ASYMMETRIC INFORMATION AND THE CLOSED-END FUND PUZZLE

by

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Working Paper 94-19

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Mutual funds have recently become one of the most popular investment vehicles in the United States. Currently there are more mutual funds than stocks traded on the NYSE, yet the standard literature in finance views mutual funds as performing a relatively simple form of financial intermediation; they mainly provide pooled diversification. Although investors attain better diversification by investing in mutual funds, this benefit alone cannot explain a number of interesting phenomena that have been documented.

The most striking example can be found in the so-called closed-end fund puzzle (for example, see Anderson and Born (1992) for a comprehensive coverage of the puzzle and Lee, Shleifer and Thaler (1991)). The closed-end fund is an investment company that raises money by issuing a fixed number of shares and invests in marketable securities. Once the mutual fund shares are issued, they are traded in the open market. Researchers have observed that the market price is usually different from the current value of the portfolio held by the fund, or its net asset value (NAV). This discrepancy persists and funds that sell at a discount are not always terminated through either liquidating the assets of the fund or converting it to an open-end fund. None of these findings are expected if one views the mutual fund as a tool for effective diversification.

Past studies have tried to resolve this puzzle without changing the fundamental role played by the mutual funds. Agency costs, the existence of illiquid assets in the fund's assets, and tax considerations are a few such possibilities. However, Lee, Shleifer and Thaler (Lee et al. (1991)) argue that these standard explanations, taken together, cannot fully resolve the puzzle. Based on the theoretical work of Delong, Shleifer, Summers and Waldman (1990), Lee et al. suggest that investor sentiment is a driving force behind the pricing of the closed-end funds. Although their argument has some empirical support (see Lee et al. (1991) and Chopra, Lee, Shleifer and Thaler (1993)), it is not satisfactory in completely resolving the puzzle for several reasons. First of all, their theoretical underpinning rests on the existence of noise traders who trade closed-end fund shares based on an exogenously given irrational expectation. Secondly, they do not explain why dissolving the closed-end fund is not optimal given the extra volatility created by noise traders. Finally, their explanation does not depend on the organizational features of the closed-end fund but extends to all other assets, including stocks. Therefore the closed-end fund puzzle to them is a manifestation of a bigger puzzle, which is the inefficiency of the entire financial market.

In our paper, we provide a rational explanation to the puzzle by showing how the organizational features of the closed-end fund can lead to discounts or premia in equilibrium. To do this, we highlight two different roles the mutual fund can play as a financial intermediary: providing information and creating new securities. To de-emphasize the diversification benefit of the mutual fund, we consider a fund manager who invests in one risky asset and one riskless asset. The fund manager, however, has private information on the future value of the risky asset, and uses it in choosing how to manage the fund. As a consequence, the fund manager indirectly reveals her information to uninformed investors who learn from the price of the risky asset. This means that the payoff produced by the informationally superior fund manager is not redundant. In other words, the closed-end fund is an entirely new security which is not simply a linear combination of the underlying assets. Since the closed-end fund has a fixed number of shares, its price is determined in equilibrium by the uninformed investor's assessment of the final payoff it produces. As a result, discounts or premia can occur in equilibrium.

If the closed-end fund is a new security, one might wonder whether an uninformed investor learns more from the mutual fund price than from the price of the risky asset. If this was the case, an uninformed investor might be able to completely unravel the fund manager's private signal from mutual fund and risky asset prices. In our model, if there was such a fully revealing equilibrium, the closed—end fund would be a redundant asset whose price should be determined by a no—arbitrage condition, and the price of the closed-end fund would equal the NAV of the fund and could not convey further information. That is, the price of the closed-end fund becomes functionally dependent on the price of the risky asset, and thus uninformed investors can learn only from the price of the risky asset. With the aid of some parametric assumptions, which we will employ in Section I, we indeed characterize a closed-form pricing formula for the closed-end fund as a constant function.

Our closed-form solution shows how the price of the closed-end fund depends on the underlying parameters of the model. There are three important coefficients: one represents the precision of the private signal; another represents how aggressively the fund manager rebalances the fund based on the signal; and the last represents supply shocks in the risky asset. Although these three parameters determine the price of the closed-end fund, the first two are particularly important because they determine whether or not the fund sells at a discount. This indicates that the role of noise traders represented by the supply shock to the risky asset is rather limited in explaining the pricing pattern of the closed-end funds.

Finally, we show how a discounted closed-end fund does not always lead to the termination of the fund. In our model, the fund manager provides private information and creates a new security by managing the fund. As a result, an uninformed investor does not want to eliminate the fund just because the fund sells at a discount.

The paper is organized as follows. In Section I we present a model. We then characterize how equilibrium prices of the risky asset and the closed-end fund are determined in Section II. Some parametric examples of our model are provided in Section III. Section IV explains how our conclusion can resolve the closed-end fund puzzle and Section V concludes.

I. The Model

The model we consider is similar to that in Admati and Pfleiderer (1990), except that it explicitly includes a closed-end fund. The model involves three time periods (period zero, one, and two) and three assets (one risky asset, one riskless asset and one closed-end fund). Trading in the securities market takes place in period one, yielding equilibrium prices of these three assets, denoted by P, R, and C, respectively. The risky asset pays off a random amount $\tilde{F}$ in period two. The supply of the risky asset is a random variable $\tilde{Z}$, which realizes in period one, but is not directly observable to agents. One might attribute this supply shock to the existence of noise traders in the trading of the risky asset. The riskless asset is the numeraire asset and, without loss of generality, the riskless rate is set to zero, or equivalently, $R=1$ and a riskless asset pays one unit for sure. The closed-end fund is defined by three attributes: the private information the fund manager holds, the investment strategy employed by the manager, and the number of shares of the closed-end fund issued. In period zero, a fund manager develops an information technology, which will provide her with private information $\tilde{S}$, about $\tilde{F}$ in period one, where

$$\tilde{S} = \tilde{F} + \tilde{\theta}$$

and $\tilde{\theta}$ denotes noise. Based on this technology, the fund manager starts up a closed-end fund in period zero by announcing her period-one investment strategy and issuing a fixed number of shares for free.[1] The period-one investment strategy we consider here is a linear one: the fund manager will have zero initial wealth in period one (i.e., NAV is zero), but will invest $\alpha\tilde{S}$ shares in the risky asset, financed by an offsetting position in the riskless asset, where α is a positive number.[2] The α represents how aggressively the fund manager exploits her private information in period one. As a result of this strategy, the closed-end fund produces a final payoff of $\alpha\tilde{S}(\tilde{F} - P)$ in period two, a payoff that is then distributed to mutual fund holders. The total number of shares issued in period zero is set to one without loss of generality and no further shares are issued afterwards. Unlike the risky asset, we assume no supply shock to the fund shares in period one. Finally, we assume that the fund manager trades neither the risky asset, nor the closed-end fund shares on her own account in period one.[3]

Other than the fund manager, we assume there is one representative uninformed investor who purchases the initial shares of the closed-end fund and makes a portfolio choice between the three assets in period one. The uninformed

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[1]In this paper we do not consider what is the best way for the fund manager to sell her information. However, any IPO price of the fund in period zero does not change our results. For the fund manager's maximization problem in period zero, see Admati and Pfleiderer (1990).

[2]This implies that the fund manager does not behave in a contrary fashion.

[3]In fact, we can allow the fund manager to trade the risky asset on her own account without changing our results.

investor does not observe $\tilde{S}$ nor $\tilde{Z}$ directly, but is aware of the linear investment strategy of the fund. He also observes the prices of all three assets in period one. Conditional on this information, he chooses a portfolio to maximize his expected utility. In other words, he takes the price distribution and price realization as given and invests $-q$ in the risky asset, x shares in the closed-end fund and the remaining wealth in the riskless asset. As a result, his consumption in period two will be

$$W_2 = W_1 + x[\alpha\tilde{S}(\tilde{F} - P) - C] - q(\tilde{F} - P)$$

where W_1 denotes his initial endowment in period 1, including the market value of the closed-end fund shares that he holds.

Because the prices are determined in a competitive rational expectation equilibrium, as in Grossman and Stiglitz (1980), P and C satisfy the following market clearing conditions in period 1

$$x = 1$$

$$\alpha\tilde{S} - q = \tilde{Z}$$

for an optimal demand of x and $-q$ by the uninformed investor.

Now, we add the following parametric assumptions for analytical tractability.

A1) Constant Absolute Risk Aversion: the uninformed investor has a negative exponential utility function with an absolute risk aversion coefficient set to one.

A2) Normal Distributions: the random variables $(\tilde{F}, \tilde{\theta}, \tilde{Z})$ are jointly normally distributed with zero means and a variance-covariance matrix given by

$$\Omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_Z^2 \end{bmatrix}.$$

A3) Linear Pricing Rule: the price of the risky asset follows a linear pricing rule, or

$$P = a\tilde{S} - b\tilde{Z},$$

where constants a and b are properly determined by the equilibrium conditions.

A4) Second-Order Condition for the Maximization of the Uninformed Trader: the parameters, α, σ_θ^2 and σ_Z^2, satisfy

$\{\alpha^2(1 + \sigma_\theta^2) + (1 + \alpha)\sigma_Z^2\}^2 - \alpha^2(1 + \sigma_\theta^2)\sigma_Z^2(\alpha^2\sigma_\theta^2 + \sigma_Z^2) > 0.$[4]

Before we describe how the closed-end fund is priced in equilibrium, we want to resolve one important issue regarding the informational efficiency of the model. Since the uninformed investor observes two prices, P and C, in period one to distinguish $\tilde{S}$ and $\tilde{Z}$, one might wonder whether a fully revealing equilibrium might arise in our model. Our first theorem proves it cannot.

Theorem 1: In our model there is no fully revealing equilibrium.

Proof: We prove this by producing contradiction. Suppose there is a fully revealing equilibrium, so that the uninformed investor can unravel $\tilde{S}$ and $\tilde{Z}$ completely by observing P and C. Then the payoff generated by the fund can be replicated by the uninformed investor. Since the fund manager produces the final payoff of the fund with NAV equal to zero, so can the uninformed trader due to the no-arbitrage condition. This implies that C = 0 for all realizations of $\tilde{S}$

[4]See p.22 in the Appendix for a discussion of this assumption.

and $\tilde{Z}$. In this case, C cannot convey any information on $\tilde{S}$ and $\tilde{Z}$, and P alone cannot result in a fully revealing equilibrium. ■

Theorem 1 has several important implications for how the uninformed investor will behave in our model. First, because the information asymmetry remains in equilibrium, he cannot replicate the final payoff of the closed-end fund through his portfolio choice and regards the closed-end fund as a new security. Second, the uninformed investor assesses the risk profile of the fund differently according to what he learns about $\tilde{S}$ from P. Third, although the closed-end fund is a new security, he cannot learn more information from C than he can from P. In other words, C is a function of P, or $C = f(P)$. Therefore, the uninformed investor's information set can be reduced from $\{C,P\}$ to $\{P\}$.

Before we proceed, we want to point out how we capture the organizational features of the closed-end fund in our model. The number of shares issued is fixed and shares are traded in period one in the market place. As the uninformed investor learns about $\tilde{S}$ by observing price, his assessment of the final payoff of the fund will affect the price of the fund.

II. Characterization of the Equilibrium

In this section, we completely characterize the equilibrium prices, C and P, by identifying the constants a, b in the linear rule of assumption (A3), and the functional form f. We start with the portfolio choice problem faced by the uninformed investor in period one. He solves

$$\max_{x,q} \; E\left[-e^{-W_2} \mid P\right]$$

subject to the budget constraint

$$W_2 = W_1 + x[\alpha\tilde{S}(\tilde{F} - P) - C] - q(\tilde{F} - P)$$

$$= W_1 - xC + (x\alpha\tilde{S} - q)(\tilde{F} - P),$$

where, from Theorem 1, we know that his information set contains only P. Note that the closed-end fund is not a redundant security so that the maximization is with respect to both x and q.

The budget constraint shows that the uninformed investor regards the closed-end fund as an indirect way of holding the risky asset. Because both his total position in the risky asset, $x\alpha\tilde{S} - q$, and the net payoff from one unit investment in the risky asset, $\tilde{F} - P$, are stochastic, he updates the distributions of both $\tilde{F}$ and $\tilde{S}$ conditional on P. The following lemma, whose proof is omitted, shows how his updating is made.

Lemma 1: Suppose the risky asset's price is given by $P = a\tilde{S} - b\tilde{Z}$ for some constants a and b. The distribution of $\tilde{F}$ and $\tilde{S}$ conditional on P is normal with

$$\begin{pmatrix} \tilde{F} \\ \tilde{S} \end{pmatrix} \mid P \sim N\left(\begin{pmatrix} h^{-1}aP \\ h^{-1}a(1+\sigma_\theta^2)P \end{pmatrix}, \begin{pmatrix} h^{-1}(a^2\sigma_\theta^2+b^2\sigma_z^2), & h^{-1}b^2\sigma_z^2 \\ h^{-1}b^2\sigma_z^2 & , h^{-1}(1+\sigma_\theta^2)b^2\sigma_z^2 \end{pmatrix}\right),$$

where $h = a^2(1 + \sigma_\theta^2) + b^2\sigma_z^2$.

Using Lemma 1 one can calculate the conditional moments of both the total position in the risky asset and the net payoff from one unit investment in the risky asset. Conditional expectations of the total position and the net payoff of the risky asset, denoted by μ_1 and μ_2 respectively, are

$$\mu_1 = h^{-1}(1 + \sigma_\theta^2)a\alpha Px - q = k_1Px - q,$$

$$\mu_2 = (h^{-1}a - 1)P = k_2P,$$

and a conditional variance-covariance matrix, Σ, is given by

$$\Sigma_{11} = h^{-1}(1 + \sigma_\theta^2)\sigma_z^2 b^2\alpha^2x^2 = k_3x^2,$$

$$\Sigma_{12} = h^{-1}\sigma_z^2 b^2\alpha x = k_4x,$$

$$\Sigma_{22} = h^{-1}(a^2\sigma_\theta^2 + b^2\sigma_z^2) = k_5,$$

where the constants k_1, k_2, k_3, k_4, and k_5 are defined above to simplify notation. Note that the conditional variance-covariance matrix does not depend on the realization of the price P.[5] Consequently, the uninformed investor cannot change the variance-covariance matrix by holding the risky asset.

Although the second-period wealth of the uninformed investor involves the product of two normal distribution random variables, $\tilde{S}$ and $\tilde{F}$, the expected utility of the uninformed investor conditional on P can be computed as a function of μ_1, μ_2 and Σ and is given in Lemma 2 (this lemma comes from Admati and Pfleiderer (1990)).

Lemma 2: The expected utility of the uninformed investor conditional on P is equal to

$$-|\Sigma|^{-\frac{1}{2}} \left|\Sigma^{-1} + \begin{pmatrix} 0, & 1 \\ 1, & 0 \end{pmatrix}\right|^{-\frac{1}{2}} \exp\left[-\mu_1\mu_2 + \frac{1}{2}(\mu_2, \mu_1)\left[\Sigma^{-1} + \begin{pmatrix} 0, & 1 \\ 1, & 0 \end{pmatrix}\right]^{-1}\begin{pmatrix} \mu_2 \\ \mu_1 \end{pmatrix} - W_1 + Cx\right].$$

Proof: See Admati and Pfleiderer (1990, p.924). ■

With the above lemmas, we can determine the uninformed investor's demand for both the risky asset and the closed-end fund from the first-order conditions

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[5]This is due to our normal distribution assumption (A2).

of the uninformed investor's maximization problem. Interestingly, as Admati and Pfleiderer (1990) point out, the optimal demand for the risky asset is linear with respect to P, despite the fact that the second-period wealth is not normally distributed conditional on P.[6]

Lemma 3: The uninformed investor's demand for the risky asset is determined by

$$q = k_1xP - \frac{k_2P}{k_5} - k_4x\frac{k_2P}{k_5}.$$

Lemma 3 shows that the uninformed investor' demand for the risky asset is composed of three components. The first component, k_1xP, exactly offsets the conditional expectation of his indirect holding of the risky asset through the closed-end fund to make his expected total position equal to

$$\mu_1 = k_1xP - q = \frac{k_2P}{k_5} + k_4x\frac{k_2P}{k_5}.$$

The second component, k_2P/k_5, would be the demand for the risky asset without the closed-end fund. The third component, k_4xk_2P/k_5, can be regarded as a hedging demand induced by the existence of the closed-end mutual fund because k_4x is the covariance between $x\alpha\tilde{S}$ and $\tilde{F}$ conditional on P.

Lemma 3 also has an interesting implication for the uninformed investor's demand for the closed-end fund. One might expect that the demand for the closed-end fund also depends on P because the payoff of the closed-end fund changes as the uninformed investor learns from the price of risky asset. However, we prove

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[6]The exact functional form in Lemma 3, however, is different from Admati and Pfleiderer (1990).

in Lemma 4 that the uninformed investor's demand for the closed-end fund does not depend on P. This implies that, although the uninformed investor's expected utility depends on μ_1, μ_2 and Σ, he mainly uses the risky asset to hedge against the changing information P and demands the closed-end fund to maximize the expected utility affected by Σ.

Lemma 4: The uninformed investor's demand for the closed-end fund satisfies

$$C = -\frac{k_6(k_6x - k_4)}{k_3k_5 - (k_6x - k_4)^2}.$$

With the above lemmas, we can determine equilibrium prices, P and C. When the supply of the closed-end fund is fixed to one, Lemma 3 implies

$$q = \gamma P,$$

where γ is a constant, $\gamma = k_1 - k_2(1+k_4)/k_5$. This shows that the uninformed investor's equilibrium demand for the risky asset does not depend on the price of the closed-end fund. As a result, we can determine the coefficients a and b in the linear pricing rule of (A3) from the market clearing condition of the risky asset without worrying about their dependence on C. That is,

$$\alpha\tilde{S} - \gamma P = \tilde{Z}$$

or equivalently,

$$P = \frac{\alpha}{\gamma}\tilde{S} - \frac{1}{\gamma}\tilde{Z}.$$

Thus, the coefficients a and b satisfy

$$a=\frac{\alpha}{\gamma} \qquad b=\frac{1}{\gamma}.$$

This leads to Theorem 2.

Theorem 2: In equilibrium, the linear pricing rule, $P = a\tilde{S} - b\tilde{Z}$ is determined by

$$a = \alpha b,$$
$$b = \frac{\alpha + \sigma_z^2}{\alpha^2(1 + \sigma_\theta^2) + (\alpha + 1)\sigma_z^2},$$

where a>0 and b>0.

Theorem 2 shows how a and b are determined by the deep parameters of the model; α, σ_θ^2 and σ_z^2. As σ_θ^2 increases, we can show that both a and b decrease, but the first derivatives of a and b with respect to α and σ_z^2 take on a variety of signs.

Now we consider the equilibrium price of the closed-end fund, C. When the supply of the fund share is fixed, Lemma 4 implies that the market price of the closed-end fund is constant and does not depend on the price of the risky asset, although the uninformed investor's evaluation of the final payoff of the fund is affected by the price of the risky asset. Theorem 3 identifies an exact closed-form solution for the price of the closed-end fund when the supply of the fund share is one. Since the NAV of the fund is zero, C being different from zero indicates that a discount or premium arises in equilibrium.[7]

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[7]Eun, Janakiramanan and Senbet (1993) show how a discount or premium arises for the closed-end country fund with respect to the fund's holding position. Their result, however, depends on market segmentation between the home country and the host country.

Theorem 3: In equilibrium, the closed-end fund share sells at

$$c = \frac{\alpha\sigma_z^2(\alpha^2\sigma_\theta^2 + \alpha^2 + \sigma_z^2)(1 - \alpha\sigma_\theta^2)}{(\alpha^2(1 + \sigma_\theta^2) + (1 + \alpha)\sigma_z^2)^2 - \alpha^2(1 + \sigma_\theta^2)\sigma_z^2(\alpha^2\sigma_\theta^2 + \sigma_z^2)}.$$

Theorem 3 shows how the equilibrium price of the closed-end fund is determined by the underlying parameters. Interestingly, whether or not the fund sells at a discount depends on only two parameters, α and σ_θ^2.[8] For example, when $\alpha\sigma_\theta^2 = 1$, the closed-end fund always trades at NAV. The fund sells at a discount when $\alpha\sigma_\theta^2 > 1$ and at a premium when $\alpha\sigma_\theta^2 < 1$. This means that the role of noise traders in our model is rather limited in explaining the pricing pattern of the closed-end funds, because σ_z^2 can only affect the magnitude of discount or premium.

Theorem 3 also has an interesting implication for why the discount arises in equilibrium. In our model the discount can arise even though the fund is managed by the informationally superior fund manager. Consequently, the discount is not an indication that the fund manager has no private information. The discount, however, is related to the investment strategy that the fund manager employs. If the fund manager rebalances the fund too aggressively relative to the precision of his private signal, then $\alpha\sigma_\theta^2 > 1$ and the fund sells at a discount.

Theorem 3 also means that the uninformed investor cannot earn a higher return from investment in the discounted closed-end fund than a direct investment in the fund's underlying asset. In our model, both the risky asset and the

[8]Due to (A4) in Section I, the denominator of c is always positive.

closed-end fund are determined by the uninformed investor's marginal rate of substitution and, as a result, both of them command an equilibrium rate of return.

III. Some Parametric Examples

In this section, we present a few parametric examples to facilitate understanding of our model. Since the coefficients in P and C are highly non-linear with respect to α, σ_θ^2 and σ_z^2, we provide some sensitivity analysis as a substitute for a comparative static analysis.

Case 1 (Benchmark Case): $\alpha=1$, $\sigma_\theta^2=1$, and $\sigma_z^2=1$.

In this case, the fund manager's private signal is contaminated by a noise with a variance equal to σ_F^2. Given signal S, the fund manager perceives $\tilde{F}$ as

$$N(\frac{1}{2}S,\ \frac{1}{2}).$$

The uninformed investor regards $\tilde{F}$ and $\tilde{S}$ as

$$N\left(\begin{pmatrix}\frac{2}{3}P\\ \frac{4}{3}P\end{pmatrix},\ \begin{pmatrix}\frac{2}{3}, & \frac{1}{3}\\ \frac{1}{3}, & \frac{2}{3}\end{pmatrix}\right),$$

and takes $-2P$ position in the risky asset. Thus, $P = \frac{1}{2}\tilde{S} - \frac{1}{2}\tilde{Z}$ and $C = 0$ occur in equilibrium.

Case 2 (Precise Signal): $\alpha=1$, $\sigma_\theta^2=0.25$, $\sigma_z^2 = 1$.

In this case, the fund manager's signal provides more precise information on $\tilde{F}$ than the benchmark case. Given signal S, the fund manager perceives $\tilde{F}$ as

$$N(0.8S,\ \ 0.2),$$

and $P = 0.615\tilde{S} - 0.615\tilde{Z}$ occurs in equilibrium. Given P, the uninformed investor regards $\tilde{F}$ and $\tilde{S}$ as

$$N \sim \left(\begin{pmatrix}0.722P\\0.903P\end{pmatrix}, \begin{pmatrix}0.556 & 0.111\\0.111 & 0.556\end{pmatrix}\right)$$

and takes $-1.625P$ position in the risky asset. The fund sells at premium, $C = 0.188$. The following Tables 1 and 2 show how these results are affected when we change α and σ_Z^2.

Table 1: Sensitivity Analysis with α
Precise Signal Case: $\sigma_\theta^2 = 0.25$, $\sigma_Z^2 = 1$

	$\alpha = 0.33$	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$
γ	1.102	1.625	3.813	6.208
a	0.299	0.615	0.787	0.805
b	0.907	0.615	0.262	0.161
C	0.171	0.188	0.047	−0.035
var(P)	0.935	0.852	0.843	0.837

Table 2: Sensitivity Analysis with σ_Z^2
Precise Signal Case: $\sigma_\theta^2 = 0.25$, $\alpha = 1$

	$\sigma_Z^2 = 0.25$	$\sigma_Z^2 = 1$	$\sigma_Z^2 = 4$	$\sigma_Z^2 = 8$
γ	1.400	1.625	1.850	1.917
a	0.714	0.615	0.541	0.522
b	0.714	0.615	0.541	0.522
C	0.097	0.188	0.245	0.258
var(P)	0.765	0.852	1.534	2.518

Case 3 (Imprecise Signal): $\alpha=1$, $\sigma_\theta^2 = 4$, $\sigma_Z^2 = 1$.

In this case, the fund manager's signal provides an imprecise information on $\tilde{F}$. Given signal S, the fund manager perceives $\tilde{F}$ as

$$N\ (0.2S,\ \ 0.8).$$

As a result, $P = 0.285\tilde{S} - 0.285\tilde{Z}$ occurs in equilibrium. Given P, the uninformed investor regards $\tilde{F}$ and $\tilde{S}$ as

$$N \sim \left[\begin{pmatrix} 0.583P \\ 2.917P \end{pmatrix}, \begin{pmatrix} 0.833 & 0.667 \\ 0.667 & 0.833 \end{pmatrix}\right]$$

and takes $-3.5P$ position in the risky asset. The fund sells at discount, $C = -0.750$. The following Tables 3 and 4 show how these results are affected when we change α and σ_z^2.

Table 3: Sensitivity Analysis with α
Imprecise Signal Case: $\sigma_\theta^2 = 4$, $\sigma_z^2 = 1$

	$\alpha = 0.33$	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$
γ	1.409	3.500	12.250	21.833
a	0.234	0.285	0.245	0.229
b	0.710	0.285	0.082	0.046
C	-0.060	-0.750	-2.063	-2.639
var(P)	0.778	0.490	0.307	0.264

Table 4: Sensitivity Analysis with σ_z^2
Imprecise Signal Case: $\sigma_\theta^2 = 4$, $\alpha = 1$

	$\sigma_z^2 = 0.25$	$\sigma_z^2 = 1$	$\sigma_z^2 = 4$
γ	4.400	3.500	2.600
a	0.227	0.285	0.385
b	0.227	0.285	0.385
C	-0.158	-0.750	-12.000
var(P)	0.271	0.490	1.331

IV. Resolving the Closed-End Fund Puzzle

In this section, we discuss how our model can provide a rational explanation to the closed-end fund puzzle. Lee et al. (1991) argue that the existing standard explanations cannot satisfactorily explain the puzzle. They instead present investor sentiment as the main driving force behind the puzzle. Since investor sentiment can affect not only the closed-end fund but also the small firm, they claim that the closed-end fund puzzle is tied to the small firm effect. To Chen, Kan and Miller (1993), this attempt is trying to "catch two elusive birds with one stone and is missing both." To us, Lee et al. (1991) use the wrong stone to catch the closed-end fund puzzle. The right stone to use is the organizational structure of the fund, combined with asymmetric information between the fund manager and the uninformed investor.

In our model, the fund manager acts like an entrepreneur. She develops an information technology which gives her an informational advantage to start up and she manages the fund when she gets her private signal. In the process, she not only makes the market more informationally efficient but also creates a new security. Because informational asymmetry remains in equilibrium, the closed-end fund is not a redundant asset to the uninformed investor and, as a result, the fund trades at a premium or discount in equilibrium. This price does not mean an arbitrage opportunity to the uninformed investor. Rather, the premium or discount *is* an arbitrage opportunity to the fund manager. As long as the fund manager cannot trade closed-end fund shares on her own account, the economy is arbitrage free. The uninformed investor does not have an incentive to dissolve the fund despite the discount because of the financial intermediation the fund performs. Therefore, we can better understand the closed-end fund puzzle when we take both the asymmetric information and the organizational structure of the closed-end fund into account.

V. Conclusion

We explain the closed-end fund puzzle as an equilibrium phenomenon which arises due to the organizational feature of the fund when the fund manager has private information. In our model, the fund manager performs two important economic roles: providing information and creating a new security. Although the closed-end fund is not a redundant asset, its price is determined contingent on the price of the underlying risky asset. Consequently, discount or premium naturally occurs in equilibrium.

This research suggests two future research topics. First, we need to determine the optimal closed-end fund contract at a set-up stage. Since this optimal contract will determine the investment rule of the fund, figuring out an optimal contract is essential for complete resolution of the puzzle. The other is the analysis of open-end funds. In an open-end fund, the uninformed investor will change the number of shares of the open-end fund as he learns from the price of the risky asset.

APPENDIX

Proof of Lemma 3: From Lemma 2, we have

$$|\Sigma| = (k_3k_5 - k_4^2)x^2 = k_6x^2$$

$$|\Sigma^{-1} + \begin{pmatrix}0, & 1\\ 1, & 0\end{pmatrix}| = \frac{k_3k_5 - (k_6x - k_4)^2}{k_6^2x^2}$$

$$-\mu_1\mu_2 = - k_2P(k_1Px - q)$$

$$(\mu_2\ \mu_1)\left[\Sigma^{-1} + \begin{pmatrix}0, & 1\\ 1, & 0\end{pmatrix}\right]^{-1}\begin{pmatrix}\mu_2\\ \mu_1\end{pmatrix}$$

$$= \frac{k_6}{k_3k_5-(k_6x - k_4)^2}\left[k_3k_2^2P^2x^2 + 2k_2Px(k_4- k_6x)(k_1Px - q) + k_5(k_1Px - q)^2\right],$$

where $k_6 = k_3k_5 - k_4^2$.

We transform the uninformed investor's maximization problem into an equivalent minimization problem by considering the log of the negative expected utility. The minimization problem is

$$L = -W_1+Cx + \frac{1}{2}\log k_6 - \frac{1}{2}\log(k_3k_5-(k_6x-k_4)^2) - k_2P(k_1Px- q)$$
$$+ \frac{1}{2}\frac{k_6}{k_3k_5-(k_6x-k_4)^2}\left[k_3k_2^2P^2x^2+2k_2Px(k_4-k_6x)(k_1Px- q) + k_5(k_1Px- q)^2\right]$$

Now, we get a first-order condition of L with respect to q:

$$\frac{\partial L}{\partial q} = k_2P - \frac{k_6}{k_3k_5 - (k_6x-k_4)^2}\left[k_2Px(k_4-k_6x) + k_5(k_1Px - q)\right] = 0$$

Solving for q, we get the result. ∎

Proof of Lemma 4: We take a derivative of L with respect to x and evaluate it at $q = \gamma P$ to get

$$0 = C + \frac{k_6(k_6x - k_4)}{k_3k_5 - (k_6x - k_4)^2} - k_1k_2P^2$$

$$+ \frac{k_6^2\ (k_6x - k_4)k_2^2}{k_5\{k_3k_5 - (k_6x - k_4)^2\}^2}\left[k_3k_5x^2 + 2(k_4x - k_6x^2)(1 + k_4x) + (1 + k_4x)^2\right]P^2$$

$$+ \frac{k_6}{k_3k_5 - (k_6x - k_4)^2}\left[k_2^2\left\{k_3x + (k_4 - 2k_6x)\frac{(1 + k_4x)}{k_5}\right\} + k_1k_2\ (1 + 2k_4x - k_6x^2)\right]P^2$$

Using $k_6 = k_3k_5 - k_4^2$, we can further simplify the above equation:

$$k_3x + (k_4 - 2k_6x)\ \frac{1+k_4x}{k_5} = \frac{(2k_4x + 1)\ (k_4 - k_6x)}{k_5},$$

$$k_3k_5 - (k_6x - k_4)^2 = k_6\ (1 + 2k_4x - k_6x^2),$$

$$k_3k_5x^2 + 2(k_4x - k_6x^2)(1 + k_4x) + (1 + k_4x)^2 = \frac{\{k_3k_5 - (k_6x - k_4)^2\}(1 + 2k_4x)}{k_6}.$$

This leads to

$$C = -\frac{k_6(k_6x - k_4)}{k_3k_5 - (k_6x - k_4)^2}. \quad \blacksquare$$

Proof of Theorem 2:

$$\gamma = k_1 - \frac{k_2}{k_5}\ (1 + k_4)$$
$$= \frac{a(1 + \sigma_\theta^2)\alpha}{a^2(1 + \sigma_\theta^2) + b^2\sigma_z^2} - \frac{1 + h^{-1}b^2\sigma_z^2\alpha}{h^{-1}(a^2\sigma_\theta^2 + b^2\sigma_z^2)}\left(\frac{a}{a^2(1 + \sigma_\theta^2) + b^2\sigma_z^2} - 1\right)$$

Now, we replace a with αb and use $\gamma = \frac{1}{b}$ to solve for b. $\blacksquare$

Proof of Theorem 3: From Lemma 4 we evaluate C at $x = 1$ to get

$$C = -\frac{k_6(k_6-k_4)}{k_3k_5-(k_6-k_4)^2} = -\frac{k_6-k_4}{1+2k_4-k_6}$$

$$= \frac{k_4+k_4^2-k_3k_5}{(1+k_4)^2-k_3k_5}.$$

Finally, we substitute the original values of k_3, k_4 and k_5 to get the result.

■

Proof of the Second-Order Conditions of the Minimization Problem:

Here, we prove that (A4) in Section I is enough to satisfy the second-order condition of the minimization problem. To see this, let

$$k_7 = k_3k_5 - (k_6 - k_4)^2$$

for notational simplicity. Evaluating at x=1 and $q=\gamma P$, we obtain

$$\frac{\partial^2 L}{\partial q^2} = \frac{k_5k_6}{k_7},$$

$$\frac{\partial^2 L}{\partial q \partial x} = -\frac{k_6}{k_7}(k_1k_5 - k_2k_4)P,$$

$$\frac{\partial^2 L}{\partial x^2} = \frac{k_6^2}{k_7}(k_3k_5 + (k_6-k_4)^2) + eP^2,$$

where a constant e is defined by

$$e = \frac{k_2^2(1 + 2k_4)k_6^2}{k_5k_7} + \frac{4k_1k_2(k_6 - k_4)k_6}{k_7}$$
$$+ \frac{k_6}{k_7}(k_2^2k_3 + k_1^2k_5 + 2k_1k_2(k_4 - 2k_6)) - 2\frac{k_2^2}{k_5}(1 + k_4)k_6.$$

It is tedious to check

$$\frac{k_5 k_6}{k_7} e - \left(\frac{k_6}{k_7}\right)^2 (k_1 k_5 - k_2 k_4)^2 = 0,$$

or, in other words,

$$e = \frac{k_6}{k_5 k_7} (k_1 k_5 - k_2 k_4)^2.$$

As a result, the second-order condition is satisfied if and only if $k_7 > 0$. Since $k_7 = k_6(1 + 2k_4 - k_6)$, and $k_6 > 0$, $1 + 2k_4 - k_6$ should be positive, and this follows from (A4). ∎

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