*	*0.095*	*0.091*
72						3.644	0.121	0.035	0.065	0.055
						2.288	*0.151*	*0.053*	*0.087*	*0.085*
84						3.644	0.124	0.036	0.069	0.056
						2.288	*0.162*	*0.057*	*0.101*	*0.097*
96						3.695	0.124	0.035	0.064	0.055
						2.373	*0.161*	*0.055*	*0.096*	*0.095*
108						3.695	0.122	0.035	0.063	0.055
						2.373	*0.157*	*0.054*	*0.094*	*0.095*
120						3.678	0.11	0.031	0.054	0.045
						2.438	*0.126*	*0.044*	*0.065*	*0.065*

In **Figure 2** we plot, for companies with initial number of significant insiders, $N(0)$, equal to 1, 2, 5 or 6, the aggregate insider stake as a function of the number of months elapsed after the IPO. We restrict ourselves to these values for $N(0)$ for the clarity of the graph. We observe that the aggregate company stake declines over time, and that the speed of adjustment towards the long run equilibrium is, on average, faster for companies with a higher initial number of significant insiders than for companies with a lower initial number of significant insiders. Finally, in REITs with a higher initial number of significant insiders, the long run aggregate insider stake tends to be higher, on average, than for companies with a smaller number of significant insiders.

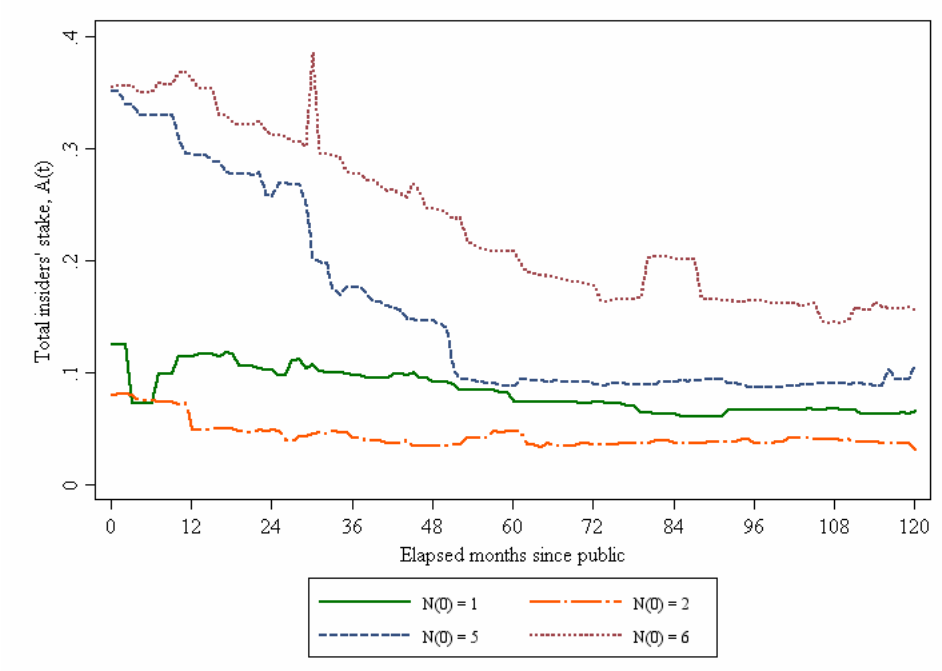

Figure 2: Dynamics of the aggregate insiders' ownership by the initial number of insiders, $N(0)$

We consider insiders that own at least 1% of shares outstanding at some point in time during the first 10 years post IPO. Our universe consists of 59 REITs that went public between January 1986 and December 1994, which appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership. In this graph, we only use data on the 35 REITs that had 1, 2, 5, or 6 significant insiders at the IPO date.

4.2. Evolution of aggregate insider ownership

Our model's first prediction is that the aggregate insider stake gradually declines over time. To test for the presence of a decline, we compare the mean aggregate insider stake across REITs at two dates: the IPO date (t=0) and at 120 months (or 10 years) after the IPO date (t=120). **Table 3** reports the results of a t-test for determining whether there is a statistically significant decline in the aggregate insider stake during the 120 months after an IPO. The null and the alternative hypotheses are, respectively Ho: $\overline{A(0)} - \overline{A(120)} \leq 0$ and Ha: $\overline{A(0)} - \overline{A(120)} > 0$, where we denote with an upper bar the mean across REITs. Given that the standard deviations of $A(0)$ and $A(120)$ differ from each other (see **Table 2**) we use a Welch's correction for different variances[20]. From the results in **Table 3** we conclude that one can reject the null hypothesis with a 99% confidence level (the p-value is equal to 0.0082). The aggregate insider ownership stake declines by 42% during the first 10 years after the IPO.[21]

[20] A description of Welch's correction can be found in Sanchs (1984), page 271. The results do not change if, instead of a Welch's correction we use a Satterthwaite's correction.

[21] Restricting the test to REITs with a large initial aggregate insider stake (say, above 30%), the obtained P-value of the test is even lower.

Because normality of A is rejected, we have regressed $A(t)$ on t and $\ln(t)$, see **Table 4**. The results of this regression also confirm the decrease in aggregate insiders' holdings as a function of the time elapsed since the IPO.

Table 3: Testing a decline in the aggregate insider stake

The table reports a *t*-test for determining whether a difference in the average aggregate insider stake at the time of an IPO, $A(0)$, is statistically different from the aggregate insider stake 120 months later, $A(120)$. We use a t-test with Welch's correction for different variances. We consider only insiders that, at some point in time during the first 10 years after the IPO own more than 1% of the shares outstanding. Our universe contains 59 REITs that went public between January 1986 and December 1994, that appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership.

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
***A*(0)**	59	0.1891	0.028	0.216	0.133	0.245
***A*(60)**	59	0.1096	0.016	0.126	0.077	0.142
Combined	118	0. 1494	0.017	0.180	0.116	0.182
Diff		0.0795	0.033		0.015	0.144
Welch's degrees of freedom:	94.76					

Ho: $\overline{A(0)} - \overline{A(60)} \leq 0$

Ha: $\overline{A(0)} - \overline{A(60)} > 0$

t = 2.4425 P-value = 0.0082

Table 4: Regression of aggregate insiders' stake on elapsed time since IPO

We consider only insiders that, at some point in time during the first 5 or 10 years after the IPO own more than 1% of the shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders' ownership, and for which at least one insider has a stake higher than 1% at some point in time in the first five or ten years after the IPO

	Subsample for which we have 5 years of data		Subsample for which we have 10 years of data	
	(1)	(2)	(3)	(4)
	$A(t)$	$A(t)$	$A(t)$	$A(t)$
$\ln(t)$	-0.009**		-0.022**	
	(0.002)		(0.002)	
t		-0.000**		-0.001**
		(0.000)		(0.000)
Constant	0.182**	0.170**	0.220**	0.171**
	(0.008)	(0.004)	(0.010)	(0.004)
Observations	7800	7930	7080	7139
R-squared	0.00	0.00	0.02	0.01

Robust standard errors in parentheses

+ significant at 10%; * significant at 5%; ** significant at 1%

The second prediction of the model can be formulated as follows: the long-run aggregate insider stake is higher for companies with a greater number of insiders and smaller for companies with a smaller number of insiders. We estimate the long-term equilibrium, A^*, as the average aggregate insiders' ownership during the 10th year after the IPO, that is

$A^* = \frac{1}{12}\sum_{t=109...120} A(t)$. In this test, we compare, for different positive integer values **N**, the mean long-run aggregate insider stake conditional on the initial number of insiders at *t*=0 being higher than **N,** $\overline{A^*_{N(0)>\mathrm{N}}}$, with the mean long-run aggregate insider stake conditional on the initial number of insiders not being larger than **N,** $\overline{A^*_{N(0)\le\mathrm{N}}}$. As before, we perform a t-test with Welch's correction in order to correct for a difference in variances to determine. The null and the alternative hypotheses are, respectively, Ho: $\overline{A^*_{N(0)>\mathrm{N}}} - \overline{A^*_{N(0)\le\mathrm{N}}} \le 0$, and Ha: $\overline{A^*_{N(0)>\mathrm{N}}} - \overline{A^*_{N(0)\le\mathrm{N}}} > 0$. In words, the null hypothesis states that the mean long-run aggregate insider stake is not larger for REITs with a greater initial number of insiders than for REITs with an initially smaller number of insiders. In **Table 5** we report the results of the *t*-test for **N**=3 to **N**=7. We find, consistent with our model, that average long-run aggregate insider stake is higher for REITs with a greater initial number of insiders than for REITs with a smaller number of insiders. The difference is statistically significant when the breakpoint for initial number of insiders is between 3 and 6, but it fails to be statistically significant if **N**=7.

Table 5: Testing a dependence of the mean long-run aggregate insider stake on the number of initial insiders $N(0)$

In this table we report the results of a t-test that aims to determine whether, for different positive integers N, long-run aggregate insider stake, A^*, depends on the initial number of insider in a REIT. The tests are performed with a Welch's correction that takes into the account difference in variances. The null and the alternative hypotheses are, respectively Ho: $\overline{A^*_{N(0)>\mathrm{N}}} - \overline{A^*_{N(0)\le\mathrm{N}}} \le 0$ and Ha: $\overline{A^*_{N(0)>\mathrm{N}}} - \overline{A^*_{N(0)\le\mathrm{N}}} > 0$. Here, $\overline{A^*_{N(0)>\mathrm{N}}}$ and $\overline{A^*_{N(0)\le\mathrm{N}}}$ are, respectively, the mean long-run aggregate insider stake in REITs with an initial number of insiders higher (respectively not higher) than N. P values are reported for different values of N=3 through 7. We estimate the long-term equilibrium, A^* as the average aggregate insiders' stake between 109 and 120 months after the IPO. We consider only insiders that, at some point in time during the 10 years after the IPO own more than 1% of the shares outstanding. Our universe contains 59 REITs that went public between January 1986 and December 1994, which appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership.

N	**3**	**4**	**5**	**6**	**7**
p-value	0.0063	0.0129	0.0064	0.0251	0.7863

In order to cross-check our results, we regress the long-run aggregate insider stake, A^*, on the initial number of insiders, *N*(0). We report the results of the regression in **Table 6**. As one can see, the coefficient for *N*(0) is positive and significant even controlling for the REIT size and the returns on the *National Association of Real Estate Investment Trusts* REITs Index during years 8 and 9 after the IPO. The coefficients for the size and index returns have the expected sign. According to this regression, the marginal contribution of an additional insider to the aggregate equilibrium holdings is around 1.3%.

Table 6: Regression of A^* on the initial number of insiders $N(0)$

Regression of the long-run aggregate insider stake, A^*, on the initial number of insiders, $N(0)$, the initial aggregate insiders' holdings, $A(0)$, natural logarithm of the REIT's size, $Size$, and the returns on the National Association of Real Estate Investment Trusts REITs Index during years 8 and 9 after the IPO, Ret. The regression is estimated using robust standard errors. We estimate the long-term equilibrium, A^* as the average aggregate insiders' stake between 109 and 120 months after the IPO. We consider only insiders that, at some point in time during the 10 years after the IPO own more than 1% of the shares outstanding. Our universe contains 59 REITs that went public between January 1986 and December 1994, which appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership.

	(1)	(2)	(3)	(4)	(5)
	A^*	A^*	A^*	A^*	A^*
$N(0)$	0.026**	0.017*	0.013+	0.017*	0.013+
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
$A(0)$		0.210**	0.219**	0.210**	0.219**
		(0.070)	(0.074)	(0.071)	(0.074)
$Size$			-0.025*		-0.025*
			(0.010)		(0.010)
Ret				0.018	0.038
				(0.151)	(0.138)
Constant	0.028	0.019	0.520*	0.015	0.515*
	(0.026)	(0.023)	(0.217)	(0.040)	(0.232)
Observations	59	59	59	59	59
R-squared	0.18	0.26	0.35	0.26	0.36

Robust standard errors in parentheses
+ significant at 10%; * significant at 5%; ** significant at 1%

The third prediction of our model is that the initial speed of adjustment of the aggregate insider stake towards the long-term equilibrium increases in the initial number of insiders, $N(0)$. We define the speed of adjustment during the first 5 years after the IPO by the expression: $\text{SpdAdj} = -(\ln(A(60)) - \ln(A(0)))$.

In **Table 7** we report the results of t-tests, for values of **N** from 3 to 7, that compare the initial speed of adjustment when the initial number of insiders is higher than **N**, $\overline{\text{SpdAdj}}_{N(0)>\text{N}}$, with the speed of adjustment when the initial number of insiders is lower than **N**, $\overline{\text{SpdAdj}}_{N(0)\leq\text{N}}$. The null and the alternative hypotheses are, respectively, Ho: $\overline{\text{SpdAdj}}_{N(0)>\text{N}} - \overline{\text{SpdAdj}}_{N(0)\leq\text{N}} \leq 0$, and Ha: $\overline{\text{SpdAdj}}_{N(0)>\text{N}} - \overline{\text{SpdAdj}}_{N(0)\leq\text{N}} > 0$. In words, the null hypothesis states that the speed of adjustment of the aggregate insider stake toward the long-run level is not larger in REITs with an initially larger number insider than in REITs with an initially smaller number of insiders. We find that the initial speed of adjustment is higher for REITs with a greater initial number of insiders and smaller for REITs with initially smaller number of insiders. In all the cases we can reject the null hypothesis with a 90% confidence level, and in all, but when **N**=5, with a confidence level of 95%.

Table 7: Performing t-tests to determine whether initial speed of adjustment depends on the initial number of insiders

T-tests for different mean of the initial speed of adjustment of the aggregate insiders' ownership towards the long-term equilibrium depending on the number of insiders at the IPO date, N(0). We perform the tests with a Welch's correction for different variances. We define the initial speed of adjustment of A as $\mathrm{SpdAdj} = -(\ln(A(60)) - \ln(A(0)))$. The null hypothesis is Ho: $\overline{\mathrm{SpdAdj}_{N(0)>\mathrm{N}}} - \overline{\mathrm{SpdAdj}_{N(0)\leq\mathrm{N}}} \leq 0$, and the alternative hypothesis Ha: $\overline{\mathrm{SpdAdj}_{N(0)>\mathrm{N}}} - \overline{\mathrm{SpdAdj}_{N(0)\leq\mathrm{N}}} > 0$, where $\overline{\mathrm{SpdAdj}_{N(0)>\mathrm{N}}}$ and $\overline{\mathrm{SpdAdj}_{N(0)\leq\mathrm{N}}}$ are, respectively, the mean speed of adjustment in REITs with more than N insiders at the IPO date and no more than N insiders at the IPO date. We consider only insiders that, at some point in time during the first 5 years of the REIT life, own more than 1% of the shares outstanding. Our universe contains 130 REITs that became public between January 1986 and December 1999, that appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership, 3 of these REITs have not been included in the regression because the aggregate ownership of significant insiders was zero at the IPO date.

N	**3**	**4**	**5**	**6**	**7**
p-value	0.0184	0.0027	0.0662	0.0296	0.0444

In **Table 8** we report the results of regressing the initial speed of adjustment on the initial number of insiders, N(0). As our model suggests, the coefficient for N(0) is positive and significant, even controlling for the REIT's size. According to this regression, the marginal contribution of one more insider to the speed of adjustment during the first five years of the REIT's life is around 17%.

Table 8: Regression of speed of adjustment of A during the first five years of the REIT's life on the initial number of insiders

We define the initial speed of adjustment of A as $\mathrm{SpdAdj} = -(\ln(A(60)) - \ln(A(0)))$. *Size* is the average, from monthly observations during the first 5 years of the REITs life, of the natural logarithm of the REIT's size. The regression has been estimated with robust standard errors. We consider only insiders that, at some point in time during the first 5 years of the REIT life, own more than 1% of the shares outstanding. Our universe contains 130 REITs that became public between January 1986 and December 1999, that appear in CRSP data files, and for which we have been able to construct time series of insiders' ownership, 3 of these REITs have not been included in the regression because the aggregate ownership of significant insiders was zero at the IPO date.

	(1)	(2)
	SpdAdj	SpdAdj
$N(0)$	0.174**	0.168*
	(0.065)	(0.067)
Size		0.196
		(0.120)
Constant	-0.495	-4.165+
	(0.320)	(2.144)
Observations	127	127
R-squared	0.04	0.06

Robust standard errors in parentheses

+ significant at 10%; * significant at 5%; ** significant at 1%

To sum up, the empirical evidence is consistent with the main empirical predictions of the model.

5. Conclusions and Future Work

This paper develops a model of optimal ownership dynamics of multiple equally risk-averse insiders facing a moral hazard problem and tests the predictions of the model on data from the U.S. REIT industry. On the theoretical side, the paper extends the related one-agent models (DU and EUW), to the situation with multiple strategic insiders. A solution for the equilibrium share price and the dynamics of the aggregate insider ownership stake is derived in two cases: when insiders can credibly pre-commit not to deviate from their optimal ownership policies, and in the more realistic case when such a commitment is not credible (i.e., the time-consistent case). In the latter case there is an additional strategic reason for a dynamic aggregate stake adjustment: A decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates a "race to diversify" and in equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. As a result: a) The aggregate insider stake declines over time; b) The long-run aggregate insider stake increases in the number of insiders and c) The speed of adjustment towards the long-run equilibrium level also increases in the number of insiders.

Theoretical predictions of the model are, then, tested on data from the U.S. REIT industry. To the best of our knowledge, no empirical work prior to this one has studied the evolution of insider ownership of REITs. Related empirical studies on REITs, for instance Cannon and Vogt (1995), Friday et all (1999), and Capozza and Seguin (2003), focus on static cross-sectional analysis of impact of ownership concentration on REIT prices, not on predicting REIT insider ownership dynamics. In addition, no paper previously studied the dependence of long-run level of insider ownership and/or the speed of adjustment towards that level on the number of insiders in a company (either for C-corporations or REITs).

There are several possible interesting extensions both of the theoretical model and the empirical analysis that would be worthwhile to pursue in the future. On the theoretical side, the model rests on a number of other simplifying assumptions that would be interesting to relax in the future. For tractability, for example, we assumed that the only strategic interaction between insiders occurs through their market risk premium impact. Including other types of interactions would make the model less tractable but, nevertheless, interesting to pursue.[22] Also, DU and, especially, EUW, demonstrated that the private benefits of control may have a significant impact on an insider's dynamic trading policy. Incorporating private benefits of control would allow one to gain insight into the dynamics of strategic corporate control issues.[23] Finally, the model is mute on the

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[22] For example, one could include in the expression for the expected dividends an interaction term proportional to $e_i e_j$. In that case, even when investors are risk-neutral, one would expect the model to exhibit a non-trivial strategic interaction between the agents.

[23] For example, it would be interesting to consider a dynamic strategic game between one owner that can extract private benefits of control (say, a manager) and another insider who cannot extract benefits of control but still is subject to a moral hazard problem (say, a worker). Such model could be of particular use when attempting to explain the evolution of ownership in transition economies. The work in these areas is currently in progress.

issue of the initial insider stake creation. These stakes appear naturally in an IPO mechanism (see Stoughton and Zechner (1998) and DU). It would be interesting to apply this model in the context of the IPO literature. On the empirical side, it is natural to extend the analysis from REITs to C-corporations. In particular, it may be interesting to pursue empirical work in the context of empirical IPO literature.

References

Admati, A., Pfeiderer, P. and Zechner, J. (1994), "Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium", *The Journal of Political Economy* **102**, 1097-1130.

Brav, A. and Gompers, P. A. (2003). The role of lockups in initial public offerings. *Review of Financial Studies*, 16(1):1–29.

Brunnermeier, M. K. and Pedersen, L. H. (2005). Predatory trading. *The Journal of Finance*, 60(4):1825–1863.

Cannon, S. E. and Vogt, S. C. (1995). REITs and their management: An analysis of organizational structure, performance, and management compensation. *Journal of Real Estate Research*, 10(3):297–318.

Capozza, D. R. and Seguin, P. J. (2003). Inside ownership, risk sharing and Tobin's q-ratios: Evidence from REITs. *Real Estate Economics*, 31(3):367.

Coase, R. (1972), "Durability and Monopoly", *Journal of Law and Economics*, **15**, 143-149.

DeMarzo, P. and Bizer, D. (1993), "Sequential Trade", (Mimeo, Kellogg School of Management).

DeMarzo, P. and Urošević, B. (2001), "Ownership Dynamics and Asset Pricing with a `Large Shareholder`", (Mimeo, Graduate School of Business (Stanford)).

Edelstein, R. H., Urosevic, B., and Wonder, N. (2005). Ownership dynamics of REITs. *The Journal of Real Estate Finance and Economics*, 30(4):447–466.

Franks, J., Mayer, C., and Rossi, S. (2005). Ownership: Evolution and regulation. ECGI - Finance Working Paper No. 09/2003; EFA 2004 Maastricht Meetings Paper No. 3205; AFA 2003 Washington, DC Meetings.

Friday, H. S., Sirmans, G. S., and Conover, C. M. (1999). Ownership structure and the value of the firm: The case of REITs. *Journal of Real Estate Research*, 17(1):71–90.

Harjoto, M. and Garen, J. (2005). Inside ownership beyond the ipo: the evolution of corporate ownership concentration. *Journal of Corporate Finance*, 11(2005):661-679.

Kihlstrom, R. (1998), "Monopoly Power in Dynamic Securities Markets", (Mimeo, The Wharton School of Business).

Mikkelson, W., Partch, M. and Shah, K. (1997), "Ownership and operating performance of companies that go public", *Journal of Financial Economics,* **44**, 281-307.

Pritsker, M. (2005). Large investors: Implications for equilibrium asset returns, shock absorption, and liquidity. Finance and Economics Discussion Series 2005-36, Board of Governors of the Federal Reserve System.

Sanchs, L. (1984). *Applied Statistics: A Handbook of Techniques*. Springer Series in Statistics. Springer-Verlag New York, LLC, second edition.

Seyhun, N. (1998), *Investment Intelligence from Insider Trading* (MIT Press: Cambridge, MA).

Stoughton, N. and Zechner, J. (1998), "IPO-Mechanisms, Monitoring and Ownership Structure", *Journal of Financial Economics,* **49**, 45-77.

Urošević, B. (2002), "Essays in Optimal Dynamic Risk Sharing in Equity and Debt Markets", (Ph.D. Thesis, University of California at Berkeley).

Vayanos, D. (2001), "Strategic Trading in a Dynamic Noisy Market", *Journal of Finance*, **56**, 131-171.

Vayanos, D. (1999), "Strategic Trading and Welfare in a Dynamic Market", *Review of Economic Studies*, **66**, 219-254.

Yung, C. (2005). Insider trading with private information and moral hazard. *Finance Research Letters*, 2(2):51–57.

Appendix

PROOF OF PROPOSITION 1: In this equilibrium, each agent maximizes her certainty equivalent taking into account other agents' equilibrium allocations. Denoting for simplicity, $\alpha^l = \alpha$ expression (7) follows from the expression for certainty equivalent (6) upon substituting the expression for the share price (5). Indeed:

$$k(\vec{\alpha}(t)) = \int_{[t,\infty]} e^{-r(\tau-t)} \left[z\left(\alpha(\tau),\beta(\tau)\right) d\tau - d\alpha(\tau) V\left(\vec{\alpha}(\tau)\right) \right]$$

$$= \int_{[t,\infty]} e^{-r(\tau-t)} z\left(\alpha(\tau),\beta(\tau)\right) d\tau - \int_{[t,\infty]} e^{-r(s-t)} \int_s^\infty e^{-r(\tau-s)} \nu\left(\alpha(\tau)+\beta(\tau)\right) d\tau \; d\alpha(s)$$

$$= \int_{[t,\infty]} e^{-r(\tau-t)} z\left(\alpha(\tau),\beta(\tau)\right) d\tau - \int_t^\infty e^{-r(\tau-t)} \left(\int_{[t,\tau]} d\alpha(s) \right) \nu\left(\alpha(\tau)+\beta(\tau)\right) d\tau$$

$$= \int_t^\infty e^{-r(\tau-t)} z\left(\alpha(\tau),\beta(\tau)\right) d\tau + \int_t^\infty e^{-r(\tau-t)} \left(\alpha(t-) - \alpha(\tau)\right) \nu\left(\alpha(\tau)+\beta(\tau)\right) d\tau \; ,$$

Given her own initial allocation α^- and the other agents' aggregate equilibrium allocation choice β, an agent's best response is given by the first order condition:

$$\alpha^c = \frac{r\gamma^I \sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}(1-\beta) + \frac{\mu + r\gamma^I\sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}\alpha^- \qquad \text{(A1)}$$

Summing up N identical equations (A1), taking into account that $\sum_l \alpha^l = A$ and $\sum_l \beta^l = (N-1)A$, and solving for the aggregate allocation A^c one obtains (8).◆

PROOF OF PROPOSITION 2: From (6), and a calculation similar to that in the proof of **Proposition 1**, it follows that if the price-taking equilibrium exists, the equilibrium allocation is given, for each insider, by the following expression (here, each insider is taking β and A^p as given):

$$\arg\max_\alpha z\left(\alpha,\beta\right) - \alpha\, \nu\left(A^p(\tau)\right).$$

This implies that, in such equilibrium, the optimality conditions $\gamma\alpha^p - \gamma^I(1-A^p) = 0$ need to be satisfied for each insider. In addition, an equilibrium condition is needed which states that the sum of each insider's holdings α^p is equal to the aggregate holdings A^p. Summing up N identical equations and solving for A^p renders (10) as well as $\alpha^p = A^p / N$. The second order condition reads: $\frac{\partial^2 z}{\partial \alpha^2}\left(\alpha,\beta\right) = (\mu - r\gamma\sigma^2) < 0$ which completes the proof. ◆

PROOF OF PROPOSITION 3: By backward induction. From (5) it follows that the share price at time T can be written as:

$$V\left(\vec{\alpha}(T)\right) \equiv V_T(\alpha_T, \beta_T) = v_{0T} + v_T A_T = v_{0T} + v_T(\alpha_T + \beta_T),$$
$$v_{0T} = -\gamma^I \sigma^2,\ v_T = \mu / r + \gamma^I \sigma^2 \tag{A2}$$

Note that the share price at the last trading date is an affine function of the aggregate insider holdings. Next, each insider chooses her holdings at time T in such a way as to maximize the certainty equivalent at time T, J_T:

$$J_T \equiv \max_{\alpha_T} z\ (\alpha_T, \beta_T)\delta_T + V_T(\alpha_T, \beta_T)(\alpha_{T-1} - \alpha_T) \tag{A3}$$

From (A3), the following first order conditions are obtained:

$$\frac{1}{r}\frac{\partial z}{\partial \alpha}(\alpha_T, \beta_T) + \frac{\partial V}{\partial \alpha}(\alpha_T, \beta_T)(\alpha_{T-1} - \alpha_T) - V(\alpha_T, \beta_T) = 0$$

Using the fact that $V_T(\alpha_T, \beta_T)$ is an affine function of α_T and β_T and that z is a quadratic function in α_T and linear in β_T (see (2)), the first order conditions become:

$$n_{1T}\alpha_T + n_{2T}\beta_T + n_{3T}\alpha_{T-1} + n_{4T} = 0, \text{ where}$$
$$n_{1T} = \mu / r - \gamma\sigma^2 - 2v_T,\ n_{2T} = \mu / r - v_T,\ n_{3T} = v_T,\ n_{4T} = -v_{0T} \tag{A4}$$

Summing up the equations (A4) and solving for the aggregate allocation, one can show that the aggregate insider stake at time T is an affine function of the aggregate insider holdings in the previous period:

$$A_T = -\frac{n_{3T}A_{T-1}}{n_{5T}} - \frac{Nn_{4T}}{n_{5T}}$$
$$n_{5T} \equiv n_{1T} + (N-1)n_{2T} \tag{A5}$$

Substituting (A5) back into (A4) leads to the following specifications of α_T and β_T in terms of the insiders' holdings at time T-1:

$$\alpha_T = l_T^{\alpha,\alpha}\alpha_{T-1} + l_T^{\alpha,\beta}\beta_{T-1} + l_T^{\alpha,0},\ \ \beta_T = l_T^{\beta,\alpha}\alpha_{T-1} + l_T^{\beta,\beta}\beta_{T-1} + l_T^{\beta,0} \tag{A6}$$

Here, coefficients l are defined as:

$$l_T^{\alpha,\alpha} = \frac{n_{3T}[n_{2T}(2-N) - n_{1T}]}{(n_{1T} - n_{2T})n_{5T}},\ l_T^{\alpha,\beta} = \frac{n_{2T}n_{3T}}{(n_{1T} - n_{2T})n_{5T}},\ l_T^{\alpha,0} = -\frac{n_{4T}}{n_{5T}}$$
$$l_T^{\beta,\alpha} = -\frac{n_{3T}}{n_{5T}} - l_T^{\alpha,\alpha},\ l_T^{\beta,\beta} = -\frac{n_{3T}}{n_{5T}} - l_T^{\alpha,\beta},\ l_T^{\beta,0} = -\frac{n_{4T}}{n_{5T}}N - l_T^{\alpha,0}$$

Therefore, the optimal holdings for each insider at time T depend on her own past holdings as well as on those of the other insiders. Importantly, this dependence is affine. In addition, each insider's optimal holdings are positively correlated with her own holdings at the preceding time period, i.e., $l_T^{\alpha,\alpha} > 0$, and negatively correlated with all of the other insiders' holdings at the preceding time period, i.e., $l_T^{\alpha,\beta} < 0$.

In order to proceed to t=T-1, note that the share price at time t=T-1 is, again, an affine function of the aggregate holdings. Indeed, from (5) and (A6) it follows that

$$V_{T-1}(\alpha_{T-1},\beta_{T-1}) = \delta v(A_{T-1}) + e^{-r\Delta} V_T(\alpha_T(\alpha_{T-1},\beta_{T-1}),\beta_T(\alpha_{T-1},\beta_{T-1})).$$

Using the definition of ν and utilizing (A2) and (A5), one can again express $V_{T\text{-}1}$ as a function of the aggregate holdings:

$$\begin{aligned}
&V_{T-1}(\alpha_{T-1},\beta_{T-1}) = v_{0T-1} + v_{T-1}A_{T-1} = v_{0T-1} + v_{T-1}(\alpha_{T-1}+\beta_{T-1})\\
&v_{0T-1} = -\delta\gamma^I\sigma^2 r + e^{-r\Delta}[v_{0T} - Nv_T n_{4T}/n_{5T}],\\
&v_{T-1} = \delta(\mu+\gamma^I\sigma^2 r) - e^{-r\Delta}(n_{3T}v_T/n_{5T})
\end{aligned} \tag{A7}$$

The value function J_T, obtained upon the substitution of the expressions for optimal holdings (A6) into the objective function (A3), is a quadratic form in $(\alpha_{T-1},\beta_{T-1})$:

$$J_T = J_T^{\alpha,\alpha}\alpha_{T-1}\alpha_{T-1} + J_T^{\alpha,\beta}\alpha_{T-1}\beta_{T-1} + J_T^{\beta,\beta}\beta_{T-1}\beta_{T-1} + J_T^{\alpha,0}\alpha_{T-1} + J_T^{\beta,0}\beta_{T-1} + J_T^0 \tag{A8}$$

Here, with some abuse of notation, we introduced on the right hand side *coefficients Js* while on the right hand side *J* is the value function. In order to establish (A8), it is sufficient to note that affine transformations map a quadratic form into another quadratic form. Then, (A8) follows immediately from (A2), (A3) and (A6).

So far, we have determined the optimal insiders' holdings at time *T* given their holdings as well as the holdings of all of the other insiders at time *T*-1. Clearly, at time *t*=*T*-1, insiders are facing a very similar problem, as portrayed in (A9):

$$J_{T-1} \equiv \max_{\alpha_{T-2}} z(\alpha_{T-1},\beta_{T-1})\delta + V_{T-1}(\alpha_{T-1},\beta_{T-1})(\alpha_{T-2}-\alpha_{T-1}) + e^{-r\Delta}J_T \tag{A9}$$

Note that the last term in (A9) is a quadratic form in $(\alpha_{T-1},\beta_{T-1})$ (see (A8)). Proceeding by backward induction one can obtain, eventually, the insiders' optimal holdings at time *t*=1 as a function of their initial holdings. Indeed, suppose that the relationships (11)-(13) and (A10)-(A13) are valid for *t*+1. One can easily see that, then, that they are valid for *t* as well. Indeed, each insider's maximization problem reads:

$$J_t \equiv \max_{\alpha_t} \delta z(\alpha_t,\beta_t) + V_t(\alpha_t,\beta_t)(\alpha_{t-1}-\alpha_t) + e^{-r\Delta}J_{t+1}$$

where, by assumption, (11) and (12) hold. Consequently, the optimal insiders' holdings are easily seen to yield (13) as well as (A10)-(A13). Utilizing these expressions it is immediate to see that the equilibrium share price at time *t*-1 is a linear function of the aggregate insider holdings at time *t* and that (A12). Using (13) and the linearity of the share price, value function J_t can be re-written as a quadratic form in variables $(\alpha_{t-1},\beta_{t-1})$. Reading off the appropriate coefficients in J_t establishes (A13). In order to establish the second order conditions, note that they are equivalent to $n_{1t} < 0$, where n_{1t} is given by the first expression in (A11). Using (A10)-(A13), as well as the fact that $\mu \geq 0$ and that volatility and the insiders risk aversion are positive constants, one shows, working backwards period by period, that $n_{1t} < 0$ and, thus, that the equilibrium is a unique sub-game perfect equilibrium under the assumptions of the model. Note that **Assumptions A-C** in DU under which they establish the existence and uniqueness of a sub-game perfect equilibrium for N=1 coincide with $\mu \geq 0$ and the constant volatility assumptions under which unique sub-game perfection exists for N>1 in this model.

The coefficients in (13) are determined as follows:

$$l_t^{\alpha,\alpha} = \frac{n_{3t}[n_{2t}(2-N)-n_{1t}]}{(n_{1t}-n_{2t})n_{5t}},\ l_t^{\alpha,\beta} = \frac{n_{2t}n_{3t}}{(n_{1t}-n_{2t})n_{5t}},\ l_t^{\alpha,0} = -\frac{n_{4t}}{n_{5t}}$$
$$l_t^{\beta,\alpha} = -\frac{n_{3t}}{n_{5t}} - l_t^{\alpha,\alpha},\ l_t^{\beta,\beta} = -\frac{n_{3t}}{n_{5t}} - l_t^{\alpha,\beta},\ l_t^{\beta,0} = -\frac{n_{4t}}{n_{5t}}N - l_t^{\alpha,0} \quad \text{(A10)}$$

The coefficients n_{it} are defined by the following relations:

$$n_{1t} = \delta(\mu_1 - \gamma\sigma^2 r) - 2v_t + 2e^{-r\Delta}J_{t+1}^{\alpha,\alpha},\quad n_{2t} = \delta\mu_1 - v_t + e^{-r\Delta}J_{t+1}^{\alpha,\beta},$$
$$n_{3t} = v_t,\quad n_{4t} = -v_{0t} + e^{-r\Delta}J_{t+1}^{\alpha,0},\quad n_{5t} = n_{1t} + (N-1)n_{2t} \quad \text{(A11)}$$

so that $A_t = -\frac{n_{3t}}{n_{5t}}A_{t-1} - \frac{n_{4t}}{n_{5t}}$.

The following are the recursive relations that define the coefficients in (12):

$$v_{0t-1} = -\delta\gamma^I\sigma^2 r + e^{-r\Delta}[v_{0t} - Nv_t n_{4t}/n_{5t}],$$
$$v_{t-1} = \delta(\mu + \gamma^I\sigma^2 r) - e^{-r\Delta}(n_{3t}v_t/n_{5t}) \quad \text{(A12)}$$

The set of 6 recursive relations that defines the coefficients in (11) is listed below and denoted by (A13):

$$J_t^{\alpha,\alpha} = \delta(\mu_1 - \gamma\sigma^2 r)(l_t^{\alpha,\alpha})^2/2 + \delta\mu_1 l_t^{\alpha,\alpha}l_t^{\beta,\alpha} + v_t(1-l_t^{\alpha,\alpha})(l_t^{\alpha,\alpha} + l_t^{\beta,\alpha}) +$$
$$+ e^{-r\Delta}[J_{t+1}^{\alpha,\alpha}(l_t^{\alpha,\alpha})^2 + J_{t+1}^{\alpha,\beta}l_t^{\alpha,\alpha}l_t^{\beta,\alpha} + J_{t+1}^{\beta,\beta}(l_t^{\beta,\alpha})^2]$$

$$J_t^{\alpha,\beta} = \delta(\mu_1 - \gamma\sigma^2 r)l_t^{\alpha,\alpha}l_t^{\alpha,\beta} + \delta\mu_1(l_t^{\alpha,\alpha}l_t^{\beta,\beta} + l_t^{\alpha,\beta}l_t^{\beta,\alpha}) + v_t(l_t^{\alpha,\beta} + l_t^{\beta,\beta} - 2l_t^{\alpha,\alpha}l_t^{\alpha,\beta} - l_t^{\alpha,\alpha}l_t^{\beta,\beta} - l_t^{\alpha,\beta}l_t^{\beta,\alpha})$$
$$+ e^{-r\Delta}[2J_{t+1}^{\alpha,\alpha}l_t^{\alpha,\alpha}l_t^{\alpha,\beta} + J_{t+1}^{\alpha,\beta}(l_t^{\alpha,\alpha}l_t^{\beta,\beta} + l_t^{\alpha,\beta}l_t^{\beta,\alpha}) + 2J_{t+1}^{\beta,\beta}l_t^{\beta,\alpha}l_t^{\bar{a},\bar{a}}]$$

$$J_t^{\beta,\beta} = \delta(\mu_1 - \gamma\sigma^2 r)(l_t^{\alpha,\beta})^2 / 2 + \delta\mu_1 l_t^{\alpha,\beta} l_t^{\beta,\beta} - v_t (l_t^{\beta,\beta} l_t^{\alpha,\beta} + (l_t^{\alpha,\beta})^2)$$
$$+ e^{-r\Delta}[J_{t+1}^{\alpha,\alpha}(l_t^{\alpha,\beta})^2 + J_{t+1}^{\alpha,\beta} l_t^{\alpha,\beta} l_t^{\beta,\beta} + J_{t+1}^{\beta,\beta}(l_t^{\beta,\beta})^2]$$

$$J_t^{\alpha,0} = \delta(\mu_1 - \gamma\sigma^2 r) l_t^{\alpha,\alpha} l_t^{\alpha,0} + \delta\mu_1 (l_t^{\alpha,\alpha} l_t^{\beta,0} + l_t^{\alpha,0} l_t^{\beta,\alpha}) + v_{0t}(1 - l_t^{\alpha,\alpha}) + v_t [l_t^{\alpha,0}(1 - l_t^{\beta,\alpha}) + l_t^{\beta,0} - 2 l_t^{\alpha,\alpha} l_t^{\alpha,0} - l_t^{\alpha,\alpha} l_t^{\beta,0}] +$$
$$+ e^{-r\Delta}[2 J_{t+1}^{\alpha,\alpha} l_t^{a,a} l_t^{\alpha,0} + J_{t+1}^{\alpha,\beta}(l_t^{\alpha,\alpha} l_t^{\beta,0} + l_t^{\beta,\alpha} l_t^{\alpha,0}) + 2 J_{t+1}^{\beta,\beta} l_t^{\beta,\alpha} l_t^{\beta,0} + J_{t+1}^{\alpha} l_t^{\alpha,\alpha} + J_{t+1}^{\beta} l_t^{\beta,\alpha}]$$

$$J_t^{\beta,0} = \delta(\mu_1 - \gamma\sigma^2 r) l_t^{\alpha,\beta} l_t^{\alpha,0} + \delta\mu_1 (l_t^{\alpha,\beta} l_t^{\beta,0} + l_t^{\alpha,0} l_t^{\beta,\beta}) - v_{0t} l_t^{\alpha,\beta} - v_t [l_t^{\alpha,0} l_t^{\beta,\beta} + 2 l_t^{\alpha,\beta} l_t^{a,0} + l_t^{\alpha,\beta} l_t^{\beta,0}] +$$
$$+ e^{-r\Delta}[2 J_{t+1}^{\alpha,\alpha} l_t^{\alpha,\beta} l_t^{\alpha,0} + J_{t+1}^{\alpha,\beta}(l_t^{\beta,\beta} l_t^{\alpha,0} + l_t^{\alpha,\beta} l_t^{\beta,0}) + 2 J_{t+1}^{\beta,\beta} l_t^{\beta,\beta} l_t^{\beta,0} + J_{t+1}^{\alpha} l_t^{\alpha,\beta} + J_{t+1}^{\beta} l_t^{\beta,\beta}]$$

$$J_t^{0} = \delta(\mu_1 - \gamma\sigma^2 r)(l_t^{\alpha,0})^2 / 2 + \delta\mu_1 l_t^{\alpha,0} l_t^{\beta,0} - v_{0t-1} l_t^{\alpha,0} - v_t ((l_t^{\alpha,0})^2 + l_t^{\alpha,0} l_t^{\beta,0})$$
$$+ e^{-r\Delta}[J_{t+1}^{\alpha,\alpha}(l_t^{\alpha,0})^2 + J_{t+1}^{\alpha,\beta} l_t^{\alpha,0} l_t^{\beta,0} + J_{t+1}^{\beta,\beta}(l_t^{\beta,0})^2 + J_{t+1}^{\alpha} l_t^{\alpha,0} + J_{t+1}^{\beta} l_t^{\beta,0} + J_{t+1}^{0}]$$

(A13)

The boundary conditions read as follows: coefficients J_{t+1} vanish at time $t=T$; the boundary values for the stock price coefficients are given by (12) and the initial insiders' allocations are given by an (exogenous) vector $\boldsymbol{\alpha}^-$. ◆