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*An Asian Option Approach to the  
Valuation of Insurance Futures  
Contracts*

by  
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Abstract: The insurance futures contracts introduced in December 1992 by the Chicago Board of Trade offer insurers an alternative to reinsurance as a hedging device for underwriting risk. These instruments have the usual features of liquidity, anonymity, and low transactions costs that characterize futures contracts. This paper addresses the issue of pricing insurance futures contracts in an arbitrage-free framework as the expectation under the risk-neutral probability measure of the terminal cash flow provided, for instance, by a long position in a futures contract. Since by definition of the contract the terminal cash flow is related to the aggregate claims incurred during a calendar quarter, the valuation problem is of the same type as the one that arises in the pricing of zero-exercise price Asian options. We propose a solution to this problem using the exact approach developed by Geman and Yor (1992, 1993).

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# AN ASIAN OPTION APPROACH TO THE VALUATION OF INSURANCE FUTURES CONTRACTS

By

J. David Cummins and Hélyette Geman

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## INTRODUCTION

While insurers have a variety of instruments readily available to hedge the risk of assets and interest-rate sensitive liabilities, until recently reinsurance was the only mechanism for hedging underwriting risk. Although reinsurance has a number of desirable characteristics, as explained below, it also has limitations. Reinsurance played a major role in exacerbating the general liability insurance crisis of 1984-1986 (Berger, Cummins, and Tennyson, 1992), and reinsurance markets are subject to periodic price and availability cycles. The insurance futures contracts introduced in December 1992 by the Chicago Board of Trade (CBOT) have a potentially important role to play in stabilizing insurance markets by providing an alternative hedging mechanism for underwriting risk. Although the present contracts are limited to property catastrophes, futures covering other types of losses are likely to be introduced if the catastrophe futures succeed.

Unlike reinsurance, hedging through futures has the advantage of reversibility since any position may be closed before the maturity of the futures contract if the overall exposure of the insurer has diminished. Although reinsurance is, in principle, also reversible, in practice reversing a reinsurance transaction exposes the insurer to relatively high transactions costs as well as additional charges to protect the reinsurer against adverse selection. Because futures contracts are anonymous rather than negotiated between two specific parties, the potential for adverse selection and the accompanying administrative costs are greatly diminished. An insurance futures market should offer

the advantages of liquidity and low transactions costs that are common to futures contracts.

Unlike most futures contracts traded on the CBOT, insurance futures are based on an accumulation of insurance loss payments over a period of time rather than the price of a commodity or asset at the end of a period of time. Consequently, the classical relationships between the spot price and the futures price do not hold. On the other hand, the fact that the futures price at maturity will reflect a sum of claim payments entails a structural similarity between this contract and an Asian option, for which the underlying asset is an average, i.e., a sum of spot prices (up to a multiplicative constant). Even though the evolution of the state variable underlying the insurance claims process is likely to be well-approximated by geometric Brownian motion, an average or accumulation of a lognormal process is not lognormal. Thus, it would be incorrect to price these instruments using standard futures pricing techniques (e.g., the Black, 1976, model) as suggested by some observers (CBOT, 1992, Cox and Schwebach, 1992).

The same difficulties arise in the pricing of Asian options and have been mostly addressed in recent years by using approximations (Kemna and Vorst, 1990, Carverhill and Clewlow, 1990, and Lévy, 1990). Geman and Yor (1992, 1993) investigate the exact solution of this problem. They are able to provide, firstly, the moments of all orders of the arithmetic average of a lognormal distribution and, secondly, a closed form expression for the Asian option price for some values of the exercise price; for other values of the exercise price they provide the Laplace transform of the Asian option price. In this paper, we apply the Geman-Yor [G-Y] approach to the valuation of the insurance catastrophe futures contracts offered by the Chicago Board of Trade (CBOT).

Although the present paper focuses on insurance futures, the methodology is quite general and could be used to price other types of insurance contracts. Most previous insurance financial pricing models have modeled insurance in a Black-Scholes framework (e.g., Cummins, 1988). Losses are modelled as following a geometric Brownian motion process, with loss settlement based on the end-

of-period value of the process<sup>1</sup>. In reality, however, payoffs under most property-liability insurance contracts are based on loss accumulations, rather than end-of-period realizations. Thus, such contracts are related to average-rate (or Asian) options rather than to European or American options.

The paper is organized as follows: In section I, we describe insurance futures contracts and discuss the merits of these hedging instruments relative to reinsurance. Section II proposes an arbitrage-free valuation of insurance futures contracts, extending the analysis in G-Y (1992, 1993). We begin by developing a pricing model for non-catastrophic losses and then extend this model by introducing a jump process to represent catastrophes. Section III presents some numerical illustrations of insurance futures prices, using quarterly data on insurance claims and daily pricing data obtained from the CBOT. Section IV concludes the paper.

## **I. Insurance Futures**

The CBOT introduced insurance futures in December 1992. The initial offerings are limited to catastrophic property-insurance losses. The two instruments initially introduced cover national and eastern property catastrophes. The latter are viewed as important because of the exposure to hurricane losses on the Eastern seaboard. In May 1993, Midwestern catastrophe futures were introduced, again motivated in part by region-specific windstorm exposure. Property catastrophes are an important source of underwriting risk, as illustrated by Hurricane Andrew, which led to substantial losses of equity capital in the industry and several insurance company insolvencies.<sup>2</sup>

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<sup>1</sup>A similar approach, also involving the assumption that losses are based on an end-of-period realization rather than an average is based on the risk-neutral valuation models developed by Brennan (1979) and Stapleton and Subramanyam (1984). See Doherty and Garven (1986) for an insurance application of this method.

<sup>2</sup>Gross losses from Hurricane Andrew were \$13.7 billion, of which conventional catastrophe reinsurance covered only about \$3 billion. Snyder (1993). During the same twelve month period, insurers were also hit by major catastrophe losses from Hurricane Iniki and the Los Angeles riot.

In addition to the importance of property catastrophes as a risk exposure, another reason for the CBOT's focus on property losses is that such losses settle relatively quickly and thus are not subject to the lengthy payout period and accompanying loss estimation errors that characterize other risky coverages such as commercial liability insurance and workers' compensation. Property losses are relatively insulated from errors due to misstatements and manipulations of loss reserves.

Due to the relatively high loss volatilities characterizing property coverages, insurers should have a strong interest in hedging underwriting risk arising from property coverages. Nevertheless, trading in insurance futures has been light. This is most likely attributable to the fact that most insurers lack experience with financial hedging. The opposite side of the market (sellers of futures) consists primarily of speculators.

In addition to catastrophe futures, the CBOT is also developing homeowners insurance futures that would not be limited to catastrophic losses but would cover homeowners property losses from all sources. Our methodology could also be used to price these contracts.

### **Catastrophe Futures**

A unique characteristic of insurance futures is that there exists no market price, published index value, or yield rate on which to base settlement values. Accordingly, the CBOT has had to create an underlying instrument to form the basis for futures trading. For catastrophe futures, the instrument consists of losses reported each quarter to the Insurance Services Office (ISO), a well-known statistical agent. Approximately 100 companies report property loss data to the ISO. The settlement values for insurance futures are based on losses incurred by a pool of at least ten of these companies selected by the ISO on the basis of size, diversity of business, and quality of reported data. The list of reporting companies included in the pool for any given futures contract is announced by the CBOT prior to the beginning of the trading period for that contract. The CBOT also announces

the premium volume for companies participating in the pool prior to the start of the trading period for each catastrophe contract. Thus, the premiums in the pool are a known constant throughout the trading period, and price changes are attributable solely to changes in the market's expectations of loss liabilities.

Catastrophe insurance futures trade on a quarterly cycle, with contract months March, June, September, and December. A contract for any given quarter is based on losses occurring in the prior calendar quarter as reported by the participating companies at the end of the contract quarter. For example, the September 1993 contract covers losses from events occurring during the second quarter of 1993 (April through June) as reported by the end of September. The three additional months following the close of the "event quarter," are to allow for loss settlement and data processing lags that are common in insurance. Although not all losses will be reported by the end of the two quarter reporting period, reported pool losses should represent a high proportion of eventual paid losses, particularly in view of the fact that companies are allowed to report estimated losses in addition to those already paid.<sup>3</sup> Of course, the use of estimated rather than paid losses introduces potential errors into the contract settlement values and may create incentives for moral hazard (see below).

Unlike most insurance and reinsurance arrangements, insurance futures do not focus on a particular type of policy (such as homeowners or automobile insurance) but rather on particular types of losses. Losses included in the pool consist of all property losses incurred by the reporting companies arising from the perils of windstorm, hail, earthquake, riot, and flood. Reported losses can arise from eight different lines of insurance including homeowners, commercial multiple peril, earthquake, and automobile physical damage.<sup>4</sup> Even though the contracts are called catastrophe

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<sup>3</sup>In insurance terminology, the pool is based on incurred (paid plus estimated unpaid) losses rather than solely on paid losses.

<sup>4</sup>Other lines of insurance included in the pool are fire, allied lines, farmowners, and commercial inland marine (see CBOT, 1992).

futures, in fact all losses (i.e., not just catastrophe losses) for the specified perils and lines of business are included in the loss pool. However, the losses in the pool are expected to be highly correlated with property catastrophe losses because the included perils were chosen as those most susceptible to catastrophes. The use of a proxy approach rather than true catastrophe losses seems to have been motivated by the need to limit data processing costs. Trading begins as soon as a contract is listed and ends on the fifth day of the fourth month following the contract month. Thus, settlement on the September 1993 futures takes place on January 5, 1994.

Contract settlement is based on the loss ratio of the business reported to the ISO pool, i.e., the ratio of reported incurred losses to earned premiums.<sup>5</sup> The contracts trade in units of \$25,000 with prices quoted in percentage points and tenths of points. E.g., a price of 11.2 corresponds to a loss ratio of 11.2 percent and an expected settlement value of \$25,000\*. 112 = \$2,800. The average trading prices during March 1993 were 10.2 for the March 1993 contract, 9.2 for the June 1993 contract, 17.1 for the September 1993 contract, and 25.4 for the December 1993 contract. These ratios are much less than the total loss ratios for the covered lines of business because of the limitation of reported losses to the specified perils proxying for catastrophes. Contract settlement is subject to a maximum loss ratio of 200 percent.

A more precise statement of the final settlement value for a contract is provided below:

$$F(T) = \$25,000 \text{ Min}\left(\frac{L(T)}{\Pi}, 2\right) = \$25,000 \left[ \frac{L(T)}{\Pi} - \text{Max}\left(\frac{L(T)}{\Pi} - 2, 0\right) \right] \quad (1)$$

where  $F(T)$  = futures price at maturity,  $L(T)$  = losses incurred during the six month payment period, and  $\Pi$  = premiums earned during the three month exposure (loss event) period. Thus, the

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<sup>5</sup>Premiums earned is an accrual accounting measure indicating the premiums attributable to coverage provided during a specified period of time. For example, if a policy is issued on January 1 and has an annual premium of \$1,200, \$300 of the premium will be earned during the first quarter of the year. Using premiums earned as the denominator thus matches revenues against coverage provided (as measured by loss events).

settlement value is trading unit (\$25,000) times the loss ratio (L(T)/II), capped at an amount equal to twice the trading unit (i. e., a loss ratio of 2.0). This is equivalent to a long position in the loss ratio plus a short position in a call option on the loss ratio with a strike price of 2.0. The latter relationship is used below in pricing the futures contract. The CBOT placed a maximum on the settlement value both to reduce credit risk in the event of unusually large losses and to make the contract look more like reinsurance policies, which usually have upper limits (see below).

The insurer's net gain from a long position in a futures contract is the settlement price minus the price at the inception date of the contract. The price of the futures contract at any given time reflects the market's expectation of the event quarter's catastrophic loss in relation to the earned premiums for that quarter.

To illustrate some of the unique aspects of insurance futures, consider a simple hedging example. Assume that an insurer anticipates \$5 million in earned premiums on policies subject to catastrophes during the first quarter of the year. The insurer forecasts catastrophic losses of \$600,000 for this quarter, i.e., a catastrophic loss ratio of 0.12. The firm wants to hedge against catastrophic losses greater than \$600,000 by purchasing June futures contracts. Assume that the CBOT announces that the total premium volume for companies included in the June catastrophe insurance pool is \$3 billion. The company's actuaries predict that the ISO companies will incur catastrophic losses of \$345 million during the event quarter (the fourth quarter of the preceding year) for an expected loss ratio of 11.5 percent and that 80 percent of these losses will be reported by the end of March. The insurance futures price at the beginning of the trading period,  $F(0)$ , will reflect the market's expectations regarding the loss ratio of the homeowners pool. If the expectations of the company's actuaries are shared by the market, the price of an insurance futures contract is equal to:

$$\$25,000 * .115 * .8 = \$2,300.$$

To simplify the example, we assume that the expectations of the hedging company and the

market are identical. We also assume that the company's catastrophic loss ratio is perfectly correlated with the loss ratio of the pool. Under these assumptions, the insurer can fully hedge its catastrophe risk by taking a long position in insurance futures contracts. The number of contracts is determined by the following formula:

$$N_i = \frac{P_i}{V} \frac{h_i}{R_i} = \frac{5,000,000}{25,000} \frac{1.0}{0.8} = 250 \quad (2)$$

where  $N_i$  = number of contracts purchased by company i,

$h_i$  = proportion of its anticipated catastrophe losses firm i desires to hedge,

$P_i$  = premium volume of firm i,

$R_i$  = expected proportion of pool losses reported to ISO by end of reporting period,

$V$  = insurance futures contract value.

Assume that the futures contracts are held to expiration and both the pool's and the insurer's loss ratio are 5 percentage points higher than expected. At maturity, the gain on a long position in the futures contracts will be  $250 * [F(T) - F(0)] = 250 * 25,000 * 0.05 * 0.8 = \$250,000$ , where T = the expiration date. The gain on the futures contracts obviously equals 5 percent of the insurer's earned premiums and makes up for the extra losses encountered. Of course, if the loss ratio of the pool is lower than expected, the insurer incurs a loss from the long position.<sup>6</sup> If the insurer's and pool's loss ratio movements are not perfectly correlated, the hedge provided by the futures contract will not be perfect.<sup>7</sup>

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<sup>6</sup>However the insurer's initial expected loss ratio would still be achieved under the assumption of perfect correlation between the insurer's and the pool's loss ratios because the loss on the futures position would be offset by lower losses on the insurer's book of business.

<sup>7</sup>If the insurer only wants to protect against higher than anticipated loss ratios but wants to benefit from lower than expected ratios, it can buy futures call options (also offered by the CBOT) rather than futures. As with other futures call options, there is a cash settlement at the maturity of the option if the option is exercised.

We summarize this discussion by listing some of the unique and unusual characteristics of insurance futures:

1. There is no market price for insurance losses. The CBOT has created an index based on losses reported by a sample of insurers.
2. The loss ratio of the pool is not perfectly correlated with any given insurer's catastrophe loss ratio. Thus, a hedge created with insurance catastrophe futures will not be perfect.
3. Because the pool is based on all losses for certain types of perils and policies especially vulnerable to catastrophes, the pool loss ratio is actually a catastrophe proxy and thus is not perfectly correlated with the true catastrophe loss ratio of insurers reporting to the pool .
4. Because insurers report incurred rather than paid losses to the pool, the contract settlement value is subject to loss estimation error.
5. For regulatory reasons, insurers are unlikely to take short positions in futures contracts. Thus, the supply side of the market is represented almost exclusively by speculators.

### **Futures and Reinsurance**

As mentioned above, insurance futures are very similar to certain types of reinsurance contracts that are widely used in the property-liability insurance industry. Reinsurance, which is insurance among insurance companies, provides a way for insurers to efficiently diversify risk. The market for reinsurance is international, reflecting the economic principle that it is efficient to subdivide risks as finely as possible, subject to limitations imposed by transactions costs (e.g., Samuelson, 1963). Transactions costs traditionally have been low in reinsurance markets, facilitating diversification during normal periods. However, within the past two decades, reinsurance markets have sustained severe loss shocks (e.g., oil tanker disasters and rising liability losses in the U. S.) as well as increasing problems with asymmetric information caused by high-risk, low-price market entrants, many of which failed during the mid-1980s. As a result, reinsurance markets have experienced severe price fluctuations and availability problems. This creates a potentially important role for futures markets, which may be less affected by some of these problems than reinsurance markets.

The types of reinsurance most comparable to the insurance futures are known as non-proportional contracts. Such contracts have a mathematical structure similar to options and other types of financial-market derivative securities.<sup>8</sup> A general specification, fitting most types of non-proportional reinsurance, is the following:

$$V(L | M, R) = L - \text{Max}(L - M, 0) + \text{Max}(L - R, 0) \quad (3)$$

where  $V(L | M, R)$  = the primary insurer's loss at the expiration date of the reinsurance,  $L$  = total losses incurred by the primary insurer,  $M$  = the primary insurer's retention (or reinsurance point of attachment), and  $R$  = the upper limit of the reinsurance contract,  $R > M$ . The reinsurance contract has the effect of a deductible of  $M$ , i.e., the primary insurer bears the losses if they are less than  $M$  but is limited to a loss of  $M$  if losses are between  $M$  and  $R$ . If losses exceed  $R$ , the difference between  $L$  and  $R$  also must be borne by the primary insurer. It is not uncommon for the primary insurer to buy a reinsurance contract from another reinsurer that attaches at  $L = R$  and has an upper limit  $> R$ .

Although the mathematical structure of reinsurance is similar to that of financial contingent claims, there are important differences between reinsurance and financial claims. While the parties to a futures contract are anonymous to one another, reinsurance is negotiated between the parties to the contract. The negotiation process rather than an auction market determines the price of reinsurance. Although the primary company may negotiate with several reinsurers before making a decision, the buyer of reinsurance is less assured of receiving an informationally efficient, arbitrage-free price than the buyer of actively traded futures and options. Reinsurance prices and contract terms are specific to

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<sup>8</sup>The other major type of reinsurance is *proportional* reinsurance. This type of reinsurance involves the proportionate division of premiums and losses between the primary insurer and the reinsurer according to a pre-arranged constant proportion between 0 and 1. Proportional contracts do not have the same mathematical structure as the CBOT's futures offerings.

the buyer of the contract, unlike financial claims which are standardized to all buyers and sellers. Thus, reinsurance can be tailored to the specific needs of the buyer, but transactions costs are likely to be higher than in futures markets. Due to the buyer specific nature of reinsurance, it is usually not possible to close out a position in the reinsurance market by taking an opposite position, as it is in futures markets. Thus, futures markets provide better liquidity than reinsurance.

The reinsurer *underwrites the* buyer, in the sense of investigating the quality of the buyer's book of business and management, to prevent adverse selection. Contractual terms such as cost-sharing provisions are often used to reduce moral hazard. Other than guarding against manipulation of the pool by participating insurers, the costs of adverse selection and moral hazard are minimal in futures markets. Credit risk also is of minimal importance in futures markets, because of the daily margin adjustment, the existence of the clearing house, and daily trading limits.<sup>9</sup> On the other hand, buyers of reinsurance need to be concerned about the solvency of their reinsurers. If the reinsurer defaults, the primary insurer is still fully liable to its policyholders for reinsured policies.

An advantage of reinsurance over futures is that reinsurance covers the primary insurer's own loss experience, whereas futures are based on a policy pool that is not perfectly correlated with the hedger's losses. In addition, reinsurers often provide "real" services to primary companies such as advice and assistance in underwriting, primary market pricing and contract design, and loss settlement (Mayers and Smith, 1990). Such services obviously are not available in futures markets. Thus, reinsurance and futures are similar but are not perfect substitutes; there is likely to be a continuing role for both types of contracts in hedging underwriting risk.

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<sup>9</sup>Daily trading limits are ten points (\$2,500). If the daily limit goes into effect for two days in a row, the limit is raised to 15 points (\$3,750). The higher limit stays in effect until the first day that prices move by less than 10 points, at which time the initial ten point limit is reinstated.

## II. The Valuation of Insurance Futures Contracts

The objective of this section is to develop pricing models for insurance futures. We begin with a brief discussion of the traditional actuarial model for pricing property-liability insurance, explaining why this model is not appropriate for futures pricing. We then develop futures pricing models using the approach of Geman and Yor (1992, 1993) for two cases: (1) Futures on insurance contracts subject to non-catastrophic claims accumulations, and (2) catastrophe insurance futures.

### The Traditional Insurance Pricing Model

Actuaries traditionally have valued insurance claim accumulations such as those underlying catastrophe insurance futures as random sums (see, for example, Panjer and Willmot, 1992):

$$C = \sum_{i=1}^n X_i \quad (4)$$

where  $C$  = total claims accumulation over some period of time,

$n$  = the number of claims, and

$X_i$  = the amount of loss for claim  $i$ .

The number of claims ( $n$ ) is a random variable modeled by a discrete probability distribution such as the Poisson or negative binomial, while the claim amounts (also random) usually are modeled by a continuous distribution such as the gamma distribution. The  $X_i$  are assumed to be independent and identically distributed and independent of the claim number process ( $n$ ). The probability distribution of  $C$  is:

$$f_C(x) = \sum_{k=0}^{\infty} p(k) f_x^{k*}(x) \quad (5)$$

where  $p(k)$  = a discrete probability distribution such as the Poisson,

$f_x^{k*}(x)$  = the  $k$ th convolution of the probability distribution of claims severity.

The price of insurance based on this model is typically equal to the expected value plus an additive function of the second moment of the distribution (e.g., a constant times the variance or standard deviation). This pricing approach is inconsistent with the value-additivity principle which prevails in financial markets, as has been observed by some leading actuaries (e.g., Buhlmann, 1980). In contrast, our pricing approach is based on the avoidance of arbitrage opportunities<sup>10</sup> and on the modeling of the *instantaneous* claim process as a geometric Brownian motion (with a jump component in a second stage). Since aggregating claims is equivalent, up to a scale factor, to averaging claims, the approach we propose in the following sections is based on the exact Asian option pricing methodology developed by Geman and Yor (1992 and 1993). As explained above, the use of an averaging approach differentiates our models from previous insurance financial pricing models, which value losses as end-of-period realizations.

### **General Assumptions**

This section discusses the general assumptions applicable to both the non-catastrophe and catastrophe insurance futures models. The general assumptions are as follows:

**A<sub>0</sub>.** The premium volume of the pool is known with certainty. Hence, randomness of the pool's experience is entirely attributable to losses.

**A<sub>1</sub>.** There is no adverse selection or moral hazard. Adverse selection is precluded because the pool of contracts is constituted before the futures contracts begin trading. Moral hazard, e.g., manipulation of pool loss experience by company's contributing data to the pool, which seems to be a potential problem, is also unlikely, for the following reasons:

(a) Assume that an insurer participating in the pool takes a long position in the futures

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<sup>10</sup>Although our contingent claim (the insurance futures contract) cannot be duplicated with traded instruments; this is also true of other options and futures written on non-traded underlying assets such as various types of economic indices, where arbitrage free valuation seems quite reasonable.

market and overstates the amount of claims on its share of the pool. Since the pool experience is based on property losses, which settle quickly, it should be relatively easy to detect systematic claim overstatements through periodic audits of pool members.

(b) Assume that the insurer takes a short position in the futures market and understates or delays the reporting of claims incurred prior to the expiration of the futures contract. The extra gain on the futures contract would be offset by the loss of the hedge on the unreported claims. Even though this situation is technically more feasible than that discussed in case (a), it is unlikely on the part of insurers, who are naturally hedgers rather than speculators.

A<sub>2</sub>. There are no transactions costs, i.e., the insurer incurs no fees in taking either a short or long position in a futures contract.

A<sub>3</sub>. Futures contracts are infinitely divisible; any fraction can be bought or sold.

A<sub>4</sub>. Default risk is non-existent due to the clearing house, daily marking to market, and trading limits.

A<sub>5</sub>. Insurers participating in the pool report loss experience continuously, and the pool's loss experience at any given time is known to all market participants.

A<sub>6</sub>. There is no arbitrage in the sense defined by Harrison and Kreps (1979) and Harrison and Pliska (1981). I.e., if we represent the uncertainty in the economy by a probability space  $(\Omega, \mathcal{F}, P)$  and the flow of information occurring over time by a right-continuous filtration  $\mathcal{F}_t$ , there exists a "risk-neutral" probability measure  $Q$  equivalent to  $P$  under which the price of any security (expressed in the money market accumulation factor as numéraire) is a martingale. Moreover, financial markets are complete, i.e., any contingent claim has a unique price, which verifies the same martingale property.

### **Futures Pricing for Non-Catastrophic Loss Processes**

In addition to the general assumptions specified above, our futures pricing model for non-

catastrophic loss processes is based on the following additional assumption:

**A<sub>7</sub>**. The amount of claims  $S(t)$  incurred on the policies in the pool at any time  $t$  belonging to the listing period  $[0, T]$  of the futures contract is a diffusion process driven by the stochastic differential equation:

$$dS(t) = \mu S(t) dt + \sigma S(t) dW(t) \quad (6)$$

where  $S(0)$  is given,  $W(t)$  is a P-Brownian motion, and  $\mu$  and  $\sigma$  are constants representing, respectively, the expected growth and instantaneous standard deviation of the claims process. Even though  $S(t)$  does not represent the value of a traded asset, we assume (as in Shimko, 1992) that assumption **A<sub>6</sub>** entails the existence of an equilibrium relationship of the type  $\mu = \alpha + \rho\sigma$ , where  $\rho$  represents the market price of claim level risk, taken as a constant on the interval  $[0, T]$  and  $\alpha$  is the risk-adjusted drift of the claim level (sometimes called the risk-adjusted growth rate). Consequently, under  $Q$ , the process  $S(t)$  is driven by:

$$dS(t) = \alpha S(t) dt + \sigma S(t) d\tilde{W}(t) \quad (7)$$

where  $\tilde{W}(t)$  is a Q-Brownian motion and  $\alpha$  is constant.

It is clear that standard option pricing formulas cannot be used to value insurance futures contracts, contrary to some discussions in the actuarial literature (e.g., Sherman, 1991). This is precluded for two reasons: (1) The settlement value is an average of a Brownian process rather than an end-of-period value, and (2) there is no premium to pay to take a position in a futures process, but at the same time there is significant downside risk, which is never the case with an option. However, a European call option and a futures contract share a major common feature: they are both contingent claims which give the right to a random cash flow at a well-defined time  $T$ , and consequently, their pricing should be addressed with the same methodology.

However, the price of a futures contract does not exactly satisfy the property in assumption 7

(A<sub>7</sub>) since it is not the price of an asset (e.g., opening a long position in a futures contract requires a null investment). But the price of a futures contract is equal to the expectation under the risk-neutral probability Q of its settlement value (see Jamshidian, 1989, Geman, 1989). Consequently, at any time  $t \in [0, T]$ , the market price of the futures contract is equal to:

$$\begin{aligned} F(t) &= E_Q[F(T) | \mathcal{F}_t] \\ &= \frac{\$25,000}{\Pi} \{ E_Q[L(T) | \mathcal{F}_t] - E_Q[\text{Max}(L(T) - 2\Pi, 0) | \mathcal{F}_t] \} \end{aligned} \quad (8)$$

To obtain the futures price, it is necessary to evaluate the two quantities:

$$V(t) = E_Q[L(T) | \mathcal{F}_t] \quad (9)$$

$$Y(t) = E_Q[\text{Max}(L(T) - 2\Pi, 0) | \mathcal{F}_t] \quad (10)$$

We can observe that

$$L(T) = \int_0^T S(s) ds = T A(T) \quad (11)$$

where A(t) is defined as the averaging process:

$$A(t) = \frac{1}{t} \int_0^t S(s) ds \quad (12)$$

This averaging process is exactly the one whose value at time T is compared to the strike price to define the payoff of an Asian option. As discussed earlier, there is no reason to believe that this process is a geometric Brownian motion, and the calculation of V(t) is not straightforward. However, observing that  $V(t) = TE_Q[A(T) | \mathcal{F}_t]$ , we can interpret V(t) up to a constant factor as the value of an Asian option with maturity T and exercise price zero.

At this point it may be useful to recall some properties and results regarding Asian options that

have been derived in recent years. Asian options have become extremely popular, especially on thinly traded assets such as gold and some other commodities. Comparing at maturity the strike price with an average of spot prices over a time interval prevents price manipulations on maturity day by institutions with significant shares of the market for the underlying asset. A high proportion of currency options and most options on oil traded today are Asian options. However, the pricing of such instruments is not straightforward since, with the standard assumption of a geometric Brownian motion for the asset price dynamics, the distribution of the average is unknown and has no reason to be of the same type. That is why Asian option pricing has been addressed primarily through approximations, e.g., the approximation of the arithmetic average by the geometric average (Kemna and Vorst, 1990) and numerical procedures using fast Fourier transforms (Carverhill and Clewlow, 1990). However, prices calculated using these numerical procedures are subject to unknown errors of approximation. Other authors (e.g., Lévy, 1990) have suggested approximating the distribution of the average by a lognormal distribution. This leads naturally to a Black-Scholes type formula, but this formula is not an accurate reflection of the revelation of the average over time.

Consider an asset  $S(t)$  whose dynamics are driven, under the risk-neutral probability measure  $Q$ , by the stochastic differential equation (7). This process is averaged on the interval  $[0, T]$  through formula (12). Assuming constant interest rates on the interval  $[0, T]$ , G-Y (1993) establish that the price at time  $t$  of the Asian option with maturity  $T$  and exercise price  $k$ , i.e.,

$$C_{t,T}(k) = E_Q[e^{-r(T-t)} \text{Max}(A(T) - k, 0) | \mathcal{F}_t] \quad (13)$$

can be written in the following form:

$$C_{i,T}(k) = \frac{e^{-r(T-t)}}{T} \frac{4S(t)}{\sigma^2} C^{(v)}(h, q) \quad (14)$$

$$\text{where } v = \frac{2\alpha}{\sigma^2} - 1; \quad h = \frac{\sigma^2}{4}(T-t); \quad q = \frac{\sigma^2}{4} \left[ \frac{kT - \int_0^t S(s) ds}{S(t)} \right]$$

and  $\alpha$  is the constant risk-neutral drift of (4).

Since G-Y assume that interest rates are constant, we can immediately derive from equation (14) that

$$E_Q[\text{Max}(A(T) - k, 0) | \mathcal{F}_t] = \frac{4S(t)}{\sigma^2 T} C^{(v)}(h, q) \quad (15)$$

Equation (15) is what we need to value an insurance futures contract. The quantity  $C^{(v)}(h, q)$  in G-Y has a very simple expression when  $q$  is negative since it only involves the first-order moment of  $A(T)$ , and from G-Y (1992),

$$C^{(v)}(h, q) = \frac{1}{2(1+v)} [e^{2(1+v)h} - 1] - q \quad (16)$$

Observing that  $V(t) = TE_Q[A(T) | \mathcal{F}_t] = TE_Q[\text{Max}(A(T), 0) | \mathcal{F}_t]$ , i.e., that  $V(t)$  is, up to a multiplicative constant, the value of a zero-exercise price Asian option, we derive from (14), (15), and (16) that

$$\begin{aligned} V(t) &= \int_0^t S(s) ds + \frac{2S(t)}{\sigma^2(v+1)} [e^{\frac{(v+1)}{2}\sigma^2(T-t)} - 1] \\ &= \int_0^t S(s) ds + \frac{S(t)}{\alpha} [e^{\alpha(T-t)} - 1] \end{aligned} \quad (17)$$

Formula (17) clearly shows that claims already incurred over the interval  $[0, t]$  contribute a non-random amount to the total amount that will be observed at time  $T$ , and hence to the expected value at time  $t$  of this total amount, which is exactly  $V(t)$ .

When  $q$  is positive, G-Y need to use Bessel processes to calculate  $C^{(v)}(h,q)$ . They do not provide a closed-form expression for this quantity but only its Laplace transform with respect to the variable  $h$ :

$$\int_0^{\infty} e^{-\lambda h} C^{(v)}(h, q) dh = \frac{\int_0^{\frac{1}{2q}} e^{-t} t^{\frac{\mu-v}{2}-2} (1-2qt)^{\frac{\mu+v}{2}+1} dt}{\lambda(\lambda-2-2v)\Gamma(\frac{\mu-v}{2}-1)} \quad (18)$$

where  $\mu = \sqrt{2\lambda + v^2}$

and  $\Gamma$  denotes the gamma function. As we did for  $V(t)$ , we can write  $Y(t)$  using equation (15), as

$$Y(t) = TE_Q[\text{Max}(A(T) - \frac{2\Pi}{T}, 0) | \mathcal{F}_t] = \frac{4S(t)}{\sigma^2} C^{(v)}(h, q_1) \quad (19)$$

where  $q_1 = \frac{\sigma^2}{4} \left[ \frac{2\Pi - \int_0^t S(s) ds}{S(t)} \right]$

and  $v$  and  $h$  are defined as before. Most generally, at any time  $t \in [0,T]$ ,  $\int_0^t S(s)ds$  will be smaller than  $2\Pi$  and the quantity  $q_1$  will be positive. This necessitates the evaluation of the Laplace transform to obtain  $C^{(v)}(h,q_1)$ . In the rare cases where  $q_1$  is negative, we obtain  $C^{(v)}(h,q_1)$  directly from equation (16).

The value of the futures contract  $F(t)$  at any time  $t$  is obtained by inserting  $V(t)$  and  $Y(t)$  into equation (8):  $F(t) = (\$25,000/\Pi)[V(t)-Y(t)]$ . Our valuation of  $F(t)$  does not require deterministic interest rates since formula (8) holds even when interest rates are stochastic, i.e., the assumption of constant interest rates in G-Y (1993) is not needed in the use we make of their results.

### The Case of Catastrophe Insurance Futures Contracts

To deal with the valuation of catastrophe insurance futures, we retain assumptions  $A_1$  to  $A_6$  but change assumption  $A_7$  to  $A_7'$  in order to incorporate a jump component into the pure diffusion process

we previously had used to represent the amount of claims  $S(t)$  incurred for the policies in the pool:

A<sub>7</sub>. The amount of claims  $S(t)$  incurred at any time  $t$  in the listing period  $[0, T]$  is driven by the following stochastic differential equation:

$$dS(t) = \alpha S(t_-) dt + \sigma S(t_-) dW(t) + k dN(t) \quad (20)$$

where  $k$  = a positive constant representing the severity of the loss jump component due to catastrophes, and

$N(t)$  = a Poisson process with intensity  $\lambda$ .

We can observe at this point that the addition of an extra source of randomness makes A<sub>6</sub> a stronger assumption; since we do not believe that the risk of the jump component can be diversified away, this means that we assume that a contingent claim related to this risk is traded, which preserves the market's completeness.

Denoting  $d\Gamma(t) = \alpha dt + \sigma dW(t)$  and  $dH(t) = k dN(t)$ , we know that the equation:

$$dS(t) = S(t_-) d\Gamma(t) + dH(t) \quad (21)$$

has the general solution:

$$S(t) = Z(t) e^{[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} \quad (22)$$

where the process  $Z(t)$  needs to be specified. Using Ito's rule in equation (22), we obtain:

$$dS(t) = e^{[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} [dZ(t) + Z(t) d\Gamma(t) + d\langle \Gamma, Z \rangle_t] \quad (23)$$

where  $\langle \cdot \rangle$  denotes the bracket (see, for instance, Protter, 1990).

The equality between the two expressions given above for  $dS(t)$  provides the following relationship:

$$dZ(t) + d\langle \Gamma, Z \rangle_t = e^{-[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} dH(t) \quad (24)$$

Taking the bracket with  $\Gamma(t)$  on both sides, the second term cancels out since  $d\langle \Gamma, Z \rangle_t$  is by definition a bounded variation process and  $\Gamma(t)$  is a continuous process. After this operation, we have:

$$d\langle Z, \Gamma \rangle_t = e^{-[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} d\langle H, \Gamma \rangle_t \quad (25)$$

But  $d\langle H, \Gamma \rangle_t$  is itself null since the process  $H$  is purely discontinuous and  $\Gamma$  is continuous.

Equation (24) now reduces to:

$$dZ(t) = e^{-[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} dH(t) \quad (26)$$

We can solve equation (26) to obtain:

$$Z(t) = Z(0) + \int_0^t e^{-[(\alpha - \frac{\sigma^2}{2})u + \sigma W(u)]} dH(u) \quad (27)$$

and

$$S(t) = e^{[(\alpha - \frac{\sigma^2}{2})t + \sigma W(t)]} [Z(0) + \int_0^t e^{-[(\alpha - \frac{\sigma^2}{2})u + \sigma W(u)]} dH(u)] \quad (28)$$

Introducing the process  $X(t) = (\alpha - \sigma^2/2)t + \sigma W(t)$ , we can write  $S(t)$  as

$$S(t) = [S(0) + \int_0^t e^{-X(u)} dH(u)] e^{X(t)} \quad (29)$$

We now consider the aggregate claim amount at time  $T$ :

$$TA(T) = \int_0^T S(s) ds = \int_0^T S(0) e^{X(s)} ds + \int_0^T e^{X(s)} ds \int_0^s e^{-X(u)} dH(u) \quad (30)$$

where  $u < s < T$ . The last integral can also be written as:

$$\int_0^T dH(u) \int_u^T e^{[X(s) - X(u)]} ds \quad (31)$$

For the sake of simplicity, we first compute the expectation under  $Q$  of the quantity  $TA(T)$  at time 0 when the contract starts trading. This expectation will be the sum of two terms  $U_1$  and  $U_2$

where

$$U_1 = E_Q \left[ \int_0^T S(0) e^{X(s)} ds \right] \quad (32)$$

$$U_2 = E_Q \left[ \int_0^T dH(u) \int_u^T e^{[X(s) - X(u)]} ds \right] \quad (33)$$

Keeping in mind that  $X(s) = (\alpha - \sigma^2/2)s + \sigma W(s)$ , the computation of  $U_1$  is straightforward and gives  $U_1 = S(0)[e^{\alpha T} - 1]/\alpha$ .

In order to get simple closed-form expressions, we now need to assume that the Poisson process  $H$  is independent of  $\exp[X(s) - X(u)]$ . Assuming the independence of the two components of the claim process seems quite reasonable and allows us to write

$$U_2 = k \lambda E \left[ \int_0^T du \int_u^T e^{[X(s) - X(u)]} ds \right] \quad (34)$$

which reduces to

$$U_2 = k\lambda \int_0^T du \int_u^T e^{(s-u)\alpha} ds = \frac{k\lambda}{\alpha^2} [e^{\alpha T} - \alpha T - 1] \quad (35)$$

We now turn to the general situation where the current time  $t$  is in the interval  $[0, T]$  and where we need to calculate  $T E[A(T) | \mathcal{F}_t]$ . Clearly,

$$TE[A(T) | \mathcal{F}_t] = \int_0^t S(s) ds + E\left[\int_t^T S(s) ds | \mathcal{F}_t\right], \quad \text{and} \quad (36)$$

$$E\left[\int_t^T S(s) ds | \mathcal{F}_t\right] = E\left[\int_t^T e^{X(s)} ds \left\{1 + \int_0^s e^{-X(v)} dH(v)\right\} | \mathcal{F}_t\right]$$

where  $t < s < T$  and  $0 < v < s$ . This last expectation is in turn the sum of three terms that we denote  $U_1'$ ,  $U_2'$ , and  $U_3'$ , where

$$U_1' = e^{X(t)} \int_t^T e^{(s-t)\alpha} ds \quad (37)$$

$$U_2' = \int_0^t dH(v) e^{-X(v)} \left[ e^{X(t)} \int_t^T e^{(s-t)\alpha} ds \right] \quad (38)$$

$$U_3' = E\left[\int_t^T e^{X(s)} ds \int_t^s e^{-X(v)} dv\right] \quad (39)$$

We first observe that

$$U_1' + U_2' = \frac{e^{(T-t)\alpha} - 1}{\alpha} \left\{ e^{X(t)} \left[ 1 + \int_0^t e^{-X(v)} dH(v) \right] \right\} \quad (40)$$

where we exactly recognize  $S(t)$  as the second factor. We compute  $U_3'$  with the same arguments as

before:

$$U_3' = k \lambda \int_t^T dv \int_v^t E[e^{X(s)-X(v)}] ds = k \lambda \frac{e^{(T-t)\alpha} - (T-t)\alpha - 1}{\alpha^2} \quad (41)$$

Finally, we obtain:

$$TE[A(T) | \mathcal{F}_t] = \int_0^t S(s) ds + S(t) \left[ \frac{e^{(T-t)\alpha} - 1}{\alpha} \right] + k \lambda \frac{e^{(T-t)\alpha} - (T-t)\alpha - 1}{\alpha^2} \quad (42)$$

The volatility  $\sigma$  does not appear explicitly in the expectation of  $T A(T)$ , which involves only the forward value of the claim process (see Geman, 1989) but appears implicitly in both the observed claims and  $S(t)$ . We can also observe that, satisfactorily, this expectation is separately increasing in the first term (the accumulated claims up to time  $t$ ), in  $S(t)$  (the current level of the claim process), and in  $k$  and  $\lambda$  (severity and frequency of the jump component in the claim process dynamics).

### Modeling Loss Reporting Lags

Before turning to the examination of market prices and the estimation of the parameters in our model, we must describe another feature of the CBOT insurance futures contract that we have not mentioned before for the sake of readability. The aggregate losses  $L(T)$  that are reflected in the settlement value  $F(T)$  of the futures contract are losses **reported** over the two-quarter time period  $[0, T]$  arising from catastrophes that **occurred during the first quarter** of the life of the contract,  $[0, T/2]$ . To reflect this feature, we believe that the best representation of the instantaneous claim process over the time interval  $[0, T]$  consists of the juxtaposition of two stochastic processes. The first one, valid in the time interval  $[0, T/2]$ , has the jump component described in equation (20),

$$dS(t) = \alpha S(t_-) dt + \sigma S(t_-) dW(t) + k dN(t) \quad (43)$$

where  $S(0) = a$  given positive constant. The second process, valid in the second sub-period  $[T/2, T]$  is a pure diffusion process which has the same random term as the continuous part of (43) but a possibly

different drift  $\alpha'$ ,

$$dS(t) = \alpha' S(t) dt + \sigma S(t) dW(t) \quad (44)$$

where  $S(T/2)$  is given by equation (43). Severe catastrophes during the period  $[0, T/2]$  would be reflected in a high value of  $S(T/2)$  derived from equation (43), while there is no rationale for new jumps in the interval  $[T/2, T]$ . The drift term  $\alpha'$  would account for the way claims related to catastrophes already incurred in the period  $[0, T/2]$  are on average reported over the time interval  $[T/2, T]$ .

To support our point, we can observe that the volatility of the market prices of the March 1993 futures contract was much lower during the month of January 1993 than during the month of December 1992 and extremely low in absolute terms during the month of March 1993. The same property holds for the market prices of the June 1993 contract during the months of May and June (see section III).

Coming back to the pricing formulas of the futures contract over the time interval  $[0, T]$ , we now need to distinguish two periods. When  $t \in [T/2, T]$ , the calculations described in the first part of section II provide for the futures price  $V(t)$ :

$$V(t) = E_Q \left[ \int_0^T S(s) ds \mid \mathcal{F}(t) \right] = \int_0^t S(s) ds + S(t) \left[ \frac{e^{\alpha'(T-t)} - 1}{\alpha'} \right] \quad (45)$$

where the first term accounts for the claims reported and known up to time  $t$  and the second term accounts for the expectation of the claims to be reported during the period  $[t, T]$ .

When  $0 \leq t \leq T/2$ , we need to break the integral  $\int_0^T S(s) ds$  and hence  $V(t)$  into two terms:

$$V(t) = V_1(t) + V_2(t) = E_Q \left[ \int_0^{T/2} S(s) ds \mid \mathcal{F}_t \right] + E_Q \left[ \int_{T/2}^T S(s) ds \mid \mathcal{F}_t \right] \quad (46)$$

The first term  $V_1(t)$  is provided by formula (42), where  $T$  is replaced by  $T/2$ , everything else being unchanged.

$V_2(t)$  involves a new calculation. We know that for  $s \geq T/2$ ,

$$S(s) = S\left(\frac{T}{2}\right) e^{\alpha'(t-\frac{T}{2}) + \sigma W(t-\frac{T}{2}) - \frac{\sigma^2}{2}(t-\frac{T}{2})} \quad (47)$$

where  $S(T/2)$  itself is given by formula (29), which gives

$$S(s) = e^{(\alpha - \alpha')\frac{T}{2}} \left[ S(0) + \int_0^{T/2} e^{-X(u)} dH(u) \right] e^{X(s)} \quad (48)$$

where  $X(s) = (\alpha - \sigma^2/2)s + \sigma W(s)$ . Consequently,  $V_2(t)$  itself will be the sum of two terms. The first one, by the arguments developed in the first part of section II, is

$$S(0) \frac{e^{(\alpha - \alpha')\frac{T}{2}} e^{\alpha'T} - e^{\alpha\frac{T}{2}}}{\alpha'} = S(0) e^{\alpha\frac{T}{2}} \left[ \frac{e^{\alpha'\frac{T}{2}} - 1}{\alpha'} \right] \quad (49)$$

The second component of  $V_2(t)$ , by the arguments developed in the second part of section II, is equal to

$$\frac{k\lambda}{\alpha\alpha'} e^{(\alpha - \alpha')\frac{T}{2}} (1 - e^{-\alpha\frac{T}{2}}) (e^{\alpha'T} - e^{\alpha'\frac{T}{2}}) = k\lambda \left( \frac{e^{\alpha\frac{T}{2}} - 1}{\alpha} \right) \left( \frac{e^{\alpha'\frac{T}{2}} - 1}{\alpha'} \right) \quad (50)$$

Finally, for  $t \in [0, T/2]$ , the futures price is

$$\begin{aligned} V(t) = & \int_0^t S(s) ds + S(t) \left[ \frac{e^{\alpha(\frac{T-t})} - 1}{\alpha} \right] + k\lambda \left[ \frac{e^{(\frac{T-t})\alpha} - (\frac{T-t)\alpha - 1}{\alpha^2}} \right] \\ & + S(0) e^{\alpha\frac{T}{2}} \left[ \frac{e^{\alpha'\frac{T}{2}} - 1}{\alpha'} \right] + k\lambda \left( \frac{e^{\alpha\frac{T}{2}} - 1}{\alpha} \right) \left( \frac{e^{\alpha'\frac{T}{2}} - 1}{\alpha'} \right) \end{aligned} \quad (51)$$

where all five terms are positive whether  $\alpha$  and  $\alpha'$  are positive or negative.

### III. EMPIRICAL ANALYSIS

This section provides an empirical analysis of catastrophe insurance futures. We begin by

providing some summary information on catastrophic property losses and then turn to a discussion of the CBOT insurance futures. The section concludes with a discussion of parameter estimation.

### **Property Catastrophes**

Property catastrophes represent a significant risk for property owners and insurers. The insurance industry defines a catastrophe as “an event which causes in excess of \$5 million in insured property damage and affects a significant number of insureds and insurers” (Property Claims Services, 1993). As the definition implies, catastrophes represent a problem because they involve large losses and, even more fundamentally, constitute a violation of the most basic principle of insurance - the independence and diversifiability of risk. In principle, of course, very few events have world-wide consequences, so catastrophe losses should be diversifiable internationally through the reinsurance market. In practice, the demand for reinsurance often exceeds the supply because of information asymmetries, parameter estimation uncertainty, and the large concentration of property values in disaster prone areas such as the Eastern seaboard and Gulf coast in the United States. Futures markets thus have a potentially important role to play in hedging catastrophe risk.

During the period 1970 through mid-1993, an average of 34 catastrophes occurred each year, causing an annual average of \$2.5 billion in losses. Figure 1 shows catastrophe losses by quarter since 1980. As the figure suggests, most catastrophes are relatively small, e.g., less than \$250 million. However, the potential for much larger catastrophes clearly exists. For example, Hurricane Andrew caused twice as much damage (\$10.7 billion) as the next largest catastrophe (period since 1949) Hurricane Hugo (\$4.2 billion). Hurricane Iniki, the fourth largest catastrophe since 1949, also occurred during the second quarter of 1992, accounting for the extremely high losses during this period.

The lines of insurance subject to property catastrophes (the lines included in determining the settlement value of the CBOT'S futures contract) account for about 40 percent of total property-liability insurance premium volume. Property catastrophes thus represent a significant threat to insurers. Hurricane Andrew alone, for example, caused insured losses totalling about 20 percent of 1992 premium

volume in the affected lines of insurance. The possibility that losses of this magnitude can occur in the future provides insurers with a powerful incentive to hedge catastrophic risk.

Figure 2 provides more detail on the magnitude and volatility of catastrophe losses by quarter. The figure shows the average and standard deviation of catastrophe loss ratios for the period 1970 to 1993. During the first quarter of the year, property catastrophes average about 1.5 percent of total premiums, compared to a typical overall loss ratio for these lines of between 60 and 70 percent. Fourth quarter losses also are relatively small, averaging about 2 percent of premiums. Both the average loss and loss volatility are considerably higher during the second and third calendar quarters, when most windstorm, flood, and hurricane losses occur. Second quarter losses average about 4.3 percent of premiums, while third quarter losses amount to 6.2 percent. (Eliminating Hurricane Andrew, third quarter losses average about 3.5 percent of premiums.) It should be noted that these loss ratios are lower than the loss ratios for CBOT futures contracts because the futures cover all losses arising from the set of perils usually associated with catastrophes rather than being limited solely to catastrophic losses.

### **The CBOT Futures Contracts**

In spite of the importance of catastrophic risks to insurers, the CBOT futures contracts have been very thinly traded, as shown by the price/volume chart for the March 1993 contracts (Figure 3). Recall that these contracts cover loss events during the fourth quarter of 1992 that are reported to insurers by the end of March 1993. The contracts began trading on December 11, 1992 at 8 percent and closed on July 6, 1993 at a loss ratio of 8 percent after trading between 10 and 11 percent for most of the trading period. Trading in more recent contracts also has been light.

As discussed above, because no market price exists for insurance losses, the CBOT has created an index based on losses reported to a statistical agent (the Insurance Services Office). This index is as legitimate as other economic indices to be the underlying value of a futures contract as long as it is highly correlated with individual insurers' loss experience and there is no asymmetry of information

between buyers and sellers of the contract. Studies of insurance losses have shown that loss ratios are highly correlated across the insurance industry (e.g., Cummins and Nye, 1980), and one would expect catastrophic losses to have a stronger systematic component than losses in general. Furthermore, the CBOT releases “demographic reports” on the distribution of premiums by state that will be included in calculating the loss ratio underlying the futures contracts, and regional as well as national contracts are offered. This information should permit individual insurers to easily determine whether the synthetic loss ratio parallels their own loss exposure and to adjust their hedging strategies accordingly. Thus, a lack of representativeness is not the most likely explanation for the thin trading in the CBOT’s present futures contracts.

Information problems are the most likely explanation for the light trading in the CBOT’s insurance futures contracts. Traders can observe daily prices or yields on the commodities or instruments underlying other futures contracts. In contrast, very little information on the catastrophe insurance futures contract loss ratios is available prior to settlement except of course for the companies that belong to the pool. For the March 1993 contracts, data relating to the underlying loss ratio were released on only one day, April 4, more than three months after the end of the event quarter. The April 4 release consisted of an interim loss ratio “evaluated as of December 1992.” No additional information on the loss ratio was available until the contract settled on July 6. As explained below, the lack of more frequent data on the synthetic price significantly impedes parameter estimation and thus makes it very difficult for traders to form expectations. This additional source of uncertainty limits the hedging value of the contracts.

The reason given by CBOT officials for the limited release of loss ratio data is the time required by the ISO for recording and processing loss data. However, considering the power of modern computers and data base programs, it is very difficult to believe that a system could not be designed with the capability of releasing daily data. It is likewise difficult to believe that the ISO needs three months following the end of the reporting period to record and audit the loss reports. It is unlikely that

insurance futures contracts will succeed unless these information problems are solved.

### Parameter Estimation

To discuss parameter estimation, we begin with the simplest case, i.e., the futures contract with no jumps or reporting lags (see equation (17)). The parameters we need to estimate are those present in equations (6), (7), and (17):

- $\mu$  = the expected growth of the claims process,  $S(t)$ ,
- $\sigma$  = instantaneous standard deviation of the claims process,
- $\alpha$  = the risk-adjusted drift of the claims process =  $\alpha + \rho\sigma$ , where  $\rho$  = market price of claims risk, and

$S(0)$  = the starting value of the claims process.

In order to estimate the parameters, time series of loss ratios and futures prices are required. Currently, the sequence of loss ratios is not observed and the sequence of prices, while observed, is based on a small number of transactions. Thus, we discuss parameter estimation procedures and provide illustrations, but the estimation of parameters with sufficient precision for use in trading must await the availability of more complete data.

Recall that  $S(t)$  is a geometric Brownian motion process and that the observed loss ratios,  $L(t)/\Pi$ , are based on accumulations of this process (see equations (7) and (11)). G-Y (1992) provide formulas for the moments of accumulations based on geometric Brownian processes. Thus, if we could observe the sequence of loss ratios, we could estimate the empirical moments of  $L(t)/\Pi$  and solve for the parameters using a method of moments procedure.<sup>11</sup> The first two moments of  $L(t)$  are:

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<sup>11</sup>Maximum likelihood estimation would be very difficult due to the complexity of the probability distribution of  $L(t)$ .

$$E_p[L(T) | \mathcal{F}_t] = T E_p[A(t) | \mathcal{F}_t] = \int_0^t S(s) ds + S(t) \frac{e^{\gamma_1(T-t)} - 1}{\gamma_1} \quad (52)$$

$$E_p[L(T)^2 | \mathcal{F}_t] = T^2 E_p[A(t)^2 | \mathcal{F}_t] \quad (53)$$

$$= \left[ \int_0^t S(s) ds \right]^2 + 2 S(t) \left[ \frac{1}{\gamma_1 \gamma_2} + \frac{e^{\gamma_1(T-t)}}{\gamma_1 (\gamma_1 - \gamma_2)} + \frac{e^{\gamma_2(T-t)}}{\gamma_2 (\gamma_2 - \gamma_1)} \right]$$

where  $\gamma_1 = \mu + \sigma^2/2$ , and

$$\gamma_2 = 2\mu + 2\sigma^2.$$

Given a sequence of loss ratio observations for contracts of the same calendar quarter at the same point in time (e.g., time 0) as well as an estimate of  $S(t)$ , one could calculate the first two empirical moments of the loss ratio, set them equal to (52) and (53), respectively, and solve for  $\mu$  and  $\sigma$ .

Given a sequence of price and loss ratio data along with an estimate of  $S(t)$ ,  $\alpha$  could be estimated by minimizing the sum of squares of the difference between the left and right hand sides of (17).

Estimates of  $\mu$ ,  $\sigma$ , and  $\alpha$  yield an estimate of  $\rho$ , the market price of claims risk.

The foregoing discussion assumes that an estimate of  $S(t)$  is available. However, because the claims process itself is not observed, an estimation procedure must be specified to obtain  $S(t)$ . The derivative of the claims accumulation,  $\int_0^t S(s) ds$ , is  $S(t)$ , or  $S(t)/\Pi$  for the loss ratio. The numerical derivative of the loss ratio is:

$$\frac{S(t)}{\Pi} \approx \left( \frac{1}{\Pi} \right) \frac{\int_0^{t+\Delta t} S(s) ds - \int_0^t S(s) ds}{\Delta t} = \frac{L(t+\Delta t) - L(t)}{\Pi \Delta t} \quad (54)$$

Equation (54) could be used along with observations on the loss ratio to provide estimates of  $S(t)$ .

Because (54) is a numerical approximation to a derivative,  $\Delta t$  should be as small as possible. This

reinforces the need for daily observations on the loss index. Equation (54) also provides an approximate

expression for the rate of change of the loss ratio, based on the expected value of geometric Brownian motion, i.e.,  $E_p[S(t+\Delta t)/\Pi | S(t)] = (S(t)/\Pi)\exp(\mu\Delta t + \sigma^2\Delta t/2)$ . Substituting a numerical estimate of the expected loss ratio derivative on the left hand side of this equation, it could be solved simultaneously with (52) and (53) to estimate  $\mu$ ,  $\sigma^2$ , and  $S(t)$ .

We use the incomplete data that are presently available to illustrate the estimation of these parameters. The March 1993 national catastrophe futures contracts are used as an example. The initial evaluation of the loss ratio for these contracts was 0.0567. Using (54) with  $t = 0$  and  $\Delta t = 0.25$  (one-quarter of a year), we estimate  $S(0)/\Pi$  as  $0.0567*4 = 0.2268$ .<sup>12</sup> The first and second moments of the loss ratio were then estimated based on the catastrophe loss data provided in Property Claims Service (1993). The loss data were grouped by quarters and the estimation period was 1970-1993. The first moment was adjusted upward to recognize the fact that the CBOT contracts cover all losses from policies likely to be subject to catastrophes rather than just catastrophic losses. The initial estimate of the March 1993 contract's loss ratio (0.0567) was used as the mean. The estimate of the second moment of the loss ratio about the origin was .005. However, it proved impossible to obtain convergence for the three equation system with such a low second moment estimate. Since second moments are likely to suffer from higher estimation error than first moments, this is perhaps not surprising. To complete the illustration, we increased the second moment until convergence was attained. This occurred at a value of 0.0142 for the second moment of the loss ratio. The resulting estimates of  $\mu$ ,  $\sigma$ , and  $S(0)$  are 0.0894, 0.1428, and 0.22, respectively. Since  $S(t)$  is not observed, it is difficult to determine whether these estimates are reasonable. However, simulations of geometric Brownian motion processes using these parameters yielded loss ratio averages and time paths that do not seem unrealistic.

The reader is cautioned that the parameter estimates are very sensitive to the input parameters. However, it is likely that accurate parameter estimates could be obtained with more adequate data on the

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<sup>12</sup>Using 0.25 for  $\Delta t$  is obviously much too crude to provide an accurate estimate of  $S(0)$ , and we do not recommend using such a large value of  $\Delta t$  in practical applications of the model. However, as explained above, more detailed information on the time path of the loss ratio is not available.

synthetic loss ratio. Daily reports on the ISO loss accumulation would probably be sufficient. Even reports on the loss accumulation released after the close of the contract period would be a major improvement over the current system.

To estimate the parameters of the jump component of the model, we use a methodology similar to that developed in D’Arcy and France (1992). They point out that catastrophe losses have both an expected or “normal” and an unexpected or unusual component. The expected component is estimated by fitting a trend line to the quarterly catastrophe loss ratio data series provided in Property Claims Services (1993). An exponential trend line is estimated by regressing the natural log of the loss ratio on a linear time variable. The trend line for fourth quarter catastrophes based on the sample period 1970-1992 is given below (t-ratios are given in parentheses):

$$\mathbf{Ln(Loss\ Ratio)} = \mathbf{-6.1406 + 0.1367\ Time} \quad \mathbf{R^2 = 0.53}$$

(16.78)      (5.12)

The actual and fitted values of the trend line are shown in Figure 4. Catastrophic losses above the trend line are considered jumps. Eleven jumps occurred over the twenty-three year sample period, yielding an estimated frequency parameter for the Poisson jump process of 0.47. Jump severity is estimated as the average of the exponential of the residuals of the trend line. Thus, positive jumps are partially offset by negative jumps in measuring jump severity. This is appropriate to avoid overestimating jump loss severity. Only positive jumps are considered in estimating frequency to avoid overestimating frequency. The average jump severity is estimated as 1.36, implying that the occurrence of a jump results in catastrophe losses 36 percent above the anticipated “normal” catastrophes.

The final parameter estimate needed to fully specify the model is the risk-adjusted drift ( $\alpha'$ ) during the quarter following the event quarter. Given adequate data on futures prices and the underlying loss ratios,  $\alpha'$  can be estimated by applying non-linear least squares estimation to equation (17).

### III. CONCLUSIONS

This paper develops an Asian options model for insurance futures pricing. The Asian approach is appropriate because most insurance contracts, including the CBOT catastrophe insurance futures, have payoffs defined in terms of claims accumulations rather than the end-of-period values of the underlying state variables. In our model, the state variable is assumed to be a geometric Brownian motion -- the claims process. The payoff on the insurance futures contract is determined by the accumulation or integral of the state variable.

The model is illustrated in terms of the CBOT catastrophe insurance futures. The payoff on the CBOT futures is determined by the ratio of property insurance losses to premiums (the loss ratio) for lines of insurance most likely to be affected by property catastrophes. Because there is no market price for insurance losses, the CBOT created an index based on losses reported to a statistical agent by a sample of insurers. The contracts cover loss events occurring during a specified calendar quarter (the event quarter) that are reported by the end of the calendar quarter following the event quarter.

Catastrophic losses are modelled as consisting of a geometric Brownian motion component plus a jump component. The Brownian motion component represents losses arising from “expected” or “normal” catastrophes, while the jump component represents unusually large catastrophes that occur less frequently. The model allows for separate Brownian drift parameters for the event quarter and the reporting quarter following the event quarter. The jump component, which applies only during the event quarter, is modelled using a Poisson process.

Property catastrophes represent a significant risk to property owners and the insurance industry. Reinsurance, the conventional hedging mechanism for insurance claims, does not provide a perfect hedge for insurers because of illiquidity and periodic capacity shortages in the reinsurance market. Thus, insurance futures have significant potential as a hedging device for insurers. However, insurers have not yet shown much interest in the CBOT catastrophe futures.

Insurer reluctance to trade insurance futures is due in part to the conservatism of the insurance

industry and most insurers' lack of experience with derivative financial instruments. Another potential explanation is that the synthetic loss process is not perfectly correlated with the losses of individual insurers. However, there is a significant systematic component to insurance losses, especially those involving catastrophes. Therefore, insurers should be able to reduce risk by trading futures contracts.

We believe that the primary reason for the limited trading of insurance futures is the lack of information on the loss index. While traders in commodities and financial futures can observe the underlying prices or yields at least daily, information on the loss ratio underlying the insurance futures settlement price is released only once prior to the settlement date. The release occurs after the conclusion of the six-month reporting period. Thus, there is very little information to support parameter estimation or to assist traders in forming expectations. Although insurance futures are a promising approach to hedging insurance risk, the CBOT's current offerings are unlikely to be successful unless the information problem is solved.

Topics for future research include the implementation of our parameter estimation methodology using more complete data and the extension of our model to include stochastic jump severity. The investigation of futures options and the option-like cap on the current futures contracts would also be useful. These problems can be solved currently using Monte-Carlo simulation, but the development of a closed form solution would be of both theoretical and practical interest. Finally, because most insurance settlements are determined by loss accumulations, our model could be applied to the pricing of other types of insurance contracts.

Figure 1  
**CATASTROPHE LOSSES: 1980-1993**

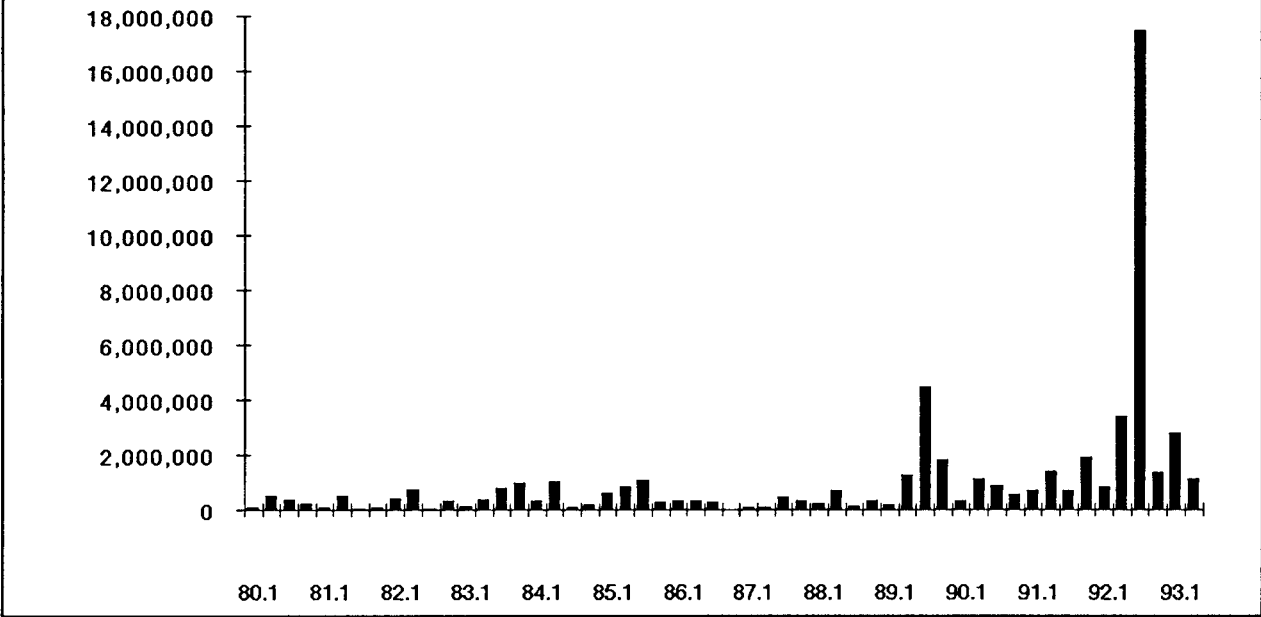


Figure 2  
**QUARTERLY CATASTROPHE EXPERIENCE: 1970-1993**

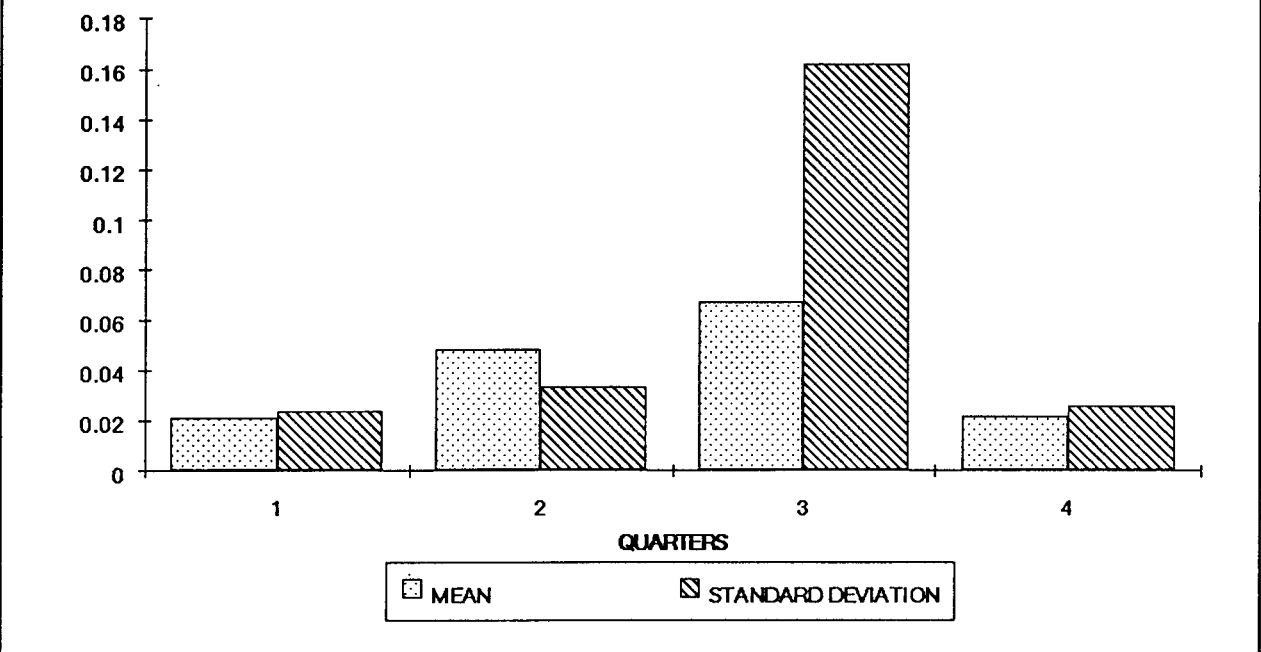


Figure 3

PRICE/VOLUME: MARCH 1993 CONTRACTS

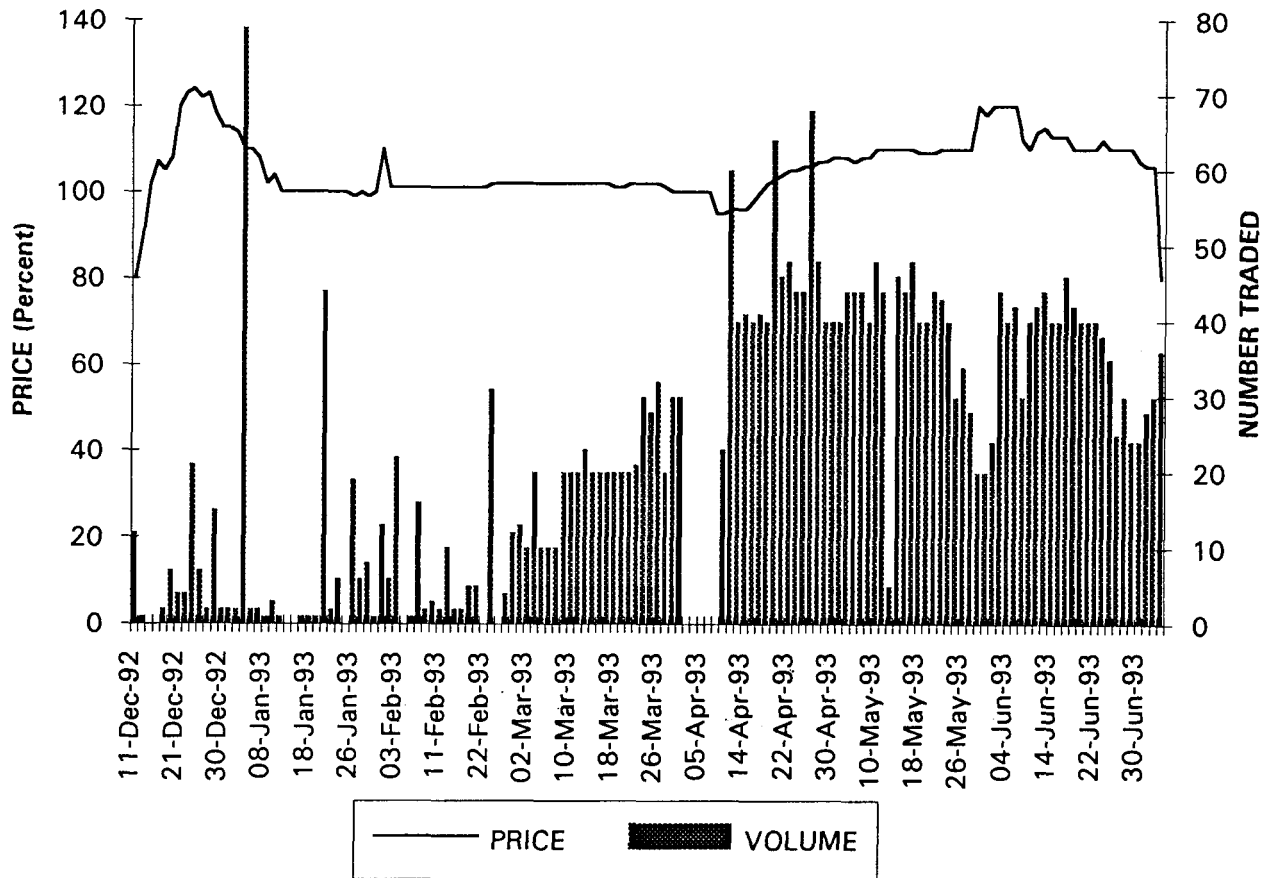
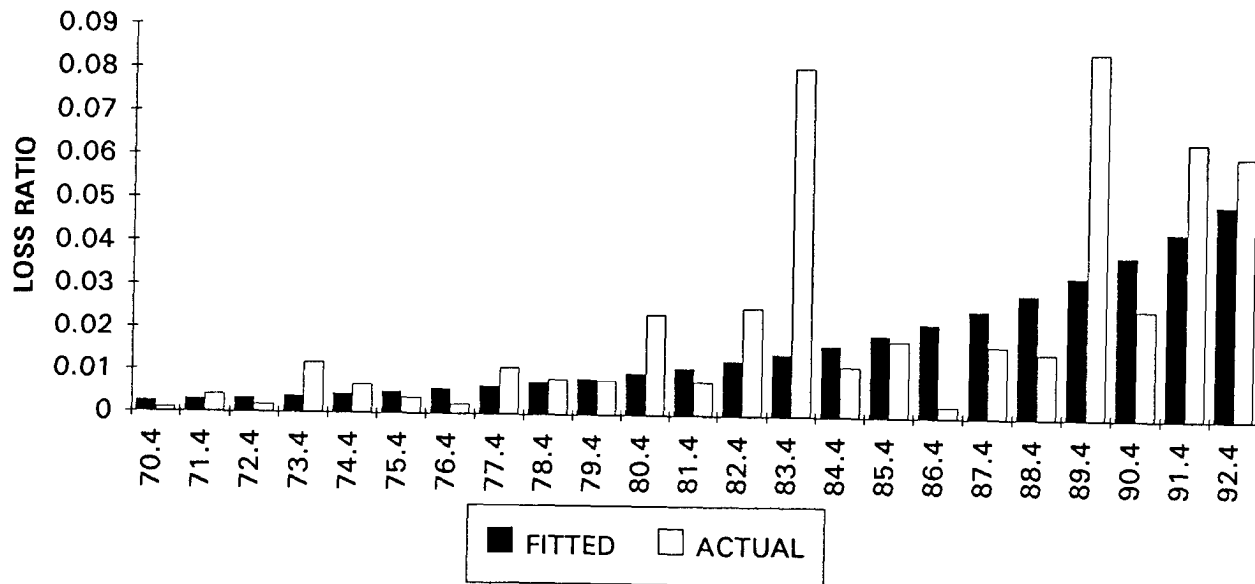


Figure 4

4TH QUARTER ACTUAL V. FITTED:  $\ln(L) = A + B \cdot \text{TIME}$



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