Wharton

Financial
Institutions
Center

Financial Markets, Intermediaries and Intertemporal Smoothing

by
Franklin Allen
Douglas Gale

95-02

The Wharton School
University of Pennsylvania

THE WHARTON FINANCIAL INSTITUTIONS CENTER

The Wharton Financial Institutions Center provides a multi-disciplinary research approach to the problems and opportunities facing the financial services industry in its search for competitive excellence. The Center's research focuses on the issues related to managing risk at the firm level as well as ways to improve productivity and performance.

The Center fosters the development of a community of faculty, visiting scholars and Ph.D. candidates whose research interests complement and support the mission of the Center. The Center works closely with industry executives and practitioners to ensure that its research is informed by the operating realities and competitive demands facing industry participants as they pursue competitive excellence.

Copies of the working papers summarized here are available from the Center. If you would like to learn more about the Center or become a member of our research community, please let us know of your interest.

Anthony M. Santomero
Director

The Working Paper Series is made possible by a generous grant from the Alfred P. Sloan Foundation

Financial Markets, Intermediaries and Intertemporal Smoothing [1]

January 1995

Abstract: The returns of assets that are traded on financial markets are more volatile than the returns offered by intermediaries such as banks and insurance companies. This suggests that individual investors are exposed to more risk in countries which rely heavily on financial markets. In the absence of a complete set of Arrow-Debreu securities, there may be a role for institutions that can smooth asset returns over time. In this paper, we consider one such mechanism. We present an example of an economy in which the incompleteness of financial markets leads to underinvestment in reserves whereas the optimum, for a broad class of welfare functions, requires the holding of large reserves in order to smooth asset returns over time. We then argue that a long-lived intermediary may be able to implement the optimum. However, the position of the intermediary is fragile; competition from financial markets can cause the intertemporal smoothing mechanisms to unravel, in which case the intermediary will do no better than the market.

[1]Franklin Allen is at the University of Pennsylvania. Douglas Gale is at Boston University. We thank seminar participants at Indiana University, the Federal Reserve Bank of Cleveland and the University of Toulouse for helpful comments. Financial support from the NSF and the Sloan grant to the Wharton Financial Institutions Center is gratefully acknowledged.

1. Introduction

In the early nineteen-seventies most industrialized countries were adversely affected by a sharp rise in oil prices. This "oil shock" had a dramatic effect on the value of U.S. firms. As illustrated in Figure 1, the real value of shares listed on the New York Stock Exchange fell by almost half compared to their value at the peak in 1972. This collapse in share prices had a severe negative impact on the wealth of any investor whose portfolio contained a significant amount of stocks. Any investor who was forced to liquidate stocks after market prices fell would have suffered from lower consumption over the remainder of his life. Retirees in particular might have been affected in this way.

In Germany, where savings are mostly placed with intermediaries such as banks and insurance companies,[1] the effect was rather different. Since their claims on intermediaries were fixed in nominal terms, these individuals did not suffer a fall in wealth like their counterparts in the U.S. and would not have been forced to reduce their consumption. Somehow the German financial system was able to smooth the oil-price shock rather than passing it on to investors.

In the nineteen-eighties, the situation was reversed. The economies of most industrialized countries performed relatively well. In the U.S., the stock market boomed, as shown in Figure 1. Investors who held stocks were able to achieve higher than expected returns and could use these returns to finance a higher level of consumption. The dissaving generation in Germany did less well, by comparison. Since their savings were placed with intermediaries, such as banks, on which they held fixed claims, there was no windfall gain for them.

The effect of the "oil shock" on the U.S. market is an example of what is usually considered a **non-diversifiable risk.** The shock causes highly correlated changes in most asset values, so investors cannot avoid the risk by holding a diversified portfolio. Nonetheless, these episodes illustrate that the risks borne by individuals in two countries may be very different, even though the countries are subjected to similar shocks. This raises the interesting question of whether and how different financial systems can cope with this sort of risk.

Traditional financial theory has little to say about hedging non-diversifiable risks. It assumes that the set of assets is given and focuses on the efficient sharing of these risks through exchange. For example, the standard diversification argument requires individuals to exchange assets so that each individual holds only a small amount of any risk. We call this kind of risk sharing **cross-sectional risk sharing,** because it is achieved through exchanges of risk among individuals at a given point in time. This diversification strategy has no effect on macroeconomic shocks which affect all asset prices in a similar way.

Departing from the traditional approach, this paper focuses on the **intertemporal smoothing** of risk. Risks which cannot be diversified at a given point in time can nevertheless be averaged over time in a way that reduces their impact on individual welfare. One hedging strategy for non-diversifiable risks is **intergenerational risk sharing.**[2] which spreads the risks associated with a given stock of assets across generations with heterogeneous experiences. Another strategy involves **asset accumulation** in order to reduce fluctuations in consumption over time. Both of these strategies are examples of the intertemporal smoothing of asset returns.

In standard financial models with fixed asset supplies and a single period, it is usually argued that somebody must bear the non-diversifiable risk. Such models implicitly assume away possibilities for intertemporal smoothing. At the other extreme, in an ideal Arrow-Debreu world, cross-sectional risk sharing and intertemporal smoothing are undertaken automatically if markets are complete and participation in those markets is complete. Neither the standard models, which assume a fixed set of assets, nor the idealized Arrow-Debreu model, which does not explicitly deal with institutions, provide much insight into the relationship between the structure of a country's financial system and the stock of assets accumulated. In particular, they don't tell us how a country's reliance on financial markets or intermediaries affects its ability to smooth asset returns by changing its dynamic accumulation path. Yet the example of the differing responses of the German and U.S. economies to the oil shock of the nineteen-seventies and the boom of the nineteen-eighties suggests that intertemporal smoothing may be of some importance in practice.

The purpose of this paper is to consider the consequences of intertemporal smoothing for welfare and for positive issues such as asset pricing in a model with incomplete markets. In practice, markets may not be complete in an Arrow-Debreu sense for a wide variety of reasons including moral hazard, adverse selection, transaction costs and incomplete participation. For simplicity and tractability, we consider the case where participation is incomplete because of an overlapping generations structure. This is a tractable paradigm for the analysis of intertemporal smoothing and captures many of the features common to a wide range of models where markets are incomplete.

Our analysis is related to a number of strands of the literature. The first is concerned with what happens when a long lived asset such as land is incorporated into a standard overlapping generations model. Scheinkman (1980), McCallum (1987) and others have showed that incorporating a long-lived asset rules out the possibility of over-accumulation. None of these papers is concerned with risk. In contrast, our paper analyzes how the risk arising from the dividend stream of long lived assets is not eliminated by financial markets but can be eliminated by an intermediary. The second strand of the literature is concerned with liquidity risk. Qi (1994) extends the Diamond-Dybvig model to an overlapping generations model. In his model there is no aggregate risk and no role for intertemporal smoothing. Finally, Fulghieri and Rovelli (1994) and Bhattacharya and Padilla (1994) also compare the performance of markets and intermediaries in achieving an efficient intertemporal allocation of resources in an overlapping generations model. There is again no aggregate uncertainty in their models and they do not consider intertemporal smoothing.

In Section 2 we describe a simple model with two assets, a risky asset in fixed supply and a safe asset that can be accumulated over time. In Section 3, we show that in the market equilibrium, the safe asset is never held; in fact, it is dominated by the risky asset. Then we show, in Section 4, that for almost all social welfare functions in a broad class, welfare can be increased by the introduction of intertemporal smoothing, something the market has no incentive to do. In Section 5 we interpret intertemporal smoothing as the product of intermediation and suggest that the contrasting performance of the U.S. and German economies may be understood in terms of this intertemporal smoothing mechanism. In Section 6 we show that this mechanism is fragile and that competition from financial markets can lead to disintermediation, which causes the smoothing mechanism to unravel. Extensions and other applications of these ideas, to social security, occupational pensions and investment in housing, are discussed in Section 7. Formal proofs are contained in the appendix.

2. The Model

Our purpose is to develop a benchmark model for considering intertemporal risk smoothing. We are interested in the case where there is a long horizon and no discounting. For the analysis to be tractable, we assume there is a finite horizon and analyze what happens in the limit as the horizon tends to infinity.

There are T dates indexed $t = 1,...,T$. At each date $t < T$ there are two generations, an old generation born the previous period and a new generation born in the current period. Apart from the initial old generation, the generations are identical and live for two periods. Without loss of generality, we assume that each generation consists of a single, representative individual.

At each date there are a consumption good and two assets. One of the assets is safe and the other is risky. There is a fixed supply of the risky asset, which lasts forever. We normalize the supply of the risky asset to unity and assume the entire amount is owned by the old generation at the first date. Each period, it produces a random amount of the consumption good. Let $\tilde{y}_t > 0$ denote the amount produced at date t. For some results we assume that the random variables $\{\tilde{y}_t\}$ are i.i.d.; in other cases, general stochastic processes are allowed. For simplicity, we always assume that the yield of the risky asset is bounded above and bounded away from 0, that is, for some $A > 1$, $1/A \leq \tilde{y}_t \leq A$ for all $t = 1,...,T$. We also assume that the variance of the risky asset's dividend is bounded away from zero.

There is a variable supply of the safe asset: one unit of consumption can be converted costlessly into one unit of the safe asset. For simplicity, the return on the safe asset is normalized to zero, so one unit of consumption invested in the safe asset at date t yields one unit of consumption at date t+1. The amount of the safe asset owned by the old generation at the first date is zero.

The generation born at date t has an initial endowment of $e > 0$ units of consumption at date t and nothing at date t+1. Agents consume only when old. In order to provide for consumption when old, generation t can purchase some of the long-lived asset, consume its output at date t+1 and sell the asset to the next generation. Agents choose their investments to maximize the expected value of their von Neumann Morgenstern utility function $u(\tilde{c}_{t+1})$, where $u:\mathbb{R} \to \mathbb{R}$ is twice continuously differentiable and satisfies $u'(c) > 0$ and $u''(c) < 0$.

The special features of this model are chosen for the sake of simplicity. In particular, the overlapping generations structure is a metaphor for the wide range of sources of incompleteness that arise in practice. Extensions are discussed in Section 7 below.

3. Market Equilibrium

Let $x_t \geq 0$ denote the amount of the risky asset and $m_t \geq 0$ the amount of the safe asset held by the young generation at date t. By requiring these quantities to be non-negative we rule out short sales and borrowing. A young agent at date t chooses a portfolio (m_t, x_t) to maximize the expected value of utility $\mathcal{E}_t[u(\tilde{c}_{t+1})]$, where $\mathcal{E}_t$ denotes the expectation operator conditional on information available at date t. The first-period budget constraint restricts the value of the portfolio to be equal to the agent's endowment: $p_t x_t + m_t = e$; the second-period budget constraint restricts his consumption to be equal to the terminal value of the portfolio:

$$c_{t+1} = x_t(y_{t+1} + p_{t+1}) + m_t. \qquad (1)$$

Since the agent's decision can at most depend on the information available to him, his choice of (x_t, m_t) is a function of the history of asset returns up to and including date t. Prices satisfy the same condition, since they must depend on agents' decisions. In other words, portfolios and prices are **adapted** to the stochastic process of the risky asset's returns. In equilibrium, each agent chooses a portfolio to maximize his expected utility, subject to the period budget constraints, and the market for the risky asset clears, that is, the demand equals one unit, at each date.

Definition: **An equilibrium consists of a sequence of portfolios $\{(\tilde{x}_t, \tilde{m}_t)\}$ and prices $\{\tilde{p}_t\}$,** adapted to the stochastic process $\{\tilde{y}_t\}$, satisfying the following conditions:

(i) for every $t = 1, \ldots, T-1$, $(\tilde{x}_t, \tilde{m}_t)$ solves

$$\text{Max } \mathcal{E}[u(\tilde{c}_{t+1})] \tag{2}$$

$$\text{s.t. } \tilde{p}_t \tilde{x}_t + \tilde{m}_t = e$$

$$\tilde{c}_{t+1} = \tilde{x}_t(\tilde{y}_{t+1} + \tilde{p}_{t+1}) + \tilde{m}_t,$$

where $\tilde{p}_T \equiv 0$, and

(ii) for every $t = 1, \ldots, T-1$,

$$\tilde{x}_t = 1. \tag{3}$$

Although the safe asset would seem to be a useful hedge against uncertainty about the yield on the risky asset, it turns out that agents will almost never hold the safe asset in equilibrium. More precisely, we can find a fixed number k, independent of the horizon T, such that no agent invests in the safe asset at any date $t < T - k$. Thus , the fraction of periods in which the demand for the safe asset is positive becomes negligible as the time horizon T approaches infinity.

Proposition 1

There exists a fixed k, independent of the horizon T, such that in any **equilibrium $\{(\tilde{x}_t, \tilde{m}_t, \tilde{p}_t)\}$, $\tilde{m}_t = 0$ and $\tilde{p}_t = e$ for all $t = 1, \ldots, T-k$.** If we **assume that the dividends $\{\tilde{y}_t\}$ are i.i.d., then as $T \to \infty$ the long-run average expected utility converges to $\mathcal{E}[u(\tilde{y}_{t+1} + e)]$.**

The proof can be explained heuristically as follows. Since the safe asset has a zero yield and the risky asset has a positive yield, the safe asset will be dominated unless there is some risk of capital loss. More precisely, at any date t in which the safe asset is held, agents must expect some probability of a fall in the value of the risky asset that more than offsets the dividend. Then $\tilde{m}_t > 0$ implies that $\tilde{p}_t > \tilde{p}_{t+1} + \tilde{y}_{t+1}$ with positive probability. This in turn implies that for some $\epsilon > 0$, the asset price falls by at least ϵ with probability at least ϵ. Furthermore, if $\tilde{p}_{t+1} < e$ then the budget constraint implies that $\tilde{m}_{t+1} > 0$. So we can use the same argument repeatedly to show that with positive probability the price continues to fall by ϵ every period. The budget constraint and the market-clearing condition imply that

$$\tilde{p}_t \leq \tilde{p}_t\tilde{x}_t + \tilde{m}_t = e. \tag{4}$$

Since the price of the risky asset is never greater than e, the price cannot fall for more than $k = e/\epsilon$ periods. In other words, the demand for the safe asset can only be positive when the horizon is less than k periods away.

To complete the proof of the first part of the theorem, note that whenever $\tilde{m}_t = 0$, the budget constraint and the market-clearing condition imply that

$$p_t = \tilde{p}_t\tilde{x}_t + m_t = e. \tag{5}$$

The last statement in the theorem follows immediately from the law of large numbers and the assumption that returns $\tilde{y}_t$ are i.i.d., since for all but a negligible fraction of periods,

$$\begin{aligned} \tilde{c}_{t+1} &= \bar{\tilde{x}}_t(\bar{\tilde{y}}_{t+1} + \tilde{p}_{t+1}) + \tilde{m}_t \\ &= \tilde{y}_{t+1} + e. \end{aligned} \tag{6}$$

To illustrate the properties of the market equilibrium, it is helpful to consider a simple example with a two-point distribution of returns.

Example 1: Suppose that $\{\tilde{y}_t\}$ is an i.i.d. dividend process with

$$\tilde{y}_t = \begin{cases} 0.5 \text{ w.pr. } 0.5 \\ 1.5 \text{ w.pr. } 0.5 \end{cases}$$

for every t, the endowment is e = 5 and the utility function is u(c) = in c.

The path of prices is shown in Figure 2. It can be seen that k = 6. During the first phase consisting of periods t = 1,...,T-6 the price is constant at $\tilde{p}_t = 5$. During the final phase from T-5 until the horizon T, the price falls toward zero. Consider the final phase first. At the horizon T, $p_T = 0$ since there is no subsequent generation to purchase the asset. At date T-1, the last generation but one can sell the asset to the final generation. Although $p_T - 0$, the asset will pay a dividend at T and hence at T-1 its price will equal the discounted present value of the dividend, which is $p_{T-1} = \mathcal{E}[u'(\tilde{c}_T)\tilde{y}_T] = 0.95$. At date T-2, the amount the young generation will be prepared to pay will reflect the dividend at T-1 and the price of the asset at T-1, so that $p_{T-2} = \mathcal{E}[u'(\tilde{c}_{T-1})\tilde{y}_{T-1}] + p_{T-1} = 1.90$. Similarly, in each period in the final phase the current price is determined by the next period's dividend and price. Going backwards through time the price rises as the stream of future dividends becomes longer. At date T-6, the price calculated in this way is 5.60. However, the endowment of the young generation is only e = 5. This is the most they can pay for the asset and so $p_{T-6} = 5$ rather than 5.60. In all the periods before date T-6 it is similarly the case that the young put their entire wealth into the asset and $p_t = 5$. They would like to invest more but cannot; they are already investing all their endowment and they are unable to borrow. The risky asset strictly dominates the safe asset, since these generations sell the asset for an amount equal to what they paid for it and, in addition, they receive a positive dividend. The expected return on the risky asset is $\mathcal{E}[r_t] = 1/5$.

During the initial phase, the price is determined not by "the discounted stream of future dividends" but instead by "the cash in the market." The equilibrium price and the expected return depend on the endowment of the younger generation rather than the future stream of dividends. If the endowment had been e = 3, then in the first and typical phase of the equilibrium, it would be the case that $p_t = 3$ and $\mathcal{E}[r_t] = 1/3$. Likewise, during the first phase when the price is constant, individuals bear no price risk but they do bear dividend risk, since the dividend can be 0.5 or 1.5. Allen and Gale (1994) discuss the importance of "cash in the market" for asset pricing in another context.

The fact that the safe asset is almost never used in equilibrium is striking and raises the question of whether some sort of market failure has occurred. However, it is easy to see that the equilibrium allocation is weakly Pareto-efficient: since the initial old generation consumes the yield from the risky asset and the endowment of the young, it is impossible to make everyone better off. Furthermore, under the additional assumption that the dividend process is i.i.d., we can show that the equilibrium is strongly Pareto-efficient, that is, it is impossible to make some agents better off without making other agents worse off.

Proposition 2

If the dividend process $\{\tilde{y}_t\}$ is i.i.d., the market equilibrium allocation is strongly Pareto-efficient.

Under the assumption that the returns to the risky asset are i.i.d., we can show that the equilibrium price of the risky asset $\tilde{p}_t$ is uniquely determined and independent of the history of returns at each date t. Then using backward induction, we can show that a generation can be made better off only by increasing the quantity of the safe asset it holds. Similarly, a generation must be worse off if it holds less of the safe asset. But this shows that any weak Pareto improvement must violate a feasibility condition, thus showing that the equilibrium allocation is strongly Pareto-efficient.

4. Intertemporal Smoothing

In the preceding section we saw that the market equilibrium allocation is technically Pareto-efficient. Nonetheless, we shall argue that there is in fact a serious form of market failure in the market equilibrium. The definition of Pareto efficiency disguises the potential for achieving a substantial increase in welfare through intertemporal smoothing. While it is not possible to make everyone better off compared with the equilibrium allocation, the main result in this section shows that by accumulating reserves in the form of the safe asset and using them to “smooth” the returns to the risky asset, it is possible to increase the welfare of all but a negligible set of agents by a non-negligible and uniform amount. The fact that the improvement is non-negligible and uniform is crucial since it is always trivially possible to make almost everyone better off by making transfers at the expense of a negligible set of agents. In the remainder of the section, we argue that this refinement of the standard Pareto criterion is in fact very mild and that the “efficient” allocations that are eliminated by it may well be regarded as pathological.

It is easy to see that without holding the safe asset there cannot be any change in welfare at all. The reason is that if the safe asset is never held, then for most agents their allocation will be exactly the same as in the equilibrium allocation. More precisely, if the safe asset is not held, then at each date it is obviously optimal to give the old generation the entire dividend from the risky asset and the current young generation’s endowment, which is exactly what they receive in equilibrium at any date $t < T - k$. Then as T approaches infinity, the new allocation, in which the safe asset is never held, differs from the equilibrium allocation at most for a negligible fraction of periods. So holding reserves in the form of the safe asset is a necessary condition for any increase in welfare. What is somewhat surprising is that any increase in welfare can be achieved by holding a dominated asset.

As the preceding argument shows, we can restrict attention to allocations which give all consumption at each date to the current old generation. Then a **feasible policy** can be represented by a non-negative sequence of random variables $\{\tilde{m}_t\}$, adapted to $\{\tilde{y}_t\}$, such that the corresponding consumption levels are non-negative:

$$\tilde{c}_{t+1} = \tilde{y}_{t+1} + \tilde{m}_{t+1} - \tilde{m}_t + e \geq 0, \quad (7)$$

for every date t, and

$$c_0 = y_1 - m_1 + e.$$

The next result demonstrates the potential for increasing the welfare of almost every generation.

Proposition 3

Suppose that the dividend process $\{\tilde{y}_t\}$ is i.i.d. For any $\epsilon > 0$ and all T sufficiently large, **there exists a feasible policy $\{\tilde{m}_t\}$ which guarantees a fraction $(1 - \epsilon)$ of the generations a utility level greater than or equal to $u(\mathcal{E}\tilde{y}_t + e) - \epsilon$.**

This proposition is essentially an application of a theorem of Schechtman (1976). Schechtman studied the problem of an individual who has a risky income $\tilde{\omega}_t$ and cannot borrow but wants to self-insure by holding a safe asset $\tilde{m}_t$. Consider the following policy: at each date t, the individual has accumulated savings of $\tilde{m}_{t-1}$, consumes $\mathcal{E}\tilde{y}_t + e$, if this is feasible, and consumes $\tilde{y}_t + e + \tilde{m}_{t-1}$ otherwise. Then the individual's savings will be

$$\tilde{m}_t = \max(\tilde{y}_t + \tilde{m}_{t-1} - \mathcal{E}\tilde{y}_t, 0). \qquad (8)$$

The renewal theorem tells us that the fraction of time that this process spends at the boundary $\tilde{m}_t = 0$ becomes vanishingly small as T approaches infinity. The individual's consumption is less than $\mathcal{E}\tilde{y}_t + e$ only when $\tilde{m}_t$ equals 0, so we conclude that the individual's consumption is equal to $\mathcal{E}\tilde{y}_t + e$ for all but a negligible fraction of the time.

The same policy is feasible in our framework. Applying Schechtman's result, we can conclude that with probability one, the fraction of generations receiving consumption less than $\mathcal{E}\tilde{y}_t + e$ converges to zero as T approaches ∞. Since agents are risk averse (u is strictly concave) and $\tilde{y}_t$ is non-degenerate, they prefer receiving the expected value of the dividend for sure to receiving the random dividend:

$$\mathcal{E}[u(\tilde{y}_t + e)] < u(\mathcal{E}\tilde{y}_t + e). \qquad (9)$$

The improvement allowed by intertemporal smoothing can be illustrated in the context of Example 1, where we see that in the market equilibrium, expected utility converges to

$$\ln(5.5) + \ln(6.5) = 1.7883,$$

as $T \to \infty$. In contrast, with intertemporal smoothing, the long-run average expected utility converges to

$$\ln(6) = 1.7918,$$

as $T \to \infty$.

Although the technical details of the proof of Proposition 3 are somewhat involved, the significance is clear. By transferring income between periods to smooth consumption as much as possible, almost every generation will be made better off by a non-negligible and uniform amount. We call this an almost uniform Pareto improvement.

It remains to make precise the sense in which an almost uniform Pareto improvement represents an increase in social welfare. We shall argue that under mild refinement of the standard concept of Pareto efficiency, an allocation is not efficient if it is possible to make an almost uniform Pareto improvement. We make use of the well known fact that if the utility functions are concave, every Pareto-efficient allocation maximizes a weighted sum of utilities. More precisely, a feasible policy $\{\tilde{m}_t\}$ is Pareto-efficient if there exists a non-negative sequence of numbers $\{\lambda_t\}$ such that $\{\tilde{m}_t\}$ solves the problem

$$\text{Max} \sum_{t=1}^{T} \lambda_t u(\tilde{c}_t) \tag{10}$$

$$\text{s.t. } \tilde{c}_t = \tilde{y}_t + \tilde{m}_t - \tilde{m}_{t-1} + e,$$

$$\tilde{m}_t \geq 0,\ \tilde{c}_t \geq 0,\ \tilde{m}_0 = 0.$$

One can think of the objective function as a Bergson-Samuelson welfare function

$$W(u_1, u_2, u_3, \ldots) = \sum_{t=1}^{T} \lambda_t u_t, \tag{11}$$

in which the welfare weights $\{\lambda_t\}$ determine how much each generation's welfare "counts" in the calculation of social well being. The welfare function W defined by equation (11) is said to be **uniformly weighted** if there exists a number $B \geq 1$ such that

$$1/B \leq \lambda_t \leq B \text{ for every } t. \tag{12}$$

At one extreme, when B equals 1, the implied welfare function is equally weighted. As B becomes larger a wider range of weights is allowed and as B becomes arbitrarily large, most schemes with positive weights are allowed. The crucial restriction is that the ratios of weights λ_t/λ_{t+h}, for different generations t and t+h, remain bounded and bounded away from 0. Now this is precisely what is not possible if the market equilibrium allocation is to be a solution of a welfare maximization problem like (10). If the market equilibrium allocation solves (10) it can be shown that the welfare weights must be extremely unequal. In fact, we shall shortly see that the ratio of the weights λ_t/λ_{t+h} converges to 0 as the distance between generations h diverges to ∞. This is the sense in which the Pareto efficiency of the market equilibrium allocation appears to be somewhat pathological and this is why we consider the restriction to uniform welfare functions rather mild.

This refinement of the Pareto criterion motivates two immediate corollaries. The first makes precise the sense in which the equilibrium allocation is not efficient. A feasible policy $\{\tilde{m}_t\}$ is called <u>uniformly</u> **Pareto-efficient** if it solves the maximization problem (10) and the welfare function is uniformly weighted. To ensure that this restriction has some cutting power, we keep B fixed as the horizon T is allowed to increase without bound.

Corollary 3.1

Suppose that equilibrium allocation described in Proposition 1 is a solution of (10) for the welfare weights $\{\lambda_t\}$. Then there exists a number $0 < \rho < 1$, independently of T, such that for every generation $t < T - k$,

$$\rho\lambda_t < \lambda_{t+1}. \qquad (13)$$

Consequently, for any B, and all T sufficiently large, the market equilibrium allocation is not uniformly Pareto-efficient.

The proposition shows that as the distance between generations grows large the relative disparity in their welfare weights grows without bound. More precisely, $\lambda_{t+h}/\lambda_t \leq \rho^h \to 0$ as $h \to \infty$. Clearly this is inconsistent with the uniform weighting property (12), which requires that

$$1/B^2 \leq \lambda_t/\lambda_{t+h} \leq B^2,$$

for all values of t and h, and this explains why the market equilibrium allocation fails to be uniformly Pareto-efficient.

In Example 1, considered in the previous section, $\rho = 0.92$ so that the weighting of each generation in the social welfare function underlying the market equilibrium allocation falls quite rapidly and it is not necessary for generations to be far apart to have widely different weights. There seems to be no good rationale for this property and, from a welfare point of view, it might be argued by many that a more equal weighting would seem to be at least as reasonable.

The second corollary shows that the intertemporal smoothing policy is uniformly Pareto-efficient, at least in the limit as $T \to \infty$. To show this, we first need to extend our definition slightly. Call a feasible policy $\{\tilde{m}_t\}$ **uniformly Pareto-efficient** in the limit if there is some bound B, and a uniform weighting scheme $\{\lambda_t\}$ such that for any ϵ and all T sufficiently large,

$$\frac{1}{T}\mathcal{E}[\sum_{t=1}^{T} u(\tilde{c}_t)] > \frac{1}{T}\mathcal{E}[\sum_{t=1}^{T} u(\tilde{c}'_t)] + \epsilon, \tag{14}$$

where $\tilde{c}_t = \tilde{y}_t + \tilde{m}_t - \tilde{m}_{t-1} + e$ and $\tilde{c}'_t = \tilde{y}_t + \tilde{m}'_t - \tilde{m}'_{t-1} + e$, for $t = 1,\ldots,T$, and as usual $\tilde{m}_0 = \tilde{m}'_0 = 0$. Note that two properties are being combined in our notion of uniform efficiency in the limit. First, although a policy may be dominated for each finite horizon T it catches up in the limit as T diverges to ∞. Second, we are comparing long-run weighted **averages** of utility, rather than weighted **sums** of utilities. With this definition, we can now state the result.

Corollary 3,2

The policy $\{\tilde{m}_t\}$ described in Proposition 3 is uniformly Pareto-efficient in the limit.

The corollary follows from the fact that the policy described in Proposition 1 maximizes the long-run average expected utility of all the generations. Putting $\lambda_t = 1$, for all T obviously satisfies the required bounds for any B > 1 and, for this specification of the welfare function, the policy must eventually overtake any other.

5. Implementation of Intertemporal Smoothing Using Intermediaries

In the preceding section we studied the uniformly Pareto-efficient allocations, without specifying the institutional framework that implements them. One possibility is that a long-lived intermediary provides insurance against swings in asset prices by averaging gains and losses over time. Such an intermediary would hold all the assets and offer a deposit contract to each generation. By accumulating large reserves the intermediary could offer almost all generations a constant return on deposits, independently of the actual returns. The market, on the other hand, has no incentive to provide this insurance. The market does not value the safe asset's contribution to future generations' welfare through risk reduction, because it only looks at rates of return. The price of the risky asset in the market equilibrium is always so low that its return dominates the safe asset, which is never held. As a result, each generation bears the full dividend risk on the risky asset.

In this interpretation, financial markets and intermediaries are not simply veils thrown over a fixed set of assets. They actually determine, in conjunction with other factors, the set of assets accumulated by the agents in the economy. By adopting one or another set of institutions, the economy is placed on a different trajectory, with important implications for the aggregate risks to be shared.

At a theoretical level it seems that an intermediated financial system can achieve a higher level of welfare than a market-based system. It is tempting, then, to compare the U.S. and German financial systems in the light of this example. Does the greater emphasis on financial markets in the U.S.

imply that U.S. investors are exposed to more risk than their counterparts in Germany, with its heavier reliance on intermediation? The answer depends on a number of practical considerations, a few of which we mention here.

First among them is the question of the objective that German intermediaries pursue in practice. This is, of course, an empirical issue. As discussed in Allen and Gale (1995), among universal banks only the commercial banks which together constitute about a quarter of the country's banking assets are profit maximizing entities. Savings banks, which account for about a third of total banking assets, do not maximize profits. They are operated in the public interest. The remaining banks are either cooperatives and have a mutual structure in which the depositors are the shareholders or are specialist banks, some of which are profit maximizing while others are not.

Whether they are operated in the public interest or not, it can be argued that German intermediaries are innately cautious. One possible explanation of this is the regulations imposed by the Bundesbank. Another is that asymmetric information makes it optimal for them to develop a reputation for financial stability. Whatever the reason, German banks are generally believed to hold high levels of hidden reserves which they draw down in bad times and build up in good ones. So even if their performance is less than optimal in the sense used here, it may nevertheless be a significant improvement over the market in terms of intertemporal risk smoothing.

How close an intermediary gets to the welfare optimum depends on the exact specification of the objective function. In the best of all possible worlds, where its objective function coincides with the social welfare function, the intermediary will choose a uniformly efficient policy. For example, we might suppose that the intermediary is an enterprise run in the public interest that seeks to maximize the long-run average expected utility of its depositors:

$$\ell im_{T\to\infty} T^{-1} \mathcal{E}[\textstyle\sum_{t=1}^{T} u_t]. \tag{15}$$

This is an objective that would be appealing from the point of view of an initial position, behind the "veil of ignorance", before any individual knew the generation into which he would be born. Alternatively, one might argue that this objective function would be appropriate for an intermediary which had a long time horizon and safeguards the future of the institution by taking into account the interests of future as well as present depositors. This might be the result of regulatory pressure, rent-seeking behavior, or corporate culture supported by a repeated-game equilibrium. We do not expect that short-term profit or value maximization will lead to this kind of behavior, but our view is that neoclassical economics puts too much emphasis on the profit-maximizing firm as the only alternative to a welfare-maximizing government. In practice, there is a broad spectrum of institutions with intermediate objectives and there is room in a mature theory for institutions that maximize a welfare function like the one above (see Allen and Gale (1995) for a more detailed discussion of this issue).

6. Competition between Intermediaries and Financial Markets

A commonly heard argument is that financial markets of the type observed in the U.S. are desirable because of the risk sharing opportunities they provide. It has been suggested that this argument is correct as far as cross-sectional risk sharing opportunities are concerned but ignores intertemporal risk smoothing. We have shown in the context of a simple overlapping generations model that an intermediated financial system can lead to almost everybody being better off than with financial markets. A natural question that arises is whether it is possible to combine the cross-sectional risk sharing advantages of U.S.-style markets with the intertemporal risk smoothing advantages of a German-style intermediated system.

In this section we show that there is a significant problem from trying to combine the two types of system. The reason is that risk sharing of the kind discussed in the last few sections implies some form of arbitrage opportunity. So it will be easier to perform this feat in an economy which does not have active markets. Taking advantage of arbitrage opportunities is rational for the individual, but it undermines the insurance offered by the intermediary. This is why the U.S. financial system cannot provide intertemporal risk smoothing of the type described, although it provides a tremendous variety of financial instruments. Increased openness to competition and the opportunity to trade in new financial markets may have a similar impact on the German system.

One way to see the effect of competition from financial markets is to consider the effect of opening up a relatively **small** closed and intermediated financial system to **global** financial markets. The small country's financial system now faces the constraint that individuals can opt out and invest in global markets instead. The assumption that the country is small relative to the rest of the world implies that prices in the global market are not affected by the financial system of the small country or its investors' decision to participate in the risk sharing mechanism provided by the intermediary.

Let $\{(\bar{p}_t, \bar{x}_t, \bar{m}_t)\}$ be the equilibrium in the global market and let $\{\bar{m}'_t\}$ be the optimal policy implemented in the small country. The global equilibrium represents a benchmark for the welfare of investors in the absence of a long-lived intermediary, as well as an outside option for the individuals when the intermediary is in operation. We assume that all investors in the small country make use of the intermediary. Since the intermediary can always replicate the investment opportunities available through the market, there is no loss of generality in this assumption.

Disintermediation can take several forms, depending on whether investors are able to make side trades while taking advantage of the intermediary. We assume that the intermediary can enforce **exclusivity,** which means that an agent who wants to trade in the market is unable to make use of the intermediary at all. This assumption makes disintermediation less attractive and hence produces a weaker constraint on the intermediary's problem of designing a risk smoothing scheme. We can show that even this weak constraint on the intermediary is sufficient to rule out any welfare improvement from intertemporal risk smoothing. Alternative (stronger) specifications of the disintermediation constraint would only strengthen this result.

The **disintermediation constraint** (DC), which ensures that people do not abandon the risk sharing mechanism in the small country, can be stated as follows. For any history $y^{t-1} = (y_1,\ldots,y_{t-1})$ it is necessary that

$$\mathcal{E}[u(\tilde{c}'_t)\,|\,y^{t-1}] \geq \text{Max}_{(x,m)}\ \mathcal{E}[u(x(\tilde{y}_t + m)\,|\,y^{t-1}], \qquad (16)$$

where $\{\tilde{c}'_t\}$ is the consumption sequence generated by the optimal policy. The expression on the right is the maximum expected utility an agent born at date t could obtain from trading on the open market. The expression on the left is the expected utility offered by the risk sharing mechanism. The crucial point is that both expressions are conditioned on all the information available at date t-1. An agent makes his decision whether to join the risk sharing mechanism after he has observed y^{t-1}.

The possibility of disintermediation implies that an intermediated financial system in a small open country does not allow any improvement in average expected utility over that obtained by investors in global financial markets.

Proposition 4

If the policy $\{\bar{m}'_t\}$ is feasible and satisfies the disintermediation constraint (15), then each agent is ex post no better off under $\{\bar{m}'_t\}$ than he would be in the market equilibrium $(\bar{p}_t, \bar{x}_t, m_t)$.

To understand Proposition 4, it is helpful to think about the policy described in Proposition 3. That policy gives each generation a return on its deposits equal to the lesser of the expected return to the risky asset and the sum of the actual return and the reserves held by the intermediary so that the consumption of each generation is:

$$\tilde{c}'_t = \mathrm{Min}(\mathcal{E}[\tilde{y}_t], \tilde{y}_t + \tilde{m}'_{t-1}) + e. \tag{17}$$

If the reserves held by the intermediary are very low (close to zero), the agent must be worse off than in the market equilibrium, because compared to the equilibrium he loses the high returns when $\tilde{y}_t > \mathcal{E}\tilde{y}_t$ and still suffers the probability of loss when $\tilde{y}_t < \mathcal{E}\tilde{y}_t$. So any generation will only be better off if it inherits a large reserve from the previous generation. This will be true most of the time, but occasionally a generation will be born when reserves are low and that generation will be worse off **ex post.** If that generation can opt out of the risk sharing mechanism, the whole scheme will unravel, leaving us in the situation described by Proposition 4.

Propositions 1 and 3 imply that expected utility will be higher for almost every generation in an intermediated economy than in a market-based economy. Incomplete financial markets do not allow intertemporal smoothing, while intermediaries in principle can, provided investors do not have ready access to financial markets. This suggests that the German financial system, with its reliance on financial intermediaries, may have some advantages over the U.S., which relies more on markets. However, Proposition 4 suggests that opening the German financial system to foreign competition, for example by creating a single European market in financial services, could undermine the potential for intertemporal smoothing.

7. Extensions and Other Applications

Our formal analysis has focussed on a simple overlapping generations model. This benchmark is meant to illustrate the absence of intertemporal smoothing that can result from incomplete markets and to show how an intermediated financial system can eliminate the resulting inefficiencies. It is important to stress that the overlapping generations structure is chosen because of its tractability. We believe that there are many other types of incompleteness that lead to the absence of intertemporal smoothing. In this section we discuss some of these other types of incompleteness. Intermediation is one way of providing intertemporal smoothing and in our overlapping generations model is able to implement an optimal allocation. We also discuss other ways of achieving intertemporal smoothing at the end of the section.

A natural extension of our model is to the case where each generation lives for more than two periods. In this case the opportunities for intertemporal risk smoothing may be increased. For example, generations whose prime earning years were during the Great Depression of the 1930’s or the oil shocks of the 1970’s and early 1980’s had a very different experience from generations whose prime earning years were during the 1920’s or 1950’s and 1960’s. Sharing the risk associated with these types of event can increase welfare, To illustrate the type of risk sharing that could occur, suppose each generation lives for three periods and only the middle-aged and old consume. Each period the young and middle aged will have a desire to share their consumption risk in the next period when they are middle-aged and old, respectively. One form this risk sharing may take is that in high output states next period the then old will give consumption to the then middle-aged in exchange for the opposite flow when output is low. This type of risk sharing cannot be achieved by holding standard debt and equity securities. If financial markets are relied on for this type of risk sharing then individualized securities based on age, health and so on will be necessary. The problems in implementing this type of contract such as adverse selection, moral hazard and high transaction costs are well known. In contrast, implementing intertemporal smoothing with an intermediary does not face these difficulties. There is no need for the individualized contracts necessary in financial markets and all the problems with these are avoided.

The extension of the model from having generations with three-period lives rather than two-period lives naturally raises the issue of what happens when more periods are introduced. In addition to the risk sharing across generations discussed above, this extension introduces the possibility of self-insurance over time. The result that Schechtman (1976) proved shows that with a long enough time horizon, individuals may be able to considerably reduce risk on their own. Whether such self-insurance can be achieved in an individual's lifetime depends on a number of factors. In the first place, the number of independent shocks may be small. We can think of the Great Depression as being one shock and the boom in the 1950's and 1960's as another. With this interpretation the number of periods each generation lives through is small. In addition, there are life cycle considerations which may prevent households from self-insuring. For example, the desire to purchase a house and provide an education for their children means that many households do not start saving for retirement until fairly late in life. For both these reasons the possibilities for self-insurance are limited.

Finally, note that incomplete market participation will not be a problem when agents have a bequest motive that causes successive generations to act like a single infinitely-lived individual. However, since all the empirical literature we know suggests that bequest motives are either incomplete or absent, this is likely to remain a theoretical curiosity.[3]

We have so far limited our discussion of alternatives to the market to intermediaries. Another important issue is the extent to which other institutions can alleviate the effects of risk. Perhaps the most obvious example of such an institution is the government. They could, for example, provide intertemporal smoothing by investing in safe assets directly. However, in recent years, responding to the argument that the private sector is better able to make investment decisions, governments in a number of countries have reduced their direct investment and have privatised their existing holdings of assets. If there are government failures which limit the effectiveness of governments in investing directly, intermediaries may be able to smooth risk more effectively.

Other methods of alleviating risk available to the government are social security schemes and budget deficits. Pay-as-you-go social security schemes and budget deficits are concerned with reallocations at a given point in time. They are methods of sharing risks between and within generations; they do not achieve intertemporal smoothing through asset accumulation. Funded social security systems can provide intertemporal smoothing through asset accumulation, but only if the fund is invested in real assets rather than paper claims. The government may play an important role in this respect if it is interested in the long-run welfare of its citizens. Similar considerations apply to occupational pension schemes provided by industrial groups or large employers.

8. Conclusion

A number of European countries have apparently decided to encourage the development of more sophisticated financial markets. In discussing French financial reforms in the mid 1980’s Melitz (1990) makes the following observation.

> As one contemplates the panoply of measures that took effect in France from late 1984 to the end of 1986, there is no doubt that the changes were inspired by a general vision. This was no mere lifting of controls: new instruments were created; new markets were added, including markets in futures; and the importance of permitting every individual agent to hedge his risks was clearly recognized. The whole program smacks of a close acquaintance with the principles of finance.

Although cross-sectional risk-sharing advantages may be gained as a result of these reforms, intertemporal risk smoothing possibilities may also be lost.

The integration of the European Union, the desire of the U.S. to liberalize trade in financial services, and the reconstruction of financial systems in the former Soviet block, all make the trade-off between the two types of risk sharing relevant from a policy perspective. Of course, this is not the only issue that arises in the design of a country’s financial system. A fuller discussion of these issues is contained in Allen and Gale (1995).

Appendix

Proof of Proposition 1

For any history $y^t = (y_1, \ldots, y_{t-1})$ and any equilibrium, the amount of the safe asset held by the young generation $m_t = e - p_t$ is positive if and only if the following first-order condition is satisfied:

$$\mathcal{E}[(p_{t+1} - p_t + y_{t+1})u'(p_{t+1} - p_t + y_{t+1} + e)|y^t] = 0. \tag{18}$$

Let $\tilde{x}$ and $\tilde{y}$ be random variables with joint probability distribution μ. Then the first order condition is a special case of the equation:

$$\int(\tilde{x} + \tilde{y})u'(\tilde{x} + \tilde{y} + e)d\mu = 0. \tag{19}$$

The following lemma establishes an important inequality that must be satisfied by any random variables satisfying this equation.

<u>Lemma</u>: Suppose that $\tilde{x}$ and $\tilde{y}$ are random variables satisfying

$$-e \le \tilde{x} \le e \text{ and } 1/A \le \tilde{y} \le A$$

with probability 1 and that u' is a continuous, increasing function on $[0,2e + A]$. Then there exist numbers $\epsilon > 0$ and $\eta > 0$ independent of $\tilde{x}$ and $\tilde{y}$ such that $\text{Prob}[\tilde{x} < \epsilon] > \eta$.

Proof:

$$\begin{aligned} 0 &= \int(\tilde{x} + \tilde{y})u'(\tilde{x} + \tilde{y} + e)d\mu \\ &= \int\tilde{x}u'(\tilde{x} + \tilde{y} + e)d\mu + \int\tilde{y}u'(\tilde{x} + \tilde{y} + e)d\mu \\ &\ge u_{min}\int_{x\ge 0}\tilde{x}d\mu + u_{max}\int_{x<0}\tilde{x}d\mu + u_{min}\int\tilde{y}d\mu, \end{aligned} \tag{20}$$

$u_{min} = u'(2e + \bar{y})$ and $u_{max} = u'(0)$. Since the first expression is clearly non-negative, this implies that

$$\frac{u_{min}}{u_{max}}\mathcal{E}\tilde{y} \ge \int_{x<0}\tilde{x}d\mu. \tag{21}$$

For any number $0 < \epsilon < e$,

$$\int_{x<0}\tilde{x}d\mu \ge -\epsilon\,\mu(0 > \tilde{x} \ge -\epsilon) - \mu(\tilde{x} < -\epsilon)e, \tag{22}$$

since $\tilde{x} \geq - e$. Then

$$\mu(\tilde{x} < - \epsilon) \geq e^{-1}(\beta - \epsilon\ \mu(0 > \tilde{x} \geq - \epsilon)) \geq e^{-1}(\beta\epsilon), \tag{23}$$

where $\beta = \frac{u_{min}}{u_{max}} A^{-1}$. Putting $\eta = e^{-1}(\beta - \epsilon)$ for some small value of $\epsilon > 0$ completes the proof.█

Returning to the proof of the proposition, we can identify y_{t+1} with $\tilde{y}$ and $p_{t+1} - p_t$ with $\tilde{x}$. Since the conditions of the lemma are evidently satisfied, we can conclude that

$$p_{t+1} - p_t \leq - \epsilon \text{ with probability } \eta,$$

where the numbers ϵ and η are the ones given in the lemma and are independent of y^t and of the equilibrium. In particular, with probability $\eta > 0$, $p_{t+1} < e$ and $m_{t+1} > 0$, so repeating the same argument we can conclude that with probability η^2, $p_{t+2} - p_t < - 2\epsilon$. Continuing in this way, we can show that with probability η^k, $p_{t+k} - p_t < - k\epsilon$, for every k such that $t < T - k$. Since this argument must ultimately lead to negative prices for k sufficiently large, letting k be the smallest value of k greater than e/ϵ, we conclude that $m_t > 0$ is possible only if $t \geq T - k$. By construction k is evidently independent of the equilibrium and T.█

Proof of Proposition 2

Consider the decision of the last generation who are young at date T-1. Their decision is to maximize $\mathcal{E}_{T-1}u(\tilde{y}_T x_{T-1} + e - p_{T-1}x_{T-1})$ subject to $p_{T-1}x_{T-1} \leq e$. The first-order condition for an optimum is

$$\mathcal{E}_{T-1}u'(\tilde{y}_T x_{T-1} + e - p_{T-1}x_{T-1})(\tilde{y}_T - p_{T-1}) \geq 0, \tag{24}$$

with strict equality if $p_{T-1} < e$. Since $x_{T-1} = 1$ in equilibrium and u is strictly concave, the first-order condition uniquely determines the agent's portfolio and the equilibrium price and since $\{\tilde{y}_t\}$ is i.i.d. these values are independent of history. Using the same argument repeatedly shows that there is a unique equilibrium in which the price p_t at each date t is independent of the history $\tilde{y}_t$.

Now suppose that $(\tilde{m}'_t)$ is a feasible policy that achieves a Pareto-improvement compared to the equilibrium allocation. In order that generation T-1 be no worse off, it must be the case that $\tilde{m}'_{T-1} \geq m_{T-1} = e - p_{T-1}$, for all possible histories $\tilde{y}_{T-1}$. And if generation T-1 is better off this inequality must be strict. The same argument can be repeated for each preceding generation, so showing that $\tilde{m}'_t \geq m_t$, for all t, with strict inequality for those generations who are actually better off. But this is clearly not feasible, so we conclude that the equilibrium allocation is strongly Pareto-efficient.█

Proof of Proposition 3

Consider the policy $(\bar{m}_t)$ defined as follows:

$$\bar{m}_0 = 0 \quad (25)$$

$$c_t = \text{Min}(\mathcal{E}\tilde{y}_t, \bar{m}_{t-1} + \tilde{y}_t) + e$$

$$m_t = \bar{m}_{t-1} + y_t + e - c_t.$$

Define a random variable X_t by putting $X_t = 1$ if $\bar{c}_t < \mathcal{E}\tilde{y}_t + e$ and $X_t = 0$ otherwise. Shechtman (1976) has shown that $\mathcal{E}[\frac{1}{T}\sum_{t=1}^{T} X_t] \to 0$ as $T \to \infty$. Then, for any $\epsilon > 0$,

$$\#\{0 \leq t \leq T | \mathcal{E}X_t > \epsilon\}/T \to 0 \text{ as } T \to \infty. \quad (26)$$

Consequently, for any $\epsilon > 0$, there exists a $\bar{T}$ such that $T > \bar{T}$ implies that

$$\#\{0 \leq t \leq T | \mathcal{E}u(\bar{c}_t) > u(\mathcal{E}\tilde{y}_t + e) - \epsilon\} > (1 - \epsilon)T, \quad (27)$$

as required.█

Proof of Corollary 3.1

We begin by establishing the inequality (13). The sequence $\{\tilde{y}_t\}$ is initially assumed to be i.i.d. Suppose that a policy $\{\tilde{m}_t\}$ solves the problem

$$\text{Max } \mathcal{E}[\textstyle\sum_{t=1}^{T} \lambda_t u(\tilde{c}_t)] \tag{28}$$
$$\text{s.t. } \tilde{c}_t = \tilde{y}_t + e + \tilde{m}_t - \tilde{m}_{t-1}.$$

The first-order conditions for the choice of $\tilde{c}_t$ for this problem imply

$$\lambda_t u'(\tilde{c}_t) = \mu_t \tag{29}$$

and for the choice of $\tilde{m}_t$ imply

$$\mu_t \geq \mathcal{E}_t \mu_{t+1}, \tag{30}$$

where μ_t is the Lagrange multiplier associated with the constraint and $\mathcal{E}_t$ denotes the expectation operator conditional on the information available at date t. Suppose that $\{(\tilde{x}_t,\tilde{m}_t)\}_{t=1}^{T}$ is an attainable allocation with the property that $\tilde{m}_t = 0$ for every t. Recall that $\tilde{y}_t \leq A$ with probability 1. Then

$$\lambda_t u'(e + \tilde{y}_t) \geq \mathcal{E}_t \lambda_{t+1} u'(e + \tilde{y}_{t+1}) \tag{31}$$

for every realization of $\tilde{y}_t$, which implies that

$$\frac{\lambda_t}{\lambda_{t+1}} \geq \frac{\mathcal{E}_t u'(e + \tilde{y}_{t+1})}{u'(e + A)} = \frac{1}{\rho}, \tag{32}$$

where $\rho < 1$ can be chosen independently of t because the process $\{\tilde{y}_t\}$ is assumed to be i.i.d.

The preceding argument can be extended to the case in which the $\{\tilde{y}_t\}$ are not i.i.d., if it is assumed that variance of $\tilde{y}_{t+1}$ conditional on $\tilde{y}_t$ is bounded away from 0. In that case, the ratio

$$\frac{\mathcal{E}_t u'(e + \tilde{y}_{t+1})}{u'(e + A)} > 1 \tag{33}$$

is uniformly bounded away from 1 and so is bounded below by $1/\rho$ for some $\rho < 1$.

It is clear that a sequence $\{\lambda_t\}$ satisfying (13) cannot satisfy the uniform weighting property (12), and this proves that the equilibrium allocation is not uniformly Pareto-efficient. An alternative proof follows from the fact that an almost uniform Pareto improvement is possible. Let $\{\tilde{c}_t\}$ be the consumption sequence generated by the policy defined in the proof of Proposition 3. Let $\eta = u(\mathcal{E}\tilde{y}_t + e) - \mathcal{E}u(\tilde{y}_t + e) > 0$. Then for any $\epsilon > 0$ there exists a $\bar{T}$ such that $T > \bar{T}$ implies

$$\#\{0 \leq t \leq T \mid \mathcal{E}u(\tilde{c}_t) < \mathcal{E}u(\tilde{y} + e) + \eta\} < \epsilon T. \qquad (34)$$

For any B-uniform sequence $\{\lambda_t\}$, where $B \geq 1$ is fixed but arbitrary, and for all $T > \bar{T}$,

$$\begin{aligned} \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \lambda_t \mathcal{E}u(\tilde{c}_t) &\geq \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \lambda_t (\mathcal{E}u(\tilde{y}_t + e) + \eta) - \epsilon B u(\mathcal{E}\tilde{y}_t + e) \qquad (35) \\ &\geq \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \lambda_t \mathcal{E}u(\tilde{y}_t + e) + \eta B^{-1} - \epsilon B u(\mathcal{E}\tilde{y}_t + e) \\ &> \tfrac{1}{T}\textstyle\sum_{t=1}^{T} \lambda_t \mathcal{E}u(\tilde{y}_t + e) \end{aligned}$$

for ϵ sufficiently small. This shows that the equilibrium allocation is not uniformly efficient for any fixed but arbitrary B and all T sufficiently large.█

Proof of Corollary 3.2

What Schechtman (1976) actually shows is that

$$\tfrac{1}{T}\textstyle\sum_t \mathcal{E}u(\tilde{c}_t) \to u(\mathcal{E}\tilde{y}_t + e). \qquad (36)$$

For any feasible consumption sequence $\{\tilde{c}'_t\}$,

$$\begin{aligned} \tfrac{1}{T}\mathcal{E}\textstyle\sum_t u(\tilde{c}'_t) &\leq \mathcal{E}u(\textstyle\sum_t \tfrac{1}{T}\tilde{c}_t) \qquad (37) \\ &\leq u(\mathcal{E}\textstyle\sum_t \tfrac{1}{T}\tilde{c}_t) \\ &= u(\mathcal{E}\tilde{y}_t + e). \end{aligned}$$

Choosing $\lambda_t = 1$ for every t,

$$\begin{aligned} \tfrac{1}{T}\mathcal{E}\textstyle\sum_t \lambda_t u(\tilde{c}_t) &= \tfrac{1}{T}\mathcal{E}\textstyle\sum_t u(\tilde{c}_t) \qquad (38) \\ &\to u(\mathcal{E}\tilde{y}_t + e), \end{aligned}$$

which shows that the policy is B-uniformly undominated in the limit.█

Proof of Proposition 4

Let $((\tilde{p}_t,\tilde{m}_t,\tilde{x}_t))$ be the global equilibrium and $(\tilde{m}'_t)$ the policy adopted by the intermediary. Using the budget constraint of the buying generation $e = \tilde{p}_t\tilde{x}_t + \tilde{m}_t$ and the market-clearing condition $\tilde{x}_t = 1$ gives $\tilde{p}_t = e - \tilde{m}_t$. It follows that the disintermediation constraint (DC) can be written

$$\mathcal{E}[u(\tilde{c}_t)|y^{t-1}] \geq \mathcal{E}[u(\tilde{y}_t + \tilde{m}_{t-1} - \tilde{m}_t + e)|y^{t-1}], \tag{39}$$

for $t = 1,...,T-1$. In the last period T, $e = 0$ and $\tilde{m}'_T = \tilde{m}_T = 0$ so $\tilde{c}_t = \tilde{y}_T + \tilde{m}'_{T-1}$ and in this case the DC reduces to

$$\mathcal{E}[u(\tilde{y}_T + \tilde{m}'_{T-1})|y^{T-1}] \geq \mathcal{E}[u(\tilde{y}_T + \tilde{m}_{T-1})|y^{T-1}]. \tag{40}$$

Since $\tilde{m}'_{T-1}$ and $\tilde{m}_{T-1}$ are functions of y^{T-1}, this implies

$$\tilde{m}'_{T-1} \geq \tilde{m}_{T-1}. \tag{41}$$

Suppose $\tilde{m}'_{T-k} \geq \tilde{m}_{T-k}$ for some k. Then

$$\mathcal{E}[u(\tilde{y}_{T-k}+\tilde{m}'_{T-k-1}-\tilde{m}'_{T-k}+e)|y^{T-k-1}] \geq \mathcal{E}[u(\tilde{y}_{T-k}+\tilde{m}_{T-k-1}-\tilde{m}_{T-k}+e)| \; ^{T-k-1}], \tag{42}$$

implies $\tilde{m}'_{T-k-1} \geq \tilde{m}_{T-k-1}$. So by induction

$$\tilde{m}'_t \geq \tilde{m}_t \quad \text{for } 1,...,T-1. \tag{43}$$

Since $\tilde{m}'_1 \geq \tilde{m}_1$, if the initial old generation is not to be made worse off it must be the case that $\tilde{m}'_1 = \tilde{m}_1$. Substituting this into the DC implies that $\tilde{m}'_2 \leq \tilde{m}_2$. Combining this with (43) implies $\tilde{m}'_2 = \tilde{m}_2$. By induction, we can show that $\tilde{m}_t = \tilde{m}'_t$ for all $t = 1,...,T-1$, so the DC is satisfied with equality in all periods, which implies that no one is made better off ex post by the risk sharing mechanism.∎

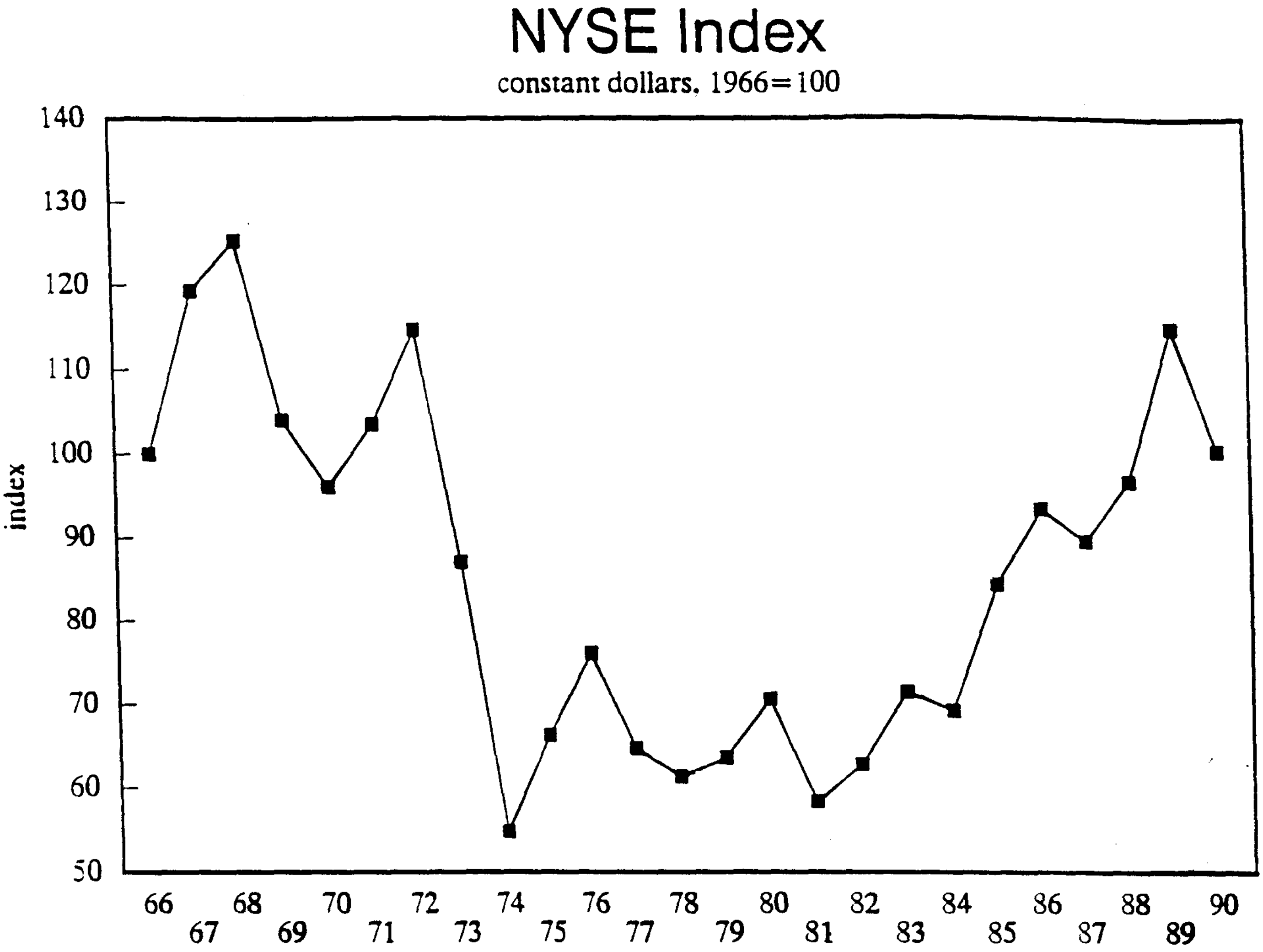

Figure 1

The variation of real U.S. stock prices 1966-1990

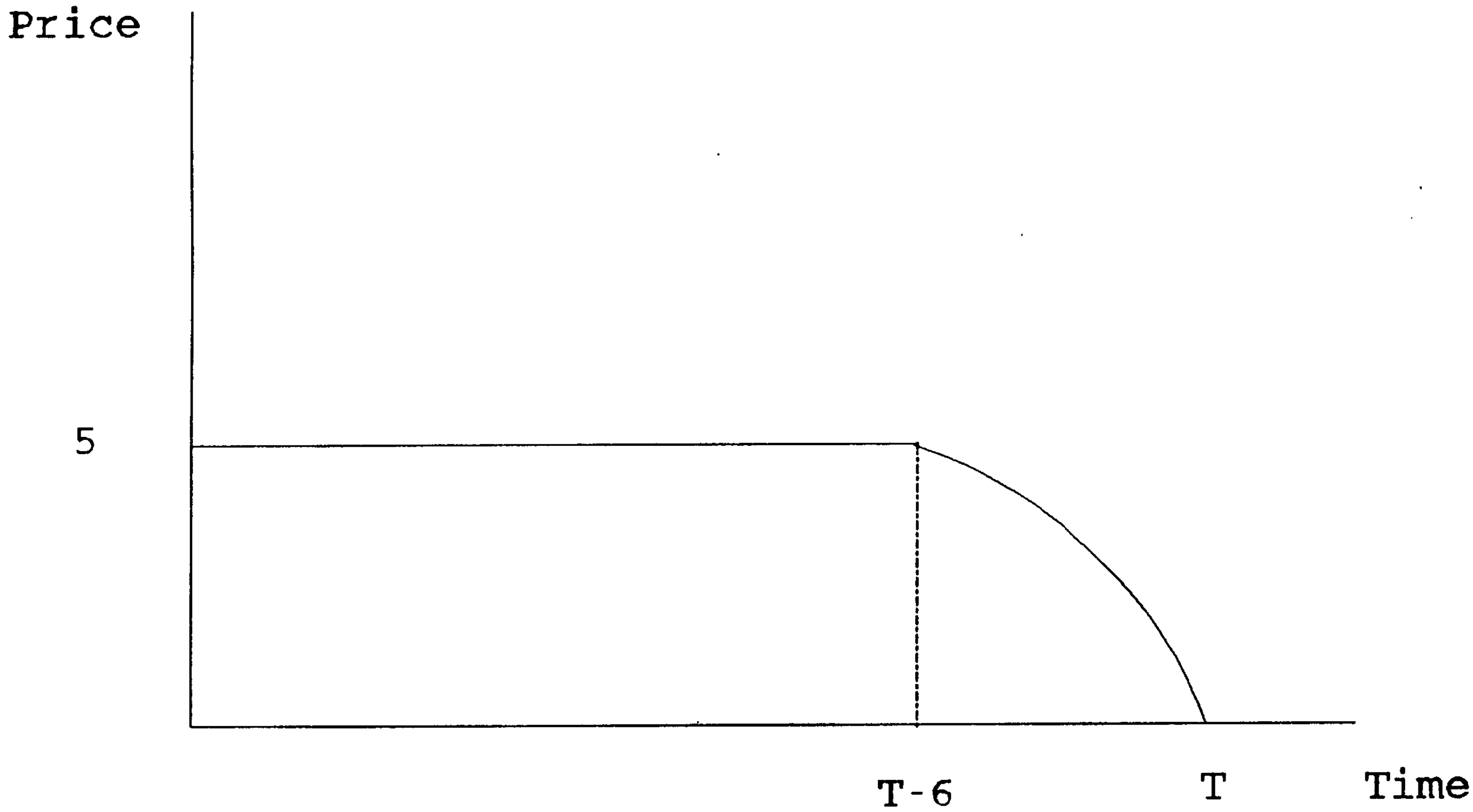

Figure 2

The path of prices over time in Example 1

References

Allen, F. and D. Gale (1994). "Limited Market Participation and Volatility of Asset Prices," American Economic Review 84, 933-955.

Allen, F. and D. Gale (1995). "A Welfare Comparison of Intermediaries and Financial Markets in Germany and the U.S.," European Economic Review, forthcoming.

Altonji, J., F. Hayashi and L. Kotlikoff (1992). "Is the Extended Family Altruistically Linked? Direct Tests Using Microdata," American Economic Review 82, 1177-1198.

Bhattacharya, S. and A. J. Padilla (1994). "Dynamic Banking: A Reconsideration," working paper, CEMFI, Madrid.

Fulghieri, P. and R. Rovelli (1994). "Capital Markets, Financial Intermediaries, and the Supply of Liquidity in a Dynamic Economy," working paper, IGIER, Milan.

Gale, D. (1994). "The Efficient Design of Public Debt," in F. Allen and D. Gale, eds., Financial Innovation and Rsik Sharing, MIT Press, Cambridge, MA.

McCallum, B. T. (1987). "The Optimal Inflation Rate in an Overlapping-Generations Economy with Land," in W. A. Barnett and K. J. Singleton, eds., New Approaches to Monetary Economics, Cambridge University Press, Cambridge, U.K.

Melitz, J. (1990). "Financial Deregulation in France," European Economic Review 34, 394-402.

Qi, J. (1994). "Bank Liquidity and Stability in an Overlapping Generations Model," Review of Financial Studies (forthcoming).

Schectman, J. (1976). "An Income Fluctuation Problem," Journal of Economic Theory 12, 218-241.

Scheinkman, J. (1980) "Notes on Asset Trading in an Overlapping Generations Economy," working paper, University of Chicago.

Notes

[1] See Allen and Gale (1995).

[2] For a further discussion of the issues involved in intergenerational risk sharing, the reader is referred to Gale (1994) and the references cited there.

[3] See Altonji, Hayashi and Kotlikoff (1992) and the references cited therein.