Governance Through Exit and Voice: A Theory of Multiple Blockholders*

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Abstract

Traditional theories argue that governance is strongest under a single large blockholder, as she has strong incentives to undertake value-enhancing interventions (engage in "voice"). However, most firms are held by multiple small blockholders. This paper shows that, while such a structure generates free-rider problems that hinder voice, the same co-ordination difficulties strengthen a second governance mechanism: disciplining the manager through trading (engaging in "exit"). Since multiple blockholders cannot co-ordinate to limit their orders and maximize combined trading profits, they trade competitively, impounding more information into prices. This strengthens the threat of disciplinary exit, inducing higher managerial effort. The optimal blockholder structure depends on the relative effectiveness of manager and blockholder effort, the complementarities in their outputs, information asymmetry, liquidity, monitoring costs, and the manager's contract.

KEYWORDS: Multiple blockholders, corporate governance, market efficiency, exit, voice, free-rider problem, Wall Street Rule, voting with your feet

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1 Introduction

Corporate governance can have substantial effects on firm value. Through ensuring that managers act in shareholders' interest, it reduces the agency costs arising from the separation of ownership and control. In turn, traditional theories argue that concentrated ownership is critical for effective governance, since only large investors have incentives to monitor the manager and, if necessary, intervene to correct value-destructive actions.

However, many firms in reality have multiple small blockholders (see, e.g., Faccio and Lang (2002), Maury and Pajuste (2005), Laeven and Levine (2007), and Holderness (2009)). Such a structure appears to be suboptimal for governance, as splitting equity between numerous shareholders leads to a free-rider problem: each investor individually has insufficient incentives to bear the cost of monitoring, and shareholders cannot coordinate to share this cost. Should policymakers encourage more concentrated stakes, as suggested by existing models, or can such a structure in fact be efficient? In addition, the evidence also demonstrates heterogeneity in blockholder structures. What causes the number of blockholders to vary across firms?

These questions are the focus of this paper. We demonstrate that a multiple blockholder structure may be optimal for governance, and identify the factors that determine the optimal blockholder structure. While splitting a block reduces the effectiveness of direct intervention ("voice"), we show that it increases the power of a second governance mechanism: "exit".[1] By trading on private information, blockholders move the stock price towards fundamental value, and thus cause it to more closely reflect the effort exerted by the manager to enhance firm value. If the manager shirks or extracts private benefits, the blockholders follow the "Wall Street Rule" of "voting with their feet" and selling to liquidity traders. This drives down the stock price, reducing the manager's equity compensation and thus punishing him ex post. However, such a mechanism only elicits effort ex ante if it is dynamically consistent. Once the manager has taken his action, blockholders cannot change it and are only concerned with maximizing their trading profits. A single blockholder will strategically limit her order to reduce the revelation of her private information. This optimizes her profit, but also lowers the extent to which prices reflect fundamental value and thus managerial effort. By contrast, multiple blockholders trade aggressively to compete for profits, as in a Cournot oligopoly. Total quantities (here, trading volumes) are higher than under monopoly, so more information is impounded in prices. Multiple blockholders thus serve as a commitment device to

[1] Prior papers on blockholder trading focus on the "Wall Street Rule" (the possibility of blockholder exit), rather than additional purchases. For example, Hirshman's (1970) book is titled "Exit, Voice, and Loyalty", and the models of Admati and Pfleiderer (2009) and Edmans (2009) only analyze block disposal, not enhancement. Although the blockholder can buy as well as sell in this paper, we use the term "exit" to describe the blockholder's influence on managerial decisions through her trading (in either direction), to be consistent with prior literature.

reward or punish the manager ex post for his actions.

The co-ordination problems and externalities created by splitting a block play opposing roles in voice and exit. For voice, the externalities are positive: intervention improves the value of other shareholders' stakes, but this effect is not internalized by the individual blockholder. Since these externalities are positive, there is "too little" intervention with multiple blockholders. For exit, the externalities are negative. Higher trading volumes reveal more information to the market maker, leading to a less attractive price for other informed traders. Blockholders trade "too much" from the viewpoint of maximizing combined profits. However, firm value does not depend on trading profits as they are a mere transfer from liquidity traders to blockholders. Instead, "too much" trading is beneficial as it increases price informativeness and induces effort ex ante. The 2007 hedge fund crisis is a prominent example of the substantial price changes that result from multiple investors trading in the same direction.

We derive an interior solution for the optimal number of blockholders that maximizes firm value. This optimum arises from a trade-off between voice and exit: fewer blocks maximize intervention incentives, but more blocks increase trading. Therefore, it is increasing in the value created by managerial effort and decreasing in the value created by blockholder intervention. If blockholders are passive and non-interventionist, as with most mutual funds, a large number is optimal. By contrast, if investors contribute significantly to the firm's operations, such as activists and venture capitalists, concentrated ownership is efficient. The optimal number is also increasing in the manager's alignment with the stock price, since this augments the importance of stock price informativeness for the manager's effort choice.

In the core model, blockholders are automatically informed about firm value. We extend the model to allow for costly information acquisition. In equilibrium some blockholders may decide to stay uninformed, because their trading profits are insufficient to justify monitoring. Since uninformed blockholders do not engage in exit, and reduce intervention by diluting ownership, they unambiguously reduce firm value. Thus, the optimal number of blockholders is bounded above, to ensure that competition in trading is sufficiently low that trading profits are adequate to motivate all blockholders to acquire information. If trading profits (net of monitoring costs) increase, this bound is loosened and so the number of blockholders rises. This in turn occurs if market liquidity and the blockholders' informational advantage increase, and monitoring costs fall.

The core model assumes that blockholder and manager efforts are substitutes, with independent effects on firm value. For example, the firm value impact of managerial efforts to launch new products is unaffected by the extent to which blockholders extract private benefits or monitor managerial perks. However, in some cases there may be positive complementarities, where the marginal productivity of one party's effort is increasing in the output level of the other party - for example, the blockholder formulates a strategy which is then implemented by the manager. Since managerial effort is only productive if it is accompanied by high blockholder effort (and vice versa), the optimal number of blockholders balances the output levels of both parties. The effect of effort productivity changes direction: the optimal number is now decreasing in the effectiveness of managerial effort and increasing in the effectiveness of blockholder effort. If blockholder effort is ineffective, a low number is necessary to “boost” blockholder output so that it is at a similar level to the manager’s output.

The opposite case is negative complementarities, where the marginal productivity of one party’s effort is decreasing in the other party’s output. This occurs if blockholders correct managerial shirking: blockholders are most effective if the manager exerts low effort or consumes private benefits, and the manager’s rent extraction decision has greatest effect on firm value if blockholders are not monitoring. We model negative complementarities by specifying that firm value depends only on the higher of the output levels of the two parties. Therefore, the optimum is determined entirely by the more effective action, and ignores trade-off considerations with the less effective action. The efficient number of blockholders is therefore either very low (if blockholder effort is relatively effective) or very high (if managerial effort is relatively effective).

We show that the firm value optimum may differ from the socially optimal number of blockholders that maximizes total surplus (firm value net of effort costs), and the private optimum that would be chosen by the blockholders if they retraded their stakes to maximize their combined net payoffs. However, the comparative statics with respect to the effectiveness of manager and blockholder effort are the same for all three optima. This is important for the paper’s empirical predictions, since the private optimum is likely to be observed in reality.

We close by discussing these implications, which fall under two broad themes. First, the model suggests a different way of thinking about the interaction between multiple blockholders, that can give rise to new avenues for empirical research. Prior models perceive blockholders as entities that compete for private benefits, and so existing empirical studies of multiple blockholders typically focus on rent extraction (e.g. Laeven and Levine (2007)). Our paper suggests that future analyses may be motivated by conceptualizing them as informed traders, competing for trading profits. The model predicts that blockholder structure impacts price efficiency and consequently firm value, and their power in exerting governance depends on microstructure factors such as liquidity. One recent example of such a research direction is Gallagher, Gardner and Swan (2008), who show that an increase in the number of blockholders reduces trading profits, augments price efficiency, and leads to subsequent improvements in firm performance. Similarly, Smith and Swan (2008) show that trading by multiple blockholders disciplines managerial compensation.

Second, the theory implies that the number of blockholders is important as both a dependent and independent variable in empirical studies. Existing research often focuses on explaining total institutional ownership or the size of the largest blockholder. This paper suggests that the number of blockholders is another important feature of governance structures and generates testable predictions for the factors that should cause blockholder structure to vary across firms, potentially explaining the heterogeneity observed empirically.

As an independent variable, the number of blockholders is a relevant determinant of market efficiency or the strength of corporate governance. Empirical papers frequently use total institutional ownership as a gauge of price efficiency, since institutions are typically more informed than retail investors. However, market efficiency requires not only that investors be informed, but that they impound their information into prices and so the number of informed shareholders is a relevant (additional) factor. Similarly, governance is typically proxied for using total block ownership, or the holding of the largest shareholder, but the number of blockholders is also an important factor.

This paper is organized as follows. Section 2 reviews related literature. Section 3 presents the model and analyzes the effect of blockholder structure on both "voice" and "exit". Section 4 derives the optimal number of blockholders that maximizes firm value, total surplus, and the blockholders' payoff. Section 5 extends the model to analyze costly information acquisition, complementarities and differences in the manager's contract, Section 6 considers empirical implications, and Section 7 concludes. The Appendix contains all proofs not in the main text.

2 Related Literature

The vast majority of blockholder models involve the large shareholder adding value through direct intervention, or "voice" as termed by Hirshman (1970). This can involve implementing profitable investment projects and strategies, or overturning an inefficient managerial action. In Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Maug (1998), Kahn and Winton (1998) and Mello and Repullo (2004), a larger block is unambiguously more desirable as it reduces the free-rider problem and maximizes incentives to intervene.

By contrast, Burkart, Gromb and Panunzi (1997) show that the optimal block size is finite if blockholder intervention can deter managerial initiative ex ante. Bolton and von Thadden (1998) and Faure-Grimaud and Gromb (2004) achieve a finite optimum through a different channel, as too large a block reduces free float. While these papers only consider a single shareholder , Pagano and Roell (1998) point out that if the finite optimum is lower than the total amount of external financing required, the entrepreneur will need to raise funds from additional shareholders. Although this leads to a multiple blockholder structure, the extra blockholders play an entirely passive role: they are merely a "budget-breaker" to provide the remaining funds. Replacing the additional blockholders by creditors or dispersed shareholders would have the same effect. In this paper, all blockholders play an active role. In Winton (1993), as in our model, all investors actively monitor, and total monitoring is highest under a single large shareholder owing to the free-rider problem. A multiple blockholder structure arises as investors face wealth constraints, rather than from concerns over price efficiency.

Two recent papers by Admati and Pfleiderer (2009) and Edmans (2009) analyze an alternative mechanism through which blockholders can exert governance: "exit". Informed trading causes prices to more accurately reflect fundamental value, in turn inducing the manager to undertake actions that enhance value.[2] The survey evidence of McCahery, Sautner and Starks (2008) finds that exit is the primary governance mechanism used by institutions, and Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) document direct evidence of this channel. However, Admati and Pfleiderer and Edmans both consider a single blockholder and do not feature "voice".

Attari, Banerjee and Noe (2006), Faure-Grimaud and Gromb (2004), and Aghion, Bolton and Tirole (2004) feature a blockholder who can only intervene and a speculative agent who can only trade. The blockholder does not trade; even though the speculator does, such trading does not exert governance as there is no managerial decision. These theories thus consider voice only. Noe (2002) features multiple blockholders who both intervene and trade. Since stock price informativeness has no effect on managerial effort, blockholder exit again does not exert governance.[3] In our model, all blockholders engage in both intervention and trading, and the latter affects the manager's incentives; blockholders thus govern through both exit and voice. Indeed, McCahery et al. find that institutional blockholders use both governance mechanisms frequently.[4] To our knowledge, this paper is the first theory that analyzes both of these major governance mechanisms, and the tradeoffs between them.

Most existing theories of multiple blockholder structures focus on the formation of coalitions to extract private benefits: examples include Zwiebel (1995), Bennedsen and Wolfenzon (2000), Müller and Wärneryd (2001), Bloch and Hege (2003), Maury and

[2] In Holmstrom and Tirole (1993), Calcagno and Heider (2008) and Ferreira, Ferreira and Raposo (2008), price efficiency is also desirable as it helps monitor management. In Fulghieri and Lukin (2001), efficient prices reduce the cost of raising funds for a high-quality firm. These papers do not analyze the effect of blockholder structure on price efficiency and there is no blockholder intervention.

[3] Similarly, the single blockholder models of Maug (1998, 2002), Kahn and Winton (1998), Mello and Repullo (2004), Brav and Mathews (2008), and Kalay and Pant (2008) allow the blockholder either to intervene or to sell her stake (in the last two papers, the intervention occurs through voting). However, exit again does not exert governance, and so these papers are theories of voice only.

[4] While exit is the primary mechanism (undertaken by 80% of institutions), 66% vote against management and 55% engage in discussions with the board. Six other channels of "voice" as used by at least 10% of respondents. Institutions can both trade freely on information and engage in voice because the above intervention mechanisms do not require them to have a board seat and become a firm insider.

Pajuste (2005) and Gomes and Novaes (2006). This paper derives a multiple blockholder structure through a quite different channel – analyzing its effect on governance through exit, rather than control contests. By studying different blockholder actions, the model generates a new range of empirical predictions. One set relates to the stock market: the number of blockholders affects price efficiency and trading profits, and the optimal structure depends on microstructure features such as liquidity and the blockholders' information advantage. Furthermore, in this model, blockholders create value indirectly through influencing managerial behavior as well as directly through exerting effort (or consuming private benefits). Therefore, the optimal shareholding structure depends on the relative effectiveness of blockholder and manager effort.[5]

We now turn from related theories to the empirical facts that motivate our model. Table 1 illustrates the prevalence of multiple blockholders using U.S. data for 2001 from Dlugosz et al. (2006). They define a blockholder as a shareholder with at least 5% of the firm's equity. The table illustrates that 70% of firms have multiple blockholders, and 26% of firms have at least four blockholders. Focusing on outside blockholders, these figures remain significant at 57% and 17%. Hence, not only do most firms have multiple blockholders, but even among such firms, the number of blockholders varies. Therefore, we seek not only to show that a multiple blockholder structure can be optimal, but also explain why blockholder numbers vary across firms. Hand-collected data from Holderness (2009) gives consistent results, showing that 74% of firms having multiple blockholders and 26% have at least four blockholders.[6]

While the data in Table 1 and from Holderness (2009) are confined to the U.S., Laeven and Levine (2007) find that 34% of European firms have more than one blockholder; Maury and Pajuste (2005) document a figure of 48% for Finnish firms. Using Western European data made available by Faccio and Lang (2002) we find a similar ratio of 39%. (Laeven and Levine and Faccio and Lang also document cross-country variation in blockholder structures, providing further evidence of heterogeneity.) These figures are also significant but somewhat lower than the U.S. data, because the above papers require an investor to have at least 10% of the voting rights to be a blockholder, in part motivated by existing theories based on control contests. While a 10% stake may be necessary to form a majority coalition, in our model a blockholder is simply a shareholder who has greater information than the market and so the lower threshold of Dlugosz et al. seems appropriate. Note that even a stake below 5% may often be suffi-

[5]Bolton and Scharfstein (1996) also demonstrate that free-rider problems among investors can improve firm value. A multiple creditor structure can dominate a single lender, since the resulting co-ordination problems hinder efficient renegotiation in default. This deters the manager from strategically defaulting, and thus makes creditors more willing to lend. In our paper, the benefits of co-ordination problems manifest through informed trading and the effect on stock prices.

[6]The Holderness (2009) paper does not contain the frequency of multiple blockholders. We thank Cliff Holderness for providing us with these figures using his underlying data.

	All blockholders		Outside blockholders	
N	Number of firms with N blockholders	% of firms with $\geq N$ blockholders	Number of firms with N blockholders	% of firms with $\geq N$ blockholders
0	152	100%	249	100%
1	217	88%	289	80%
2	287	70%	284	57%
3	264	47%	213	34%
4	170	26%	116	17%
5	88	12%	62	7%
6	40	5%	18	2%
7	17	2%	7	1%
8	4	0%	2	0%
9	1	0%	0	0%

Table 1: Frequency of multiple blockholders for 1,240 U.S. firms. This table reports the frequency of blockholder structures for U.S. firms in 2001 using data from Dlugosz et al. (2006).

cient to gain access to management or give the investor sufficient incentives to analyze the firm in detail (for example, mutual funds typically hold under 5%). Under a lower threshold, the prevalence of multiple blockholders and heterogeneity in structures will be even greater.

In sum, data from both the U.S. and Europe documents a significant prevalence of multiple blockholders. We seek to justify the prevalence of multiple blockholders as well as explain cross-sectional variation in multiple blockholder structures without featuring coalitions and control contests. Since our model does not assume control contests, it may apply to blockholders with less than 5% and suggests that in certain empirical applications, blockholder studies may wish to use 13f data to identify sizable shareholders below the 5% threshold.

3 Model and Analysis

Our model consists of a game between the manager, a market maker and the I blockholders of the firm. The game has two stages, and the timeline is given in Figure 1.

In the first stage, the manager and blockholders take actions that affect firm value. Firm value is given by

$$\tilde{v} = \phi_a \log a + \phi_b \log \sum_i b_i + \tilde{\eta}, \tag{1}$$

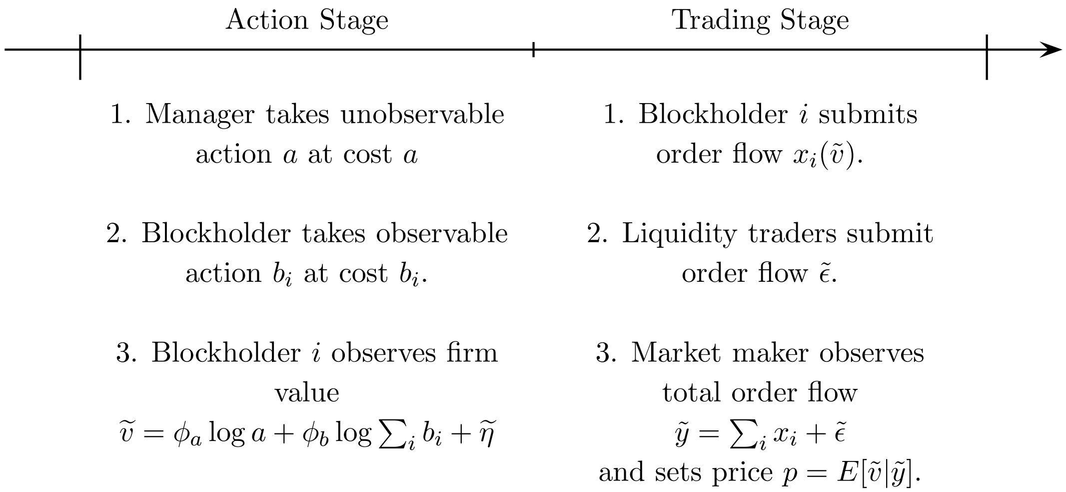

Figure 1: Timeline of the model

where $a \in [0, \infty)$ represents the action taken by the manager, $b_i \in [0, \infty)$ represents the action taken by blockholder i, and $\tilde{\eta}$ is normally distributed noise with mean zero and variance σ_η^2. The manager incurs personal cost a when taking action a, while each blockholder i incurs personal cost b_i when taking action b_i.[7] The manager's action is broadly defined to encompass any decision that improves firm value but is personally costly, such as exerting effort or forgoing pet projects and private benefits. We call these actions "initiative" and "managerial rent extraction" respectively. Similarly, the blockholder's action can involve advising the manager ("advising"), inhibiting perk consumption or pet projects ("perk prevention") or extracting private benefits ("blockholder rent extraction.")[8] Section 6 discusses which types of action will likely be most important in a particular setting, depending on the firm type and blockholder and manager characteristics. The parameter ϕ_a (ϕ_b) measures the productivity of manager (blockholder) effort. We use the term "effort" to refer to a and b_i and "output" to refer to $\phi_a \log a$ and $\phi_b \log \sum_i b_i$, i.e. effort scaled by its productivity.

In the core model, the manager's and blockholders' actions are perfect substitutes,

[7]Firm value depends on the logarithm of the combined blockholder effort level, and the action has a linear cost to each blockholder. This functional form ensures that adding blockholders does not change the available technology (in addition, it leads to substantial tractability). The common assumption of a quadratic cost and a linear effect of b_i on $\tilde{v}$ is inappropriate here: with a convex cost function, the blockholders' technology would improve if there are multiple small blockholders, since each would be operating at the low marginal cost part of the curve. A single blockholder would be able to reduce monitoring costs by dividing herself up into multiple small "units", and increase total effort. Instead, the linear cost means that the monitoring technology is constant, and so there are no mechanical reduction in monitoring costs from splitting a block.

[8]See Barclay and Holderness (1989) for a description of the private benefits that blockholders can extract. Unlike in earlier theories of multiple blockholders, here blockholders do not compete (with either each other or the manager) to consume private benefits.

i.e. have independent effects on firm value. This benchmark case is appropriate in a number of settings. For example, if the main way in which blockholders impact the firm is through rent extraction, this erodes firm value regardless of the manager's initiative or rent extraction. If the most important managerial action is initiative (e.g. designing new products or building client relationships) and the main blockholder action is perk prevention, these are also independent. However, in some situations, there may be positive or negative complementarities between the manager's and blockholders' actions. These are analyzed in Section 5.2.

Action a is privately observed by the manager, while actions b_i are publicly observed but unverifiable. The assumption that a is private is a feature of any moral hazard problem. By contrast, the assumption that b_i is public is made purely for tractability. The effects of I on competition in trading and free-rider problems in intervention are independent of whether or not b_i is observable.[9]

We normalize the number of shares outstanding to 1. The risk-neutral manager owns α shares of the firm, and each risk-neutral blockholder holds β/I shares, where $\alpha + \beta < 1$. Our model focuses exclusively on the optimal number of blockholders (I) among which a given level of concentrated ownership is divided, and thus holds the amount of concentrated ownership (β) constant. This separates our paper from previous literature that analyzes the optimal β. For example, Shleifer and Vishny (1986) and Maug (1998) show that a higher β raises incentives to intervene, but this must be traded off against the potential reduction in managerial initiative (Burkart, Gromb and Panunzi (1997)) and free float (Bolton and von Thadden (1998), Faure-Grimaud and Gromb (2004), Edmans (2009)). In this model, free float is fixed at $1 - \alpha - \beta$ and plays no role. Endogenizing β and allowing liquidity (introduced shortly) to depend on free float will lead to the same trade-off as these earlier papers.[10]

In the second stage of the game, the blockholders, noise traders, and a market maker trade the firm's equity. As in Admati and Pfleiderer (2009), each blockholder is assumed to observe firm value $\tilde{v}$ perfectly, while noise traders are uninformed. The blockholders' superior information can be motivated by a number of underlying assumptions. Their large stakes may give them greater access to information: given their voting power, management will be more willing to meet with them. Even if blockholders have the same access to information as other investors, they may be more informed as they have

[9] If b_i is private, the analysis becomes significantly more complex as it involves mixed strategies. As in Maug (1998, 2002), each blockholder randomizes between intervention and non-intervention, and the market maker's pricing rule reflects this.

[10] We could also extend the model by introducing managerial risk aversion and endogenizing α. Then, the increased price efficiency that results from a greater number of blockholders reduces the risk imposed by aligning the manager with equity value. The optimal α is greater, further inducing managerial effort. Since the effect of price efficiency on α is featured in Holmstrom and Tirole (1993) and Calcagno and Heider (2008) and further reinforces the effects in this paper, we hold α constant.

stronger incentives to engage in costly analysis of this information. For example, mutual funds undertake detailed analysis of public information to form their own financial projections and valuations. Edmans (2009) microfounds this relationship between block size and informedness. If there are short-sales constraints (or nontrivial short-sales costs), blockholders can sell more if information turns out to be negative. Since information is more useful to them, they have a greater incentive to acquire it in the first place. (Section 5.1 extends the model to costly information acquisition.) Our results are qualitatively unchanged if each blockholder obtains an imperfect signal of $\tilde{v}$: we only require that blockholders have superior information to atomistic investors.[11]

A number of empirical studies indeed find that blockholders are better informed than other investors and impound their information into prices through trading. Parrino, Sias and Starks (2003) and Chen, Harford and Li (2007) find that blockholders have superior information about negative firm prospects, which they use to vote with their feet. Bushee and Goodman (2007) show that blockholders trade on private rather than public information. Sias, Starks and Titman (2006) demonstrate that such blockholder trading has a causal effect on stock prices, and Brockman and Yan (2009) find that blockholders impound firm-specific information into prices.[12]

After observing $\tilde{v}$, each blockholder submits a market order $x_i(\tilde{v})$. Noise traders, who trade for exogenous liquidity reasons, submit market orders with a normally distributed net quantity $\tilde{\epsilon}$ with mean zero and variance σ_ε^2, where ε and η are independent. We use the term "liquidity" to refer to the standard deviation of noise trader demand, σ_ε. After observing total order flow $\tilde{y} = \sum_i \widetilde{x}_i + \tilde{\epsilon}$, the market maker determines the price $\tilde{p}$ and trades the quantity necessary to clear the market. Due to perfect competition, the market maker sets $\tilde{p}$ so that he earns zero profits, i.e. the price equals expected firm value given the order flow.

The manager's objective is to maximize the market value of his shares less the cost of effort, i.e. $\alpha\tilde{p} - a$. Each blockholder's objective is to maximize her trading profits,

[11] We could also allow signal precision to be increasing in the blockholder's individual stake and thus fall with I. This does not change any of the results as long as signal precision does not decline sufficiently rapidly with I to outweigh the beneficial effect of greater I on competition in trading. The results are in the Online Appendix.

[12] In addition, Parrino, Sias and Starks (2003), Sias, Starks and Titman (2006) and Gallagher, Gardner and Swan (2008) document that blockholders typically trade on the market rather than using a negotiated block trade. This is because only the former method allows them to trade on their information by camouflaging with noise traders (as in Kyle (1985).) Blockholders cannot trade on information in a negotiated trade because the counterparty engages in extensive due diligence since she is trading a large stake. Indeed, Barclay and Holderness (1991) find that negotiated block trades are rare and trades lead to stock price increases, inconsistent with the hypothesis that the selling blockholder is exiting on negative information. In addition, the event-study returns are independent of whether the block is traded at a premium or discount, rejecting the view that the parties in negotiated block trades have superior information to the market.

plus the fundamental value of her shares, less her cost of effort.[13]

We solve for the equilibrium of the game by backward induction.

3.1 The Trading Stage

To proceed by backward induction, we take the decisions a of the manager and b_i of the blockholders as given. (In equilibrium, these conjectures will be correct and equal the actions derived subsequently in Proposition 3.) The trading stage of the game is similar to Kyle (1985) and its extensions to multiple informed investors.[14]

Proposition 1 *(Trading Equilibrium): The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{v}) = \gamma(\tilde{v} - \phi_a \log a - \phi_b \log \textstyle\sum_i b_i) \quad \forall i \tag{2}$$

$$p(\tilde{y}) = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i + \lambda\tilde{y}, \tag{3}$$

where

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon} \tag{4}$$

$$\gamma = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}, \tag{5}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions. Each blockholder's expected trading profits are given by

$$\frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\epsilon. \tag{6}$$

Proof If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{v} = v] = (v - \mu - \lambda \sum_{j \neq i} x_j)x_i - \lambda x_i^2.$$

[13]Each blockholder thus maximizes her individual objective function. The results are unchanged if blockholders can co-ordinate (either to share the costs of intervention, or limit their trading volumes), but the cost is increasing in the number of co-ordinating parties. An increase in I reduces the co-ordination costs for both intervention and trading, with the same effects as in the core model.

[14]See, for example, Kyle (1984), Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), and Foster and Viswanathan (1993). While there are other models of informed trading to Kyle, the key mechanism through which the model justifies multiple blockholders (Cournot versus monopoly) does not depend upon the underlying trading model used.

This maximization problem yields

$$x_i(v) = \frac{1}{\lambda}[v - \mu - \lambda \sum_j x_j(v)] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(v) = \frac{1}{(I+1)\lambda}(v - \mu) \quad \forall i.$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{7}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon},$$
$$\mu = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i.$$

From this we obtain:

$$x_i(v) = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}(v - \phi_a \log \widehat{a} - \phi_b \log \textstyle\sum_i b_i) \quad \forall i,$$

$$p(y) = \phi_a \log a + \phi_b \log \textstyle\sum_i b_i + \displaystyle\frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon} y,$$

as required. Blockholder i's trading profits equal $x_i\,(p - v)$ and can computed immediately using the above expressions. ■

Trading profits are increasing in σ_η and σ_ϵ, as σ_η reflects the blockholders' informational advantage and σ_ϵ represents their ability to profit from information by trading with liquidity investors. In addition, aggregate blockholder trading profits are decreasing in the number I of blockholders. This is because multiple blockholders compete as in a Cournot oligopoly. Each blockholder chooses her trading volume to maximize individual profits. A higher volume reveals more information and makes the price less attractive to all informed traders, but she ignores this negative externality and so trades in excess of the level that would maximize combined blockholder profits.

While greater trading volumes reduce aggregate profits, they also impound more information into prices. Our definition of price informativeness is $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$, the expected change in price for a given change in firm value. This definition is particularly relevant for our setting as it captures the incentives for an agent compensated according to the stock price to improve fundamental value. It will thus later be used to derive the manager's optimal action. The common measure used in the microstructure literature is $(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p}))\,/\,\mathrm{Var}(\tilde{v})$, which measures the proportion of the variance of $\tilde{v}$ that is captured by prices. The next lemma states that these measures are identical.

Lemma 1 *The following two measures of price informativeness are equivalent:*

1. $(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \,/\, \mathrm{Var}(\tilde{v})$.
2. $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$.

Proof Using the formula for the conditional variance of a bivariate normal distribution

$$\mathrm{Var}(\tilde{v}|\tilde{p}) = (1 - \mathrm{Corr}(\tilde{v}, \tilde{p})^2)\,\mathrm{Var}(\tilde{v}),$$

we have

$$(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p})) \,/\, \mathrm{Var}(\tilde{v}) = \mathrm{Corr}(\tilde{v}, \tilde{p})^2. \tag{8}$$

Since, in equilibrium, the price is a linear function of $\tilde{v}$ and $\tilde{\epsilon}$,

$$\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right] = \frac{\mathrm{Cov}(\tilde{v}, \tilde{p})}{\mathrm{Var}(\tilde{v})}.$$

From the law of iterated expectations and (7),

$$\mathrm{Var}(\tilde{p}) = \mathrm{Cov}(\tilde{v}, \tilde{p}).$$

Therefore,

$$\mathrm{Corr}(\tilde{v}, \tilde{p})^2 = \mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]. \tag{9}$$

Combining (8) and (9) gives Lemma 1. ∎

The next proposition calculates price informativeness in the equilibrium derived in Proposition 1.

Proposition 2 *(Price Informativeness): Price informativeness, as defined by either of the above measures, is equal to $I/(I+1)$.*

Proof The result follows from equations (2), (3), (4), and (5). ∎

Both measures of price informativeness are increasing in I. As I approaches infinity, prices become fully informative. On the other hand, in the monopolistic Kyle model ($I = 1$), the blockholder fully internalizes the negative effect of a higher trading volume on profits. She limits her order, and so prices reveal only one-half of her private information.

The positive link between the number of blockholders and price informativeness does not arise because a greater number of informed agents mechanically leads to an increase in the amount of information in the market. Indeed, a single blockholder already has a perfect signal of fundamental value; since she faces no trading constraints, she could theoretically impound this entire information into prices. The amount of information in the economy is independent of the number of blockholders; the effect on price informativeness instead arises entirely from competition in trading.

As is standard in Kyle-type models, liquidity σ_ε has no effect on price informativeness. From equation (5), greater noise trading allows blockholders to trade more aggressively. This increase in informed trading exactly counterbalances the effect of increased noise and leaves price informativeness unchanged. In Section 5.1 we show that liquidity becomes relevant under costly information acquisition.

3.2 The Action Stage

We now solve for the actions of the manager and the blockholders in the first stage.

Proposition 3 *(Optimal Actions): The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I}{I+1} \right) \tag{10}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 . \tag{11}$$

Proof The manager maximizes the market value of his shares, less the cost of effort:

$$E \left[\alpha \widetilde{p} - a \right] . \tag{12}$$

When setting the price $\widetilde{p}$, the market maker uses his conjecture for the manager's action a. Therefore, the manager's actual action affects the price only through its influence on $\tilde{v}$, and thus blockholders' order flow. The manager's first-order condition is given by:

$$\alpha \left(\mathrm{E} \left[\frac{d\tilde{p}}{dv} \right] \right) \left(\frac{\phi_a}{a} \right) - 1 = 0. \tag{13}$$

From Proposition 2, his optimal action is therefore

$$a = \alpha \left(\frac{I}{I+1} \right) \phi_a .$$

Each blockholder maximizes her trading profits, plus the fundamental value of her shares, less her cost of effort. From (6), the blockholder's trading profits do not depend her first-stage action.[15] Therefore, blockholder i simply chooses b_i to maximize the fundamental value of her shares, less her cost of effort:

$$E \left[\left(\frac{\beta}{I} \right) \widetilde{v} - b_i \right] . \tag{14}$$

[15] This is because the blockholder's action is publicly observable. Informed trading profits depend on the blockholder's relative information advantage, and this is unaffected by a publicly observable variable. See Maug (1998, 2002) and Kahn and Winton (1998) for models where the blockholder's action is unobservable.

The optimal action of blockholder i is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{15}$$

■

The manager's action a is the product of three variables: the effectiveness of effort ϕ_a, his equity stake α, and price informativeness $\frac{I}{I+1}$. It is thus increasing in I as a higher I augments price informativeness. The intuition is as follows. Greater price informativeness (a higher $\mathrm{E}\left[\frac{d\tilde{p}}{dv}\right]$) means that the stock price more closely reflects the firm's fundamental value, and consequently the manager's effort. Therefore, the manager is more willing to bear the cost of working. In effect, blockholder trading rewards managerial effort ex post, therefore inducing it ex ante. The dynamic consistency of this reward mechanism depends on the number of blockholders. Critically, trading occurs *after* the manager has taken his action, at which point the action cannot be undone and shareholders are concerned only with maximizing their trading profits. A single blockholder optimizes her profits by limiting her order, at the expense of price informativeness. Therefore, the promise of rewarding effort by bidding up the price to fundamental value is not credible. By contrast, multiple blockholders trade aggressively, augmenting price informativeness, and thus constitute a commitment device to reward the manager ex post for his actions. While such aggressive trading is motivated purely by the private desire to maximize individual profits in the presence of competition, it has a social benefit by eliciting managerial effort.[16]

In sum, multiple blockholders lead to greater trading volumes. This both reduces aggregate profits and impounds more information into prices. Since firm value is increasing in price informativeness (as it induces effort ex ante) and independent of trading profits (which are a pure transfer from atomistic shareholders to blockholders), higher trading volumes lead overall to an increase in firm value.

As in earlier models, combined blockholder effort $\sum_i b_i$ is decreasing in I, owing to the free-rider problem. Therefore, there is a trade-off between the intervention and trading effects.

4 The Optimal Number of Blockholders

This section derives the optimal number of blockholders. We start by deriving the optimal number that maximizes firm value, and later analyze the social optimum (that

[16]Fishman and Hagerty (1995) also show that introducing additional informed traders is a commitment to trading more aggressively. Thus an informed agent may sell her information to other traders, rather than only exploiting it herself. Oehmke (2008) shows that competition between prime brokers in liquidating collateral reduces sale proceeds and may encourage hedge funds to concentrate collateral with a single broker.

maximizes total surplus, taking into account the costs borne by the manager and blockholder) and the private optimum (that maximizes the total payoff to blockholders).

Proposition 4 *(Firm Value Optimum): The number I^* of blockholders that maximizes firm value is:*

$$I^* = \frac{\phi_a - \phi_b}{\phi_b}.^{17} \tag{16}$$

Proof From Proposition 3, expected firm value is:

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{17}$$

The first-order condition with respect to I is given by:

$$\frac{\phi_a - \phi_b - \phi_b I}{I + I^2} = 0. \tag{18}$$

$\hat{I} = (\phi_a - \phi_b)/\phi_b$ satisfies the first order condition. Since the left hand side of (18) is positive for $I < \hat{I}$ and negative for $I > \hat{I}$, I^* is indeed a maximum. ■

The optimal number of blockholders solves the trade-off between the positive effect of more blockholders on managerial effort, and the negative effect on blockholder intervention. The optimum is therefore increasing in ϕ_a, the productivity of the manager's effort, and declining in ϕ_b, the productivity of blockholder intervention. In Section 6, we discuss potential empirical proxies for these variables. Owing to the trade-off, the optimum is typically an interior situation with a multiple blockholder structure, rather than fully dispersed ownership or a single concentrated blockholder.

While Proposition 4 is concerned with maximizing firm value, the social optimum maximizes total surplus, which also takes into account the effort costs borne by the manager and blockholders. As with firm value, total surplus is also unaffected by trading profits, since they are a transfer from liquidity traders to the blockholders. In theory, the social optimum would be chosen by a social planner. If the noise traders are the firm's atomistic shareholders (as in Kahn and Winton (1998) and Bolton and von Thadden (1998)), it will also be chosen by the initial owner when taking the firm public, since IPO proceeds will equal total surplus. The owner will have to compensate the blockholders (in the form of a lower issue price) for their expected intervention costs, and the manager for his effort in the form of a higher wage. Trading profits have no effect on IPO proceeds: while blockholders will pay a premium in expectation of trading gains, small shareholders will demand discounts to offset their future losses.

[17] In reality, the number of blockholders must be a strictly positive integer. To economize on notation, we ignore such technicalities when stating I^*. If $\frac{\phi_a - \phi_b}{\phi_b} < 1$, the optimal number is 1. If $\frac{\phi_a - \phi_b}{\phi_b}$ is a non-integer, the optimal number is found by comparing (17) under the two adjacent integers.

Proposition 5 *(Social Optimum): The number I^*_{soc} of blockholders that maximizes total surplus is the unique positive solution to*

$$\frac{\phi_a}{I\,(I+1)} - \frac{\phi_b}{I} - \frac{\phi_a \alpha}{(I+1)^2} + \frac{\phi_b \beta}{I^2} = 0, \tag{19}$$

which may be higher or lower than I^*. Moreover, I^*_{soc} is increasing in ϕ_a and β, and decreasing in ϕ_b and α.

Proof Total surplus is given by:

$$\phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)\right] - \phi_a \alpha \left(\frac{I}{I+1}\right) - \phi_b \beta \frac{1}{I}. \tag{20}$$

Taking first-order conditions yields (19). The Appendix proves that there is a unique positive solution and that it maximizes (20). It also addresses the comparative statics. ■

Compared to equation (17), equation (20) contains two additional terms. Increasing the number of blockholders raises the cost of managerial effort, but reduces the combined cost of blockholder effort. The social optimum may thus be higher or lower than the number that maximizes firm value. If β rises, total blockholder costs $\phi_b \beta \frac{1}{I}$ become more important in the social welfare function, and so I^*_{soc} rises to reduce these costs by lowering intervention. Conversely, a rise in α increases the importance of the manager's costs and thus lowers I^*_{soc}. The comparative statics with respect to ϕ_a and ϕ_b are the same as in Proposition 4.

Finally, we analyze the privately optimal division of β that would maximize blockholders' combined payoffs. In other words, we ask the question: if blockholders in aggregate hold β of the firm, do they have incentives to split or combine stakes to achieve the number that maximizes either firm value or total surplus?

Proposition 6 *(Private Optimum): The number I^*_{priv} of blockholders that maximizes total blockholders' payoff is the unique positive solution to*

$$\beta\left[\frac{\phi_a}{I\,(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}\right] - \frac{(I-1)}{2\sqrt{I}(I+1)^2}\sigma_\eta \sigma_\varepsilon = 0, \tag{21}$$

which may be higher or lower than I^*, and higher or lower than I^*_{soc}. Moreover, I^*_{priv} is increasing in ϕ_a and β, and decreasing in ϕ_b and $\sigma_\eta \sigma_\varepsilon$.

Proof Total blockholders' payoff is given by:

$$\beta\left\{\phi_a \log\left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] + \phi_b \log\left[\phi_b \beta \frac{1}{I}\right]\right\} - \phi_b \beta \frac{1}{I} + \frac{\sqrt{I}}{I+1}\sigma_\eta \sigma_\varepsilon. \tag{22}$$

Taking first-order conditions yields (21). The Appendix proves that there is a unique positive solution and that it maximizes (22). It also addresses the comparative statics. ■

The blockholders' objective function differs from firm value in three ways. They only enjoy β of any increase in firm value; bear the costs of intervention; and are concerned with informed trading profits. Increasing I above I^* therefore has an ambiguous effect: it reduces the combined costs of intervention, but also reduces total trading profits by exacerbating competition. Therefore, as with the social optimum, the private optimum may be higher or lower than the number that maximizes firm value. An increase in β causes blockholders' effort costs to become more important in the objective function and so I^*_{priv} rises. If $\sigma_\eta \sigma_\varepsilon$ increases, trading profits become more important and so shareholders combine blocks (reduce I^*_{priv}) to lower competition.

The blockholders' objective function also differs from the social welfare function in three ways. Blockholders are concerned with trading profits and only β of firm value, but ignore the cost of managerial effort. Again, the sum of these three effects is ambiguous. Increasing I above I^*_{soc} would both reduce profits and increase the manager's costs.

The comparative statics with respect to ϕ_a and ϕ_b are the same as in Propositions 4 and 5. This is particularly important since blockholders may trade away from the structure chosen to maximize firm value or IPO proceeds, and so the private optimum is most likely to be observed empirically (see also Maug (1998)).

5 Extensions

5.1 Costly Information Acquisition

In the core model, the blockholders are endowed with private information about firm value $\tilde{v}$. In this subsection, we assume that blockholders are initially uninformed but can learn $\tilde{v}$ by paying a cost c in the first stage of the game. Blockholders that do not pay this cost will remain uninformed in the second stage.

To solve this modified version of the model, we again use backward induction.

Proposition 7 *(Equilibrium With Costly Information): Let J be the number of blockholders that acquire information in the first stage of the game. Then in the unique linear equilibrium of the trading stage, the $I-J$ uninformed blockholders do not trade. The J informed blockholders submit demands as in (2) and the market maker sets the price as* *in (3) with*

$$\lambda = \frac{\sqrt{J}}{J+1}\frac{\sigma_\eta}{\sigma_\epsilon} \tag{23}$$

$$\gamma = \frac{1}{\sqrt{J}}\frac{\sigma_\epsilon}{\sigma_\eta}. \tag{24}$$

In the first stage of the game, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1}\right) \tag{25}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2. \tag{26}$$

The number J of blockholders that acquire information is

$$J = \min\{I, n\},$$

where n satisfies

$$\frac{1}{\sqrt{n}(n+1)}\sigma_\eta \sigma_\epsilon = c.$$

Proposition 7 shows that when the number of blockholders I is sufficiently large (greater than n), some blockholders choose not to acquire information. If all blockholders became informed, competition in trading is sufficiently fierce that individual trading profits are insufficient to recoup the monitoring cost c. Hence, in equilibrium, some blockholders remain uninformed and do not participate in the trading stage of the game, earning zero trading profits.

We now analyze the optimal number of blockholders that maximizes firm value. We first observe that it is never optimal to have I greater than n. If $I > n$, then from Proposition 7, some blockholders will not acquire information in equilibrium. Uninformed blockholders do not trade and thus have no effect on governance through exit. Moreover, they dilute ownership and reduce incentives to engage in voice. Uninformed blockholders are thus unambiguously detrimental to firm value, and so the optimum involves no such blockholders. This leads to the next proposition.

Proposition 8 *(Firm Value Optimum With Costly Information): The optimal number I^*_{costly} of blockholders that maximizes firm value with costly information acquisition is equal to*

$$I^*_{costly} = \min\left(\frac{\phi_a - \phi_b}{\phi_b}, n\right). \tag{27}$$

If $n < \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} and firm value are increasing in σ_η and σ_ε and decreasing in c. If $n \geq \frac{\phi_a - \phi_b}{\phi_b}$, I^*_{costly} and firm value are independent of σ_η, σ_ε and c.

The optimal number I^*_{costly} of blockholders with costly information acquisition is weakly increasing in σ_η and σ_ϵ and weakly decreasing in c. The intuition is as follows. If $n < \frac{\phi_a - \phi_b}{\phi_b}$, then the optimum with costless information acquisition I^* is so large that competition in trading reduces individual informed trading profits below the cost of monitoring. Some blockholders thus choose to remain uninformed, and their existence reduces firm value. The optimum is therefore n, the maximum number of blockholders under which competition is sufficiently low that trading profits are high enough to motivate all blockholders to become informed. A fall in the cost of information acquisition c, an increase in the informational advantage σ_η, and a rise in liquidity σ_ε all lead to an increase in trading profits, net of monitoring costs. Higher net profits in turn raise n, as they allow greater competition in trading to be sustained before net profits become negative. This in turn increases I^*_{costly} towards I^*, and thus raises firm value.

By contrast, if $n > \frac{\phi_a - \phi_b}{\phi_b}$, net trading profits are sufficiently high that all blockholders become informed. The analysis is as in the core model of Section 4, where the optimum depends only on the effectiveness of manager and blockholder effort. The constraint that the number of blockholders is sufficiently low to induce information acquisition is not binding. Changes in net trading profits, and thus changes in σ_η, σ_ε and c, have no effect on the optimal number of blockholders or firm value.

5.2 Complementarities

In the core model, the manager's and blockholders' actions are perfect substitutes, with independent effects on firm value. The marginal productivity of the manager's (blockholders') effort is unaffected by the effort level of the other party, i.e. $\frac{\partial^2 v}{\partial a \partial b_i} = 0$. This assumption likely applies to a number of settings: for example, rent extraction by the blockholders reduces firm value regardless of the manager's effort; managerial initiative is unaffected by perk prevention by the blockholders.

In some cases, there may be complementarities between the manager's and blockholders' efforts. This subsection extends the core model to these cases. If complementarities are positive, the marginal productivity of one party's action is increasing in the effort level of the other party, i.e. $\frac{\partial^2 v}{\partial a \partial b_i} \geq 0$. This arises if manager and blockholder outputs are mutually interdependent – in particular, if the main managerial action is initiative and the main blockholder action is advising. For example, venture capital investors often have expertise in devising an effective strategy, which is then executed by the manager. Both strategy formulation and implementation are necessary for the firm to become successful.

With positive complementarities, blockholders are "allies" of the manager, providing him with specialist advice. The opposite case of negative complementarities arises if blockholders are "adversaries" of the manager, e.g. if their main value added is perk

		Blockholder Action		
		Advisory	Rent Extraction	Perk Prevention
Manager Action	Initiative	Positive Complements	Substitutes	Substitutes
	Rent Extraction	Substitutes	Substitutes	Negative Complements

Table 2: Classification of blockholder and manager actions as substitutes or complements.

prevention, and rent extraction is an important managerial action. The blockholders are most productive if managerial effort is low ($\frac{\partial^2 v}{\partial a \partial b_i} \leq 0$), i.e. the manager is pursuing private benefits. Similarly, the manager's efforts to extract rents have a greater negative effect on firm value if the blockholder is not monitoring such rents. Negative complementarities are most likely in mature firms, where the optimal strategy is often clear to the manager. Inefficiencies arise not because the manager is unaware of the correct course of action and needs blockholders' advice, but because he has private incentives to depart from the efficient action. For example, managers of "cash cows" know that they should return excess cash to shareholders, but may choose instead to reinvest it inefficiently. Table 2 summaries whether actions are likely to be substitutes or positive or negative complements depending on their type.

We analyze complementarities using the boundary cases of perfect positive (negative) complementarities, where firm value depends only on the minimum (maximum) output level of the manager and blockholders, as these scenarios are most tractable within our framework and thus allow the clearest empirical predictions.[18] Reality will typically lie between these two extremes and the optimum for a particular interior level of complementarity may be inferred by interpolating between the boundary cases. For example, we will see that the perfect substitutes solution, analyzed earlier, lies between the two perfect complementarities cases.

We commence with perfect positive complementarities, which we model with a Leontief production function:

$$\widetilde{v} = \min \left[\phi_a \log a, \phi_b \textstyle\sum_i b_i\right] + \widetilde{\eta}. \tag{28}$$

The optimal actions can no longer be derived independently. The manager's optimal action depends on his conjecture $\widehat{b}_i$ for the blockholders' actions. Blockholder i's optimal

[18] An alternative way to model complementarities is to use a constant elasticity of substitution production function, e.g. $\widetilde{v} = \left[(\phi_a \log a)^\rho + (\phi_b \log \sum_i b_i)^\rho\right]^{1/\rho} + \widetilde{\eta}$. Such a production function does not yield closed-form solutions in our framework owing to the logarithmic functional form, which was necessary for tractability of the core model.

action depends on her conjecture for the manager's effort ($\widehat{a}$) and for the actions of the other blockholders ($\widehat{b}_j,\ j \neq i$). We use the Nash equilibrium solution concept, where each party chooses the optimal action given his/her conjectures, and all conjectures are correct.

Proposition 9 *(Perfect Positive Complementarities): The manager's optimal action is*

$$a = \min\left(\phi_a \alpha \left(\frac{I}{I+1}\right), \exp\left(\frac{\phi_b}{\phi_a} \log \textstyle\sum_i \widehat{b}_i\right)\right). \tag{29}$$

Similarly, blockholder i's effort level is:

$$b_i = \begin{cases} \phi_b \beta \left(\frac{1}{I}\right)^2 & \text{if } \phi_a \log \widehat{a} \geq \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)^2\right] + \phi_b \log \sum_{j \neq i} \widehat{b}_j \\ \exp\left(\frac{\phi_a}{\phi_b} \log \widehat{a} - \log \sum_{j \neq i} \widehat{b}_j\right) & \text{if } \phi_b \log \sum_{j \neq i} \widehat{b}_j \leq \phi_a \log \widehat{a} < \phi_b \log\left[\phi_b \beta \left(\frac{1}{I}\right)^2\right] + \phi_b \log \sum_{j \neq i} \widehat{b}_j \\ 0 & \text{if } \phi_a \log \widehat{a} < \phi_b \log \sum_{j \neq i} \widehat{b}_j \end{cases}. \tag{30}$$

The number I^* of blockholders that maximizes firm value is the unique positive solution to

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp\left(\phi_b - \phi_a\right). \tag{31}$$

I^* is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

As with the core case, the optimal number of blockholders I^* is typically an interior solution, i.e. involves multiple, but finite, blockholders. However, the comparative statics with respect to ϕ_a and ϕ_b are *opposite* to the core case. In the core case, I^* is increasing in ϕ_a. If managerial effort becomes more productive, it becomes increasingly important in the trade-off between exit and voice, and so I^* rises to enhance exit. With complements, the optimal number of blockholders must balance the levels of manager and blockholder outputs. If ϕ_a rises, managerial effort is more effective and so it is not necessary to "boost" it via a high I. Instead, I should be used to enhance blockholder effort so that it becomes sufficiently high to complement the manager's output. This involves reducing I.

We now turn to the case of perfect negative complementarities, i.e.

$$\widetilde{v} = \max\left[\phi_a \log a, \phi_b \textstyle\sum_i b_i\right] + \widetilde{\eta}. \tag{32}$$

We again apply the Nash equilibrium solution concept.

Proposition 10 *(Perfect Negative Complementarities): The manager's optimal action is*

$$a=\begin{cases}\phi_a\alpha\frac{I}{I+1} & \text{if } \alpha\frac{I}{I+1}\left(\phi_a\log\left[\phi_a\alpha\frac{I}{I+1}\right]-\phi_b\log\sum_i\widehat{b}_i\right)\geq a\\ 0 & \text{if } \alpha\frac{I}{I+1}\left(\phi_a\log\left[\phi_a\alpha\frac{I}{I+1}\right]-\phi_b\log\sum_i\widehat{b}_i\right)<a.\end{cases}\quad. \tag{33}$$

Similarly, blockholder i's effort level is:

$$b_i=\begin{cases}\phi_b\beta\left(\frac{1}{I}\right)^2 & \text{if } \frac{\beta}{I}\left(\phi_b\log\left(\phi_b\beta\frac{1}{I}\right)^2-\left[\phi_a\log\widehat{a}-\phi_b\log\sum\limits_{j\neq i}\widehat{b}_j\right]\right)\geq b_i\\ 0 & \text{if } \frac{\beta}{I}\left(\phi_b\log\left(\phi_b\beta\frac{1}{I}\right)^2-\left[\phi_a\log\widehat{a}-\phi_b\log\sum\limits_{j\neq i}\widehat{b}_j\right]\right)<b_i.\end{cases}\quad. \tag{34}$$

The number of blockholders I^* that maximizes firm value is

$$I^*=\begin{cases}\infty & \text{if } \phi_a\log\left(\phi_a\alpha\right)\geq\phi_b\log\left(\phi_b\beta\right)\\ 1 & \text{if } \phi_a\log\left(\phi_a\alpha\right)<\phi_b\log\left(\phi_b\beta\right)\end{cases}\quad. \tag{35}$$

In the core model of perfect substitutes, firm value depends on both manager and blockholder efforts. Since the optimal shareholder structure must trade-off both, I^* is typically an interior solution. Here, firm value depends only on the maximum output level and there are no trade-off concerns. If managerial effort is relatively productive, I^* should be chosen exclusively to maximize the potency of exit and completely ignores voice; thus the optimal number of blockholders is infinite. By contrast, if blockholder effort is relatively productive, I^* is at its minimum value of 1.

With perfect substitutes, I^* is smoothly increasing in ϕ_a. Here, ϕ_a has a discontinuous effect. If $\phi_a\log(\phi_a\alpha)<\phi_b\log(\phi_b\beta)$, I^* is independent of ϕ_a. A small increase in ϕ_a has zero effect on I^*: since blockholder effort is still relatively more productive, I^* continues to be exclusively determined by voice, irrespective of the effectiveness of managerial effort. However, when ϕ_a rises above the level for which $\phi_a\log(\phi_a\alpha)=\phi_b\log(\phi_b\beta)$, I^* jumps from 1 to ∞. For $\phi_a\log(\phi_a\alpha)\geq\phi_b\log(\phi_b\beta)$, I^* is already exclusively determined by exit considerations, and so further increases in managerial productivity have no effect on I^*. Similarly, changes in ϕ_b have either a zero or infinite effect on I^*.

Negative complementarities therefore lead to more extreme results than the core model. The optimal number of blockholders is a corner solution; ϕ_a and ϕ_b have the same directional effect as in the core model, but their impacts are discontinuous. The case of $I^*=\infty$ represents fully dispersed ownership. Since empirical studies define a blockholder as a shareholder who owns above a minimum threshold, this case will appear in the data as zero blockholders. Therefore, under perfect negative complementarities, there is either zero or one blockholder. Indeed, Table 1 shows that both of these cases are also common in the data.

Combining all of the results, with perfect negative complementarities, I^* is either 1 or ∞ and is driven entirely by the more productive action. As complementarities become less negative, I^* becomes less extreme and is to be determined by the productivity of both actions; it continues to be increasing in ϕ_a and decreasing in ϕ_b. The case of perfect substitutes is an example. Once complementarities become sufficiently high, we approach the case of perfect complements, and the effects of ϕ_a and ϕ_b change direction.

5.3 General Compensation Contract

In the core model, the manager's payoff stems from the market value of his shares, $\alpha\widetilde{p}$, as in Holmstrom and Tirole (1993). In a more general setting, the manager can be compensated according to the fundamental value $\widetilde{v}$ as well as the market value $\widetilde{p}$, for instance using stock with a long vesting period. We thus generalize the manager's objective function to

$$E\left[\alpha\left(\omega p + (1-\omega)v\right) - a\right].$$

The actual level of ω will reflect factors outside the model and introduced in earlier work, such as takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his shares for $\widetilde{p}$ before $\widetilde{v}$ is realized, e.g. to finance consumption (Stein (1989)).[19] The core model has $\omega = 1$.

Proposition 11 *(General Compensation Contract): The number I^* of blockholders that maximizes firm value is the larger root of*

$$\frac{\phi_a \omega}{(I+1-\omega)(I+1)} - \frac{\phi_b}{I} = 0 \tag{36}$$

if equation (36) has solutions. In this case, I^* is increasing in ω. If (36) has no solutions, $I^* = 1$.

Proof Proceeding as in the main model, we have

$$a = \alpha\phi_a\left[1 - \frac{\omega}{I+1}\right]. \tag{37}$$

Firm value is given by:

$$E[v] = \phi_a \ln\left[\phi_a\alpha\left[1 - \frac{\omega}{I+1}\right]\right] + \phi_b \ln\left[\phi_b\beta\frac{1}{I}\right]. \tag{38}$$

[19] Kole (1997) shows that vesting periods are short in practice, perhaps because long vesting periods would subject the manager to excessive risk.

The first-order condition is given by (36). The Appendix proves that, if there are two roots, the larger root yields a maximum. It also addresses the comparative statics. ■

As in the core model, the optimal number of blockholders represents a trade-off between the positive effect of greater blockholders on "exit", and the negative effect on "voice." The effect of I on stock price efficiency is more important when the manager is more closely aligned with the stock price, and so the optimal number of blockholders I^* increases with the manager's short-term concerns ω.

6 Empirical Implications

The paper is motivated by the empirical observation that many firms are held by multiple small blockholders, in contrast to some earlier theories that advocate highly concentrated ownership. The theory generates a number of additional empirical implications, over and above its initial motivation. It suggests different ways of thinking about blockholders that may give rise to new directions for empirical research – viewing multiple blockholders as competing for trading profits rather than private benefits, and studying the number of blockholders rather than total ownership or the stake of the largest shareholder. These two broad themes in turn generate specific predictions for the effects of blockholder structure, and the determinants of blockholder structure. We commence with the former.

The model suggests that the number of blockholders impacts both financial markets and firm value. Starting with the former implication, it predicts that a greater number of blockholders reduces total trading profits, but increases price efficiency. Gallagher, Gardner and Swan (2008) find support for both predictions, and Boehmer and Kelley (2009) show that competition among institutional traders increases price informativeness (they do not study trading profits). Turning to the latter implication, the theory suggests that multiple blockholders can improve firm value, in contrast to existing models that advocate a single concentrated blockholder.[20] Gallagher et al. find that the threat of disciplinary exit from multiple blockholders leads to superior subsequent firm performance. They use a measure of portfolio churning to specifically test governance through exit rather than control contests. Smith and Swan (2008) show that institutional trading is successful at disciplining executive pay. Multiple investors with frequent trading have greatest effect; total institutional ownership only matters insofar

[20]If I is always at the firm value optimum, there should be no relationship between I and firm value, when controlling for the joint determinants of I and firm value. Demsetz and Lehn (1985) made this point in the context of managerial ownership and firm value. However, the empirically observed I is likely to be the private optimum, which differs from the firm value optimum. Moreover, the private optimum may shift for exogenous reasons, such as a blockholder suffering a change in management or a liquidity shock.

as it affects trading activity.

The effect of the number of blockholders on prices and firm value suggests that it is an important determinant of both market efficiency and corporate governance. Many empirical papers use total institutional ownership as a measure of market efficiency, since institutions have greater information than retail traders. However, price efficiency depends not only on the amount of information held by investors, but the extent to which this information is impounded into prices. The latter in turn depends on the number of informed shareholders. Similarly, many studies use total institutional ownership or the stake of the largest investor as a proxy for corporate governance, but the model suggests that the number of blockholders is another important factor and thus may be relevant for future empirical work.

Our model also generates predictions concerning the determinants of blockholder structure. To our knowledge, none of these predictions have been tested formally as empirical studies have focused on total block ownership rather than the number of blockholders, and so they are potential topics for future research.[21] In the paper, we considered different criteria for the optimal number of blockholders. In practice, sometimes the social optimum may be observed, for instance if the firm has recently undergone an initial public offering, or lock-ups prevent blockholders from re-trading from the initial structure. For most firms, it is most likely that the private optimum will be observed. Importantly, both optima share the same predictions for ϕ_a and ϕ_b: the number of blockholders is increasing in the productivity of the manager's effort and declining in the productivity of blockholder intervention.

We first consider the core model of perfect substitutes. The magnitude of ϕ_b depends on the nature of blockholders' expertise. Using the terminology of Dow and Gorton (1997), if blockholders have forward-looking ("prospective") information about optimal future investments or strategic choices, direct intervention is particularly valuable and ϕ_b is high. For example, activist investors (e.g. Kirk Kerkorian or Carl Icahn) are typically expert at preventing perks or empire-building; venture capitalists have skills in advising. On the other hand, passive mutual funds and insurance companies typically lack specialist expertise in managing a firm, but instead are adept at gathering backward-looking ("retrospective") information to evaluate the effect of past decisions on firm value. Their primary benefit is to impound the effects of prior managerial effort into the stock price. In such a case, ϕ_b is low and I^* is high. Similarly, Section 5.1 shows that if information is costly, the optimal number of blockholders is decreasing in the monitoring cost c and increasing in blockholders' information advantage σ_η. Insti-

[21] Maury and Pajuste (2005) and Laeven and Levine (2007) report the number of blockholders, but do not relate them to cross-sectional determinants. Faccio and Lang (2002) and Dlugosz et al. (2006) do not report the number of blockholders, but made their raw data publicly available and we constructed the numbers in Section 2 from these data.

tutions skilled at gathering retrospective information have a low monitoring cost and high information advantage, further reinforcing the prediction that I^* is high. Indeed, as firms mature, active venture capitalist investors are typically replaced by passive institutional shareholders, and the number of blockholders usually increases. Note that this association could be for reasons outside the model. As firms mature, they typically become larger; if blockholder wealth constraints limit the number of dollars they can invest in a firm (e.g. Winton (1993)), this will lead to more dispersed ownership. Therefore, the above empirical observation is only tentative support for the model; a formal test will have to control for factors such as firm size.

Another determinant of ϕ_b is blockholders' control rights and thus ability to intervene (holding constant the size of their individual stakes).[22] Black (1990) and Bebchuk (2007) note that U.S. shareholders face substantial legal and institutional hurdles to intervention, compared to their foreign counterparts. This reduces ϕ_b, thus increasing I^*, and is consistent with smaller and more numerous blockholders in the U.S.

The manager's effectiveness ϕ_a will be higher if he is more talented. Talent can be measured directly using managerial characteristics, such as education, experience or past performance, or proxied by salary (see Gabaix and Landier (2008)). ϕ_a also depends on the manager's scope to use his initiative or engage in rent extraction. It is likely lower in regulated firms, and high in firms with free cash flow problems. The latter implication suggests that mature firms should be held by many blockholders, which reinforces the earlier predictions. Firm size may also affect the relative magnitude of ϕ_a and ϕ_b. Many managerial actions can "rolled out" across the entire firm (for example, if the CEO designs a new method to reduce production costs, this can be applied firmwide) and so managerial effort has a multiplicative effect on firm value (Edmans, Gabaix and Landier (2009)). By contrast, if blockholders primarily intervene to prevent perks, such actions have an additive effect on firm value. Therefore, as firms grow, ϕ_a increases relative to ϕ_b and the number of blockholders should rise.

Negative complementarities may arise if the manager has significant scope for rent extraction, which blockholders can potentially prevent. This occurs in mature firms where the agency costs of free cash flow are potentially high: the manager may have private incentives to reinvest excess cash, which can be prevented by blockholder monitoring. If investors are passive, ϕ_a will be significantly higher than ϕ_b, and so the model predicts dispersed ownership. By contrast, if blockholders are activist and skilled in perk prevention, it is efficient to have a single blockholder. Both of these predictions reinforce the earlier results for mature firms.

Positive complementarities typically occur in start-up firms. The main managerial action is initiative, and early-stage investors (such as venture capitalists) are expert at

[22]In reality, control rights will also be increasing in the size of each blockholder's individual stake β/I. This will reinforce the negative effect of I on intervention currently in this paper.

advising the manager (e.g. by devising a strategy for the manager to implement). Typically, ϕ_a will be significantly greater than ϕ_b: the manager is able to add greater value than blockholders, given his close proximity to firm operations. In such a case, Section 5.2 predicts that I^* is lower under positive complementarities than perfect substitutes. Moreover, in start-ups, the manager often has a significant equity stake (high α) which gives him strong incentives to exert effort. From equation (31), I^* should be low to ensure blockholder effort is also high. This may explain the concentrated blockholder structure in early-stage firms, even after such firms go public and the exit governance mechanism becomes available.[23]

The theory also suggests that governance through exit is most important where the manager's short-term concerns ω are highest. Therefore, the number of blockholders should be higher when the manager's stock and options have shorter vesting periods, or takeover defenses are weaker. Again, simple cross-sectional correlations will be insufficient to support this prediction, since blockholders can plausibly affect the compensation contract. Liquidity σ_ε increases both the optimal number of blockholders and firm value. Indeed, Fang, Noe and Tice (2008) find a causal relationship between liquidity and firm value. While many other papers also generate a positive effect of liquidity on firm value (e.g. Holmstrom and Tirole (1993), Maug (1998), Faure-Grimaud and Gromb (2004), Edmans (2009)), here the specific mechanism is through changing blockholder structure.

While the theory appears to generate a number of untested predictions through a different conceptualization of blockholders to prior research, we caveat that empirical testing will have to overcome a number of challenges. First, although the model yields clear, closed-form predictions for the optimal number of blockholders in terms of certain variables, a number of these parameters (such as the effectiveness of blockholder and manager effort) are difficult to measure directly. The key challenge for empiricists is to come up with accurate proxies. Second, while the model predicts that these variables have a causal impact on blockholder structure, it may be that additional factors outside the model have an effect on both. Therefore, documenting correlations will be insufficient to support the model; identification of causal effects will require careful instrumentation.

7 Conclusion

Why are so many firms held by multiple blockholders when such a shareholding structure generates free-rider problems in monitoring? This paper offers a potential explanation. The same co-ordination issues that hinder intervention increase blockholders'

[23]In many cases, lock-ups ensure that ownership remains concentrated after an IPO. This may be interpreted as an attempt to maintain the social optimum (where blockholders continue to advise), rather than allowing investors to re-trade to the private optimum of dispersed ownership which involves lower blockholder effort.

effectiveness in exerting governance through an alternative governance mechanism: exit. Multiple blockholders act competitively in their trading behavior, impounding more information into the stock price. This in turn induces higher managerial effort, particularly if the manager has high stock price concerns.

The optimal number of blockholders depends on the relative productivity of managerial and blockholder effort. If outputs are perfect substitutes, the optimum is decreasing in the effectiveness of blockholder intervention and increasing in the potency of managerial effort. It is therefore high if blockholders are mutual funds that gather retrospective rather than prospective information, and low if they are activists. This dependence becomes stronger under negative complementarities. However, if complementarities are positive, the productivity parameters have opposite effects on the optimal shareholder structure. If blockholder effort is unproductive, concentrated blockholders are necessary to augment it to a sufficient level to complement the manager's effort.

The paper suggests a number of potential avenues for future research. On the empirical side, the model highlights the importance of the number of blockholders. As an independent variable, it is a relevant determinant of both governance and price efficiency; as a dependent variable, the optimal blockholder structure depends on many underlying factors as identified by the model. On the theoretical side, the paper has assumed symmetric blockholders and the analysis has focused on their optimal number. It would be interesting to extend the analysis to introduce asymmetries and examine the optimal distribution of shares between a fixed number of blockholders.[24] Another possible asymmetry would be to feature some blockholders specializing in trading and others in intervention, as in Faure-Grimaud and Gromb (2004), Aghion, Bolton and Tirole (2004), and Attari, Banerjee and Noe (2006) (although these models feature only one type of each blockholder). Similarly, while we have focused our study on the efficient number of blockholders, the model can be expanded to consider the simultaneous determination of the manager's stake and total blockholder ownership.

More broadly, the model suggests a new way of thinking about the interactions between multiple blockholders: as competing for trading profits, rather than private benefits. Therefore, future corporate finance models of multiple blockholders could incorporate more complex effects currently analyzed in asset pricing models of many informed traders. The present paper assumes a single trading period, but in reality there may be multiple periods in which information may arrive and blockholders may

[24] Studying asymmetric blockholders will likely require a quite different framework. In the current model (as in standard Kyle-type models), block size has no effect on trading behavior as the ability to trade is independent of one's stake. Introducing short-sale contraints will allow block size to be relevant, but will require departures from normal noise distributions to obtain tractability (see, e.g., Edmans (2009)). Moreover, to obtain closed-form solutions in the trading stage, we rely on the symmetry of the equilibrium. The case of costly information acquisition currently features an asymmetry, as some blockholders may remain uninformed.

trade. Trading profits, and thus incentives to acquire costly information, then depend not only on the quality of information but its timeliness. A blockholder who receives information late may find that the price has already moved, reducing her trading profits. In addition, in the present paper, blockholders trade on information only. If blockholders are subject to liquidity shocks (as in Brunnermeier and Pedersen (2005)), the addition of multiple trading rounds may give incentives for other blockholders to "front-run" and sell in advance of an anticipated forced liquidation. This may increase the potency of governance through exit, but reduce incentives to engage in interventions with long-run benefits.

A Appendix

Proof of Proposition 5 (Social Optimum)

Putting equation (19) under a common denominator yields

$$\frac{\phi_a I\left(I+1\right)-\phi_b I\left(I+1\right)^2-\phi_a \alpha I^2+\phi_b \beta\left(I+1\right)^2}{I^2\left(I+1\right)^2}=0. \tag{39}$$

Equation (19) is thus a cubic, and has at most three roots. The function is discontinuous at $I=-1$ and approaches $-\infty$ either side of $I=-1$ (since the $-\frac{\phi_a \alpha}{(I+1)^2}$ term dominates). It is also discontinuous at $I=0$ and approaches $+\infty$ either side of $I=0$ (since the $\frac{\phi_b \beta}{I^2}$ term dominates). It is continuous everywhere else.

As $I \rightarrow -\infty$, the $-\frac{\phi_b}{I}$ term in equation (19) dominates, and so the function asymptotes the x-axis from above. Since it approaches $-\infty$ as I rises to -1, and is continuous between $I=-\infty$ and $I=-1$, there must be one root between these two points. Similarly, since the function tends to $+\infty$ as I rises from just above -1 to just below 0, and is continuous between these two points, there must be a second root within this interval. As $I \rightarrow +\infty$, the $-\frac{\phi_b}{I}$ term in equation (19) again dominates, and so the function asymptotes the x-axis from below. Since the function tends to $+\infty$ as I approaches 0 from above, and is continuous between $I=0$ and $I=+\infty$, there must be a third root $(\widetilde{I})$ between these two points. There can be no other positive roots, since there are two negative roots and three roots in total. The positive root is a local maximum, since the gradient is negative for $I<\widetilde{I}$ and positive for $I>\widetilde{I}$.

Let $F\left(I,\theta\right)$ denote the left-hand side of equation (39), where θ is a vector of parameters ϕ_a, ϕ_b, α, β. I^*_{soc} is defined by $F=0$. Differentiating with respect to θ gives

$$\frac{\partial F}{\partial \theta}+\frac{\partial F}{\partial I}\frac{\partial I}{\partial \theta}=0.$$

Since the gradient F is negative just below I^*_{soc} and positive just above I^*_{soc}, $\frac{\partial F}{\partial I}|_{I=I^*_{soc}}<0$. Therefore, the sign of $\frac{\partial I}{\partial \theta}$ equals the sign of $\frac{\partial F}{\partial \theta}$, which in turn is the cross-partial derivative of total surplus (20) with respect to I and θ. This generates the comparative statics with respect to α, β, ϕ_a and ϕ_b.

Proof of Proposition 6 (Private Optimum)

Equation (21) can be rewritten

$$2\beta\left(-\phi_b\sqrt{I}+\frac{\phi_a}{\sqrt{I}}+\frac{\phi_b}{I^{3/2}}\right)-\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon=0.$$

Let

$$F(I)=2\beta\left(-\phi_b\sqrt{I}+\frac{\phi_a}{\sqrt{I}}+\frac{\phi_b}{I^{3/2}}\right)-\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon.$$

We need only consider $I \geq 1$. Since $2\beta \left(-\phi_b\sqrt{I} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b}{I^{3/2}}\right)$ is decreasing in $I \in [1, \infty)$ and $\frac{I-1}{I+1}\sigma_\eta\sigma_\varepsilon$ is increasing in $I \in [1, \infty)$, $F(I)$ is decreasing in $[1, \infty)$. Then since $F(\infty) < 0$ and $F(1) > 0$, there exists a unique root of $F(I) = 0$ in $[1, \infty)$.

The comparative statics results follow from taking the cross-partial derivatives of the objective function. The cross-partial with respect to I and β is $\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}$, which is positive from equation (21). The other cross-partial derivatives can be immediately signed.

Proof of Proposition 7 (Equilibrium With Costly Information)

The only difference from the previous analysis is that in the action stage of the game, blockholder i now simultaneously chooses her action b_i and whether to become informed.

We proceed by backwards induction. Let J be the number of blockholders that acquire information. In the trading stage, uninformed blockholders cannot expect to make profits and thus do not trade. Therefore, only the J informed blockholders trade and the equilibrium is similar to the one derived in Proposition 1.

Now in the action stage of the game, the manager must choose an action a. Using the same arguments as in Proposition 3, the manager's optimal action is

$$a = \phi_a\alpha\left(\frac{J}{J+1}\right). \tag{40}$$

Blockholders must choose actions b_i and whether to become informed. These decisions can be taken independently since informed trading profits are independent of b_i (which is public), and the choice of b_i depends only on blockholder i's stake β/I. The optimal action of each blockholder is thus

$$b_i = \phi_b\beta\left(\frac{1}{I}\right)^2. \tag{41}$$

From equation (6), if there are I informed blockholders, then each blockholder's trading profits are given by:

$$\frac{1}{\sqrt{I}(I+1)}\sigma_\eta\sigma_\epsilon.$$

A blockholder will acquire information if and only if her trading profits are higher than c. This gives the number J of blockholders that decide to become informed in equilibrium.

Proof of Proposition 8 (Firm Value Optimum With Costly Information)

Let n and $J(I)$ be as given in Proposition 7. Using the results of Proposition 3, the expected firm value is

$$E[\widetilde{v}] = \phi_a \log\left[\phi_a\alpha\left(\frac{J(I)}{J(I)+1}\right)\right] + \phi_b \log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]. \tag{42}$$

We wish to maximize the above expression with respect to I. Since $J(I) = n$ for $I \geq n$, it is never optimal to increase I beyond n since it reduces the second term in the firm value while keeping the first term constant. Therefore, $I^*_{costly} \leq n$.

When $I \leq n$, $J(I) = I$ and the problem is the same as in Proposition 4. From (16) we obtain the desired result.

Proof of Proposition 9 (Perfect Positive Complementarities)

The manager will not exert effort above the level for which

$$\phi_a \log a = \phi_b \log \textstyle\sum_i \widehat{b}_i,$$

i.e.

$$a = \exp\left(\frac{\phi_b}{\phi_a} \log \textstyle\sum_i \widehat{b}_i\right).$$

This derives the optimal a as given in equation (29). Similarly, blockholder i will not exert effort above the level for which

$$\phi_b \log b_i = \phi_a \log \widehat{a} - \phi_b \log \sum_{j \neq i} \widehat{b}_j,$$

i.e.

$$b_i = \exp\left(\frac{\phi_a}{\phi_b} \log \widehat{a} - \log \sum_{j \neq i} \widehat{b}_j\right).$$

A Nash equilibrium requires the following three conditions to hold:

$$\begin{aligned} \phi_b \log I b_i = \phi_a \log a. \\ a \leq \phi_a \alpha \left(\frac{I}{I+1}\right) \\ b_i \leq \phi_b \beta \left(\frac{1}{I}\right)^2. \end{aligned}$$

If the first condition was violated, then the party producing the higher output would gain by reducing effort. The two inequality conditions represent the maximum levels of effort that the manager and blockholders will exert, given the marginal cost of effort.

Out of the continuum of potential Nash equilibria, we seek the one that maximizes firm value. Since firm value is increasing in both a and b_i, it is clear that at least one incentive compatibility constraint will bind. If neither constraint binds, then all parties are exerting suboptimal effort. We could raise the effort levels of all parties while maintaining the equality condition and violating neither constraint.

We now show that, in fact, *both* constraints will bind. Consider the case where $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$. (Starting with $a = \phi_a \alpha \left(\frac{I}{I+1}\right)$ leads to the same result). Then we have

$$\phi_b \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right] = \phi_a \log a$$
$$a = \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right).$$

Recall that we also require $a \leq \phi_a \alpha \left(\frac{I}{I+1}\right)$. Hence firm value is optimized by solving:

$$\max_I \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \text{ s.t. } \exp\left(\frac{\phi_b}{\phi_a} \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]\right) \leq \phi_a \alpha \left(\frac{I}{I+1}\right).$$

The constraint will bind, and so we obtain

$$\phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1}\right)\right] = \phi_b \log \left[\phi_b \beta \left(\frac{1}{I}\right)\right]. \tag{43}$$

The firm value optimum setting I to ensure all parties exert their "full" effort levels. The intuition is as follows. Consider a Nash equilibrium where the blockholders are exerting their full effort (i.e. $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$), and the manager is not (i.e. $a < \phi_a \alpha \left(\frac{I}{I+1}\right)$). b_i is thus constrained by I via the equation $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2$, and so firm value rises if I is reduced to relax this constraint and allow b_i to rise. Unlike in the core model, we do not have the side-effect that reducing I decreases a. I only determines the upper bound to a, not its level. Since $a < \phi_a \alpha \left(\frac{I}{I+1}\right)$, the upper bound is not a constraint anyway. Rather than declining, a will rise to accompany the increase in b_i and ensure that $\phi_b \log I b_i = \phi_a \log a$ still holds.

From equation (43), the optimal number of blockholders is determined implicitly by:

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp\left(\phi_b - \phi_a\right) = Z.$$

Using the quadratic formula, the unique positive solution is

$$\underline{I} = \frac{Z + \sqrt{Z^2 + 4Z}}{2},$$

which is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

Proof of Proposition 10 (Perfect Negative Complementarities)

Deriving $\widetilde{p}$ as in the main model and solving the manager's objective function, he will choose either $a = \phi_a \alpha \frac{I}{I+1}$ or $a = 0$. If $\phi_a \log \left[\phi_a \alpha \frac{I}{I+1}\right] < \phi_b \log \sum_i \widehat{b}_i$, exerting $a = \phi_a \alpha \frac{I}{I+1}$ will have no effect on $\widetilde{p}$ and so the manager will choose $a = 0$. Even if

$\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] \geq \phi_b \log \sum_i \widehat{b}_i$, it is not automatic that the manager will exert effort. Exerting effort increases $\widetilde{p}$ not by $\frac{I}{I+1}\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$, as in the core model, but by only

$$\frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log \sum_i \widehat{b}_i\right)$$

because blockholder effort "supports" firm value even if $a = 0$. Hence the manager choose $a = \phi_a \alpha \frac{I}{I+1}$ only if

$$\alpha \frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log \sum_i \widehat{b}_i\right) \geq a.$$

and so the optimal a is as given by (33). Blockholder i's effort level is derived similarly.

There are two candidates for a Nash equilibrium:

$$\begin{cases} a = 0, b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 \\ a = \phi_a \alpha \frac{I}{I+1}, b_i = 0 \ . \end{cases}$$

Firm value is thus either $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$ or $\phi_b \log\left[\phi_b \beta \frac{1}{I}\right]$. The former is monotonically increasing in I, and maximized at $\phi_a \log(\phi_a \alpha)$ for $I = \infty$. The latter is monotonically decreasing in I, and maximized at $\phi_b \log(\phi_b \beta)$ for $I = 1$. Thus I^* is as given in equation (35).

Proof of Proposition 11 (General Compensation Contract)

Putting the derivative (36) under a common denominator yields

$$F(I, \omega) = \frac{\phi_a \omega I - \phi_b (I + 1 - \omega)(I + 1)}{I (I + 1 - \omega)(I + 1)}.$$

It is therefore a quadratic, and has at most two roots. As $I \to -\infty$, the $\phi_b (I + 1 - \omega)(I + 1)$ term dominates and so the function (36) asymptotes the x-axis from above. As I approaches -1 from below, the function approaches ∞ and is discontinuous at $I = -1$. For $-1 < I < 1 - \omega$, F is negative because the $(I + 1 - \omega)$ term in the denominator now turns negative. It approaches $-\infty$ at $I = -1$ and $I = \omega - 1$. For $\omega - 1 < I < 0$, F is positive since the $(I + 1)$ term in the denominator now turns positive. It approaches ∞ at $I = \omega - 1$ and $I = 0$. Hence, there are no roots for $I < 0$.

As I approaches 0 from above, $F \to -\infty$. As $I \to \infty$, F asymptotes the x-axis from below. Therefore, the function either has zero or two roots. If F has no roots, it is negative for all $I > 0$ and so the optimal number of blockholders is its minimum value of 1. If it has two roots, the upper root I_u is the maximum since the derivative is positive below I_u and negative above I_u. As in the proof of Proposition 5, the cross-partial is sufficient to determine the sign of $\frac{\partial I}{\partial \omega}$. This cross-partial is given by

$$\frac{\partial^2 E[v]}{\partial I \partial \omega} = \frac{\phi_a}{(I + 1 - \omega)^2} > 0.$$

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