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Alternative Monies and the Demand for Media of Exchange

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Alternative Monies and the Demand for Media of Exchange [1]

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Abstract:

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I. INTRODUCTION

Prepaid cards, cash cards, electronic purse, smart cards - these are but a few of the elements in the revolution now taking place in monetary systems around the world. At that movement's heart is the emergence of a new value transfer system where alternative monies are offered to consumers through the miracle of electronics. In some cases, the new monies are merely a repackaging of existing media of exchange. For example, stored value cards that have been traditionally available for single vendors now are being offered for multiple merchant use. In other cases, technology and its acceptability are being cross-sold, as with debit cards and the expanded use of ATM cards as point-of-sale vehicles. However, the banking industry is perhaps most enthusiastic about the emerging technology of smart cards – chip-in-cards - that have been making inroads in Europe. With the expanded capability of a memory chip, alternative monies are seen to have the capability of moving to a new plateau of acceptance.

There are many stories in the business press about the general enthusiasm for these new forms of money. For example, Block (1995) and Cutler (1994) herald the new day of electronic money. They forecast the demise of both the greenback and of commercial banks that exploit the float derived from its use. According to the hyperbole from marketing reps from this side of the financial community, currency and demand deposits soon will be endangered species. Consumer acceptance is alleged to be high, and cost efficiency associated with the new technology is expected to be substantial.

Yet, as Wenninger and Laster (1995) suggest, "to succeed, an electronic purse system will need to offer enough features of value to its three constituencies - consumers, merchants, and issuers - to induce them to bear the cost." To date much has been said about the advantages of alternative monies to the institutions issuing them. In fact, most arguments in favor of these media of exchange can be viewed as a reaction to the rising costs associated with current check clearing and on-line credit card systems. Although hard data are scarce, the cost advantages to the banking system of a movement from paper checks to debit cards is alleged to be in the range of 50%.[1]

Merchants are perceived to be winners too. Off-line pre-paid cards are expected to reduce the cost of clearing of transactions, handling cash and reducing fraud. Therefore, the advent of electronic money should be a panacea welcomed by all merchants, large and small. For major retailers, cash management and on-line authorization systems become obsolete issues. For small businesses, inexpensive off-line technologies allow even the local grocery store to accept pre-paid cards with integrity assurance and minimal expense.

To read the press, the consumer is equally enthusiastic. A 1992 study by Synergistic Research Corporation found that 30% of all consumers surveyed expressed interest in pre-paid cards, with 75% of this group willing to pay a reasonable level of transaction fee for their use. Smart Card Enterprises found even more support in their 1993 survey of Delaware consumers, with 88% expressing willingness to use some form of the new technology.[2] And this is only the evidence from the US, where consumer experience is limited and provider experimentation just beginning. NatWest is receiving very positive responses in the U.K. from its Mondex product[3] and Humphrey, Pulley and Vesala (1996) report substantial European usage and acceptance of a

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[1] See Saul (1994) for example.

[2] Again, see Saul (1994) and the studies referenced therein.

[3] See Mondex press release dated October 21, 1994.

wide range of non-cash payment instruments.

However, the economic rationale for this consumer enthusiasm is a bit unclear. Each application of the emerging electronic technology has been different in important ways. The universal receptivity to alternative monies may be more a good marketing ploy than good economics. These technologies provide alternative transaction vehicles with an array of characteristics that must be evaluated by potential users as to their desirability. These features includes transfer fees, acceptability, deferential float, different implied yields on balances held in each form, and a wide range of other unique characteristics. To understand consumer reaction requires a careful analysis of the desirability of any one of these alternative technologies in terms of its impact on consumer cash management costs through changes in transaction patterns, average money holdings, and total transaction costs.

In the current paper we offer a first attempt at such an analysis. We investigate the effect of variations in the number and type of monies on consumer transactions demand using a Baumol-Tobin type model of money demand. Our analysis draws on a long literature investigating the transactions demand for money, including, but not limited to, Baumol (1956), Tobin (1956), Santomero (1979) and Romer (1987).[4] We investigate the behavior of a representative agent faced with a choice of monies with which to transact, and ask how variations in the characteristics of these monies will affect the consumer's choice of transaction vehicle, transactions frequency, and average balance in various media.

Interestingly, the results are not transparent. Variations in the cost of transfer, interest rates, and the acceptability of alternative media have surprising effects on consumer choice. In

[4] Models of this type have recently been reviewed in Milborne (1992)

general, the cost of using a medium of exchange determines whether it will be used and for which goods it will be traded. The choice of medium of exchange, then, has a direct effect on both the average holdings of different types of money and their transaction frequencies. It, therefore, suggests that efforts by banks to alter the costs of using a particular form of money may cause representative agents to change their exchange behavior, shifting it sometimes rather dramatically. As a general characterization, the model suggests that consumer monetary choice is anything but simple, and variations in the characteristics of various media of exchange will not necessarily have straightforward effects.

II. THE STATUS OF ALTERNATIVE MONIES

As noted at the outset, there are a number of types of monetary innovations currently underway in the United States. Here, we will merely touch the surface. Interested readers may wish to refer to the references we cite here for a further discussion of current implementation. With all the experimentation, however, it is important to recognize that America is somewhat behind in introducing these types of transaction vehicles. As Humphrey, Pulley and Vesala (1996) indicate in a companion paper at this Conference, and Kokkola and Pauli (1994) report, considerably more innovation has occurred in Europe. Ignoring standard credit card transactions, the Europeans are way ahead of their American brethren in the implementation of non-cash, non-paper-based money utilization. Within Europe, the Scandinavian countries and Northern Europeans clearly have the lead. It is worth noting, however, that Asia is not far behind. Both Japan and Singapore have made substantial inroads in the use of alternative monies in commerce.

Within the US, the battleground for electronic money at the consumer level centers around debit cards and various forms of prepaid plastic and paper vehicles. We have seen more than a decade pass since the banking industry proclaimed the year of the debit card. Yet, despite its slow progress there is continued high expectations and enthusiasm for an increased use in this non-paper access to a demand deposit account; see Giesseon (1994) or Casey and Sellon (1994). Prepaid cards, long a fixture in Europe, are slowly emerging in the United States as well. Their use began with transit cards and has expanded into pay telephone applications recently. The approach taken to increase the use of these prepaid cards appears to be the broadening of the acceptability of existing cards so that they can be made usable at a broader range of retail locations. For example, New York's Metropolitan Transit Authority has broadened the acceptability of its pre-paid cards, and universities around the country are following a similar route (Cutler, 1994). At the same time, more and more single purpose card applications surface. The most recent one noted in the *American Banker* is the Blockbuster Video announcement of a stored value card for member rentals this fall (Block 1995).

The frontier application, however, appears to be smart card technology. Here, a chip-in-card technology substantially expands the memory capacity of pre-paid vehicles. We are presented with the capability of carrying a fully electronic identity card which includes both personal information and stored money. This prospect conjures up images of George Orwell's *1984* for some people, but it clearly also represents an unique commercial opportunity. Merchants can access these funds through off-line, relatively inexpensive machines that would transfer ownership to the pre-paid balances from the customer to the merchant, for subsequent clearing and collection (Wenninger and Laster 1995). Pilot studies using this technology abound, with the most recent example being a November 15, 1995 announcement of a First Union smart card pilot in Atlanta (Piskora 1995). To be sure, this technology is part of the emerging payment systems trend and is clearly in the sights of the banking community (Furash and Company 1994).

III. CONSUMER REACTION TO ALTERNATIVE MONIES

Consumer acceptance of any new technology, however, will be key to its success. Yet, little work has been done to examine consumer choice of monetary vehicles and their reaction to changes in the costs associated with value transfer. We now turn to these considerations, looking at a representative agent model of the demand for money for transaction purposes. As all of the proposed monies are offered as a medium of exchange, the demand response to their availability should be determined by the same factors that determine the demand for other forms of money.

Generically, standard models of the demand for the medium of exchange view the driving factors as the volume of payments, transactions costs, yield spreads, uncertainty, illiquidity costs, and so on. These will be considered below. We confine our attention to a world of complete certainty, using the type of model recently reviewed by Milborne (1992). Here, however, we generalize the standard model to include several media of exchange and several goods purchased.

IIIA. THE THEORETICAL MODEL

We adopt the usual Baumol-Tobin assumptions, except that (1) we allow consumers to hold inventories of goods, as in Santomero (1974), and (2) we allow several goods and several media of exchange.[5]

[5] Extension of the analysis to include such sophisticated aspects as imposition of general equilibrium, as in Romer (1987) or stochastic cash flows, as in Frankel and Jovanovic (1980) would be worthwhile but is beyond the scope of the present paper.

Model Set-up. The household receives a fixed income Y every fixed payments period, and exactly exhausts that income by buying a fixed amount, X_g, of G different goods, such that

$$Y = \sum_{g=1}^{G} X_g \tag{1}$$

Consumption of goods occurs at a constant rate that just exhausts the goods purchased each period. On the other hand, consumption expenditures (i.e., purchases of goods) occur at discrete intervals, to be chosen optimally by the household. Between such "shopping trips," the household holds inventories of the various goods, which it gradually consumes until exactly exhausting them at the moment it is time to make another shopping trip. A separate shopping trip is required for each type of good. Each type of commodity inventory pays a unique rate or return, r_{Xg}, which may be negative, such as a spoilage rate.

There are L media of exchange, M_i, available to the household. The household can use any or all of them to buy each type of good. Denote the quantity of good g bought with money i by X_{gi}. The household's choice need not be exclusionary, in that it may use one medium, M_j, on some shopping trips for good g and another medium, M_k, on others. Thus,

$$X_g = \sum_{i} X_{gi} \tag{2}$$

There are Z_{gi} trips per payments period to purchase good g with money i. Each such trip has associated with it the shopping cost β_{gi}, a lump-sum cost associated with each trip but not depending on the amount spent. This cost may be explicit, such as a delivery charge or a check-cashing fee, or implicit, such as a time cost.

The household spends only a fraction of its income during any one shopping trip. Unspent income is held in a single savings asset, S, and in money balances. Savings earn the rate of return r_S, and the various kinds of money earn rates of return r_{Mi}. We suppose that

$r_S > r_{Mi} > r_{Xg}$.[6] The household periodically converts some of S into money by making a "trip to the bank." There are T_i conversion trips to obtain M_i, and each such trip has associated with it the conversion cost α_i, which is is the cost of transferring funds into the medium of exchange. This cost, like shopping trip costs, is modeled as a lump-sum amount paid explicitly or implicitly each time a conversion is made but not depending on the size of the conversion.

As in the simple Baumol-Tobin model, optimal conversions are evenly spaced. Shopping trips occur between conversion trips and also are evenly spaced.[7] There are N_{gi} shopping trips to buy good g with money i per conversion of S into M_i. The total number of shopping trips per income payments period, Z_{gi}, to buy good g with money i is thus T_iN_{gi}. Finally, each of the assets, S and M_i, carries a fixed cost F_i that must be paid if that asset is held at any time during the payments period. These fixed costs capture such things as monthly account fees.

The household seeks to maximize the profit from managing its assets over a given payments period. Because all conversion and shopping trips are evenly spaced and consumption proceeds at a constant rate, the profit function of the representative agent can be written in terms of the average values of the respective assets:

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[6] In fact, we do not need to require that all money interest rates exceed all inventory rates of return, but it simplifies the discussion to impose that requirement. In the more general case, it is trivial to show that if the rate of return r_{Xg} on good g exceeds the rate of return r_{Mi} on money I, then money I will not be used to purchase good g.

[7] The proof that optimal trips are evenly spaced is tedious and not given here. See Tobin (1956).

$$\pi = r_S \overline{S} + \sum_{i=1}^{L} r_{Mi} \overline{M}_i + \sum_{g=1}^{G} r_{Xg} \overline{X}_g - \sum_{i=1}^{L} T_i \alpha_i - \sum_{i=1}^{L} \sum_{g=1}^{G} Z_{gi} \beta_{gi} - F_S I(S) - \sum_{i=1}^{L} F_i I(M_i) \tag{3}$$

where I(X) is an indicator function that is 1 if average holdings of asset X are positive and is 0 otherwise. From the above one can derive the needed expressions for the average asset holdings. Average total assets can be written as:

$$\begin{aligned} \overline{A} &= \overline{S} + \sum_{i=1}^{L} \overline{M}_i + \sum_{g=1}^{G} \overline{X}_g \\ &= \overline{S} + \sum_{i=1}^{L} \left(\overline{M}_i + \sum_{g=1}^{G} \overline{X}_{gi} \right) \end{aligned} \tag{4}$$

Given that trips are evenly spaced and the rate of consumption is constant, we also have:

$$\overline{A} = \sum_{g} \frac{X_g}{2} \tag{4'}$$

$$\overline{M}_i + \sum_{g} \overline{X}_{gi} = \sum_{g} \frac{X_{gi}}{2T_i} \tag{5}$$

$$\overline{X}_{gi} = \frac{X_{gi}}{2Z_{gi}} \tag{6}$$

We then have:

$$\overline{M}_i = \left(\overline{M}_i + \sum_g \overline{X}_{gi}\right) - \sum_g \overline{X}_{gi} \tag{5'}$$

$$= \frac{\sum_g X_{gi}}{2T_i} - \frac{\sum_g X_{gi}}{2Z_{gi}}$$

$$\overline{S} = \overline{A} - \sum_i \left(\overline{M}_i + \sum_g \overline{X}_{gi}\right) \tag{7}$$

$$= \sum_g \frac{X_g}{2} - \sum_i \sum_g \frac{X_{gi}}{2T_i}$$

Substituting (5) through (7) into the profit equation yields:

$$\pi = r_S\left[\sum_g \left(\frac{X_g}{2} - \sum_i \frac{X_{gi}}{2T_i}\right)\right] + \sum_i r_{Mi}\left[\sum_g \left(\frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_{gi}}\right)\right] \tag{8}$$

$$+ \sum_g r_{Xg}\left(\sum_i \frac{X_{gi}}{2Z_{gi}}\right) - \sum_i T_i\alpha_i - \sum_i \sum_g Z_{gi}\,\beta_{gi}$$

$$- F_S I(S) - \sum_i F_i I(M_i)$$

IIIB. Optimal Usage Patterns.

The household must choose the T_i, Z_{gi}, and X_{gi}. It is easiest to proceed in two steps. First, find the optimal values of T_i and Z_{gi} in terms of the $X_{gi,}$ and then find the optimal values of the X_{gi}. This procedure is akin to concentrating a likelihood function, except that here we are concentrating the profit function. As usual, we treat T_i and Z_{gi} as continuous variables, even though in fact they must be integers. The integer constraints are relevant in some of the later discussion, but it is unnecessary for our purposes to include them in the analysis formally[8].

The appropriate first order conditions give the solutions:

$$T_i = \left[(r_S - r_{Mi}) \sum_g \frac{X_{gi}}{2\alpha_i} \right]^{1/2} \tag{9}$$

$$Z_{gi} = \left[(r_{Mi} - r_{Xg}) \frac{X_{gi}}{2\beta_{gi}} \right]^{1/2} \tag{10}$$

Substituting (9) and (10) into (5) through (7) yields the following expressions for average balances:

$$\overline{X}_{gi} = \left[\frac{X_{gi}\beta_{gi}}{2(r_{Mi} - r_{Xg})} \right]^{1/2} \tag{11}$$

$$\overline{M}_i = \left[\frac{\alpha_i}{2(r_S - r_{Mi})} \sum_g X_{gi} \right]^{1/2} - \sum_g \left[\frac{\beta_{gi} X_{gi}}{2(r_{Mi} - r_{Xg})} \right]^{1/2} \tag{12}$$

$$\overline{S} = \sum_g \frac{X_g}{2} - \sum_i \left[\frac{\alpha_i}{2(r_S - r_{Mi})} \sum_g X_{gi} \right]^{1/2} \tag{13}$$

Substituting these expressions into the profit function, doing some algebra, and using the adding-up identity (2) yields:

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[8] Readers interested in the formal details of integer constraints should consult Barro(1970).

$$\pi = r_S \sum_g \frac{X_g}{2} - \sum_i^{L-1} \left[\sum_g 2\alpha_i (r_S - r_{Mi}) X_{gi} \right]^{1/2} - \left[2\alpha_L (r_S - r_{ML}) \sum_g \left(X_g - \sum_i^{L-1} X_{gi} \right) \right]^{1/2} \tag{14}$$
$$- \sum_g \left\{ \sum_i^{L-1} [2\beta_{gi}(r_{Mi} - r_{Xg}) X_{gi}]^{1/2} + \left[2\beta_{gL}(r_{ML} - r_{Xg}) \left(X_g - \sum_i^{L-1} X_{gi} \right) \right]^{1/2} \right\}$$
$$- F_S I(S) - \sum_i^{L} F_i I(M_i)$$

We next consider the first-order condition for the kth good purchased with money i, X_{ki}:

$$\frac{\partial \pi}{\partial X_{ki}} = -\frac{1}{2}[2\alpha_i (r_S - r_{Mi})]^{1/2} \left[\sum_g X_{gi} \right]^{-1/2} + \frac{1}{2}[2\alpha_L (r_S - r_{ML})]^{1/2} \left[\sum_g \left(X_g - \sum_i^{L-1} X_{gi} \right) \right]^{-1/2} \tag{15}$$
$$- \frac{1}{2}[2\beta_{ki}(r_{Mi} - r_{Xk})]^{1/2} (X_{ki})^{-1/2} + \frac{1}{2}[2\beta_{kL}(r_{ML} - r_{Xk})]^{1/2} \left(X_k - \sum_i^{L-1} X_{k1} \right)^{-1/2}$$
$$= 0$$

There are G(L-1) such conditions to be solved simultaneously for the G(L-1) different X_{ki}.

Not much insight can be gained from the general case, so we restrict attention to a simpler model where G=L=2.[9] In this case, only X_{11} and X_{21} need to be chosen; X_{12} and X_{22} are determined by the adding-up identity. The profit function then can be written as:

$$\pi = r_S \frac{X_1 + X_2}{2} - [2\alpha_1 (r_S - r_{M1})(X_{11} + X_{21})]^{1/2} - [2\alpha_2 (r_S - r_{M2})\{(X_1 - X_{11}) + (X_2 - X_{21})\}]^{1/2} \tag{14'}$$
$$- \{[2\beta_{11}(r_{M1} - r_{X1}) X_{11}]^{1/2} + [2\beta_{21}(r_{M1} - r_{X2}) X_{21}]^{1/2}$$
$$- \{[2\beta_{12}(r_{M2} - r_{X1})(X_1 - X_{11})]^{1/2} + [2\beta_{22}(r_{M2} - r_{X2})(X_2 - X_{21})]^{1/2}\}$$
$$- F_S I(S) - F_1 I(M_1) - F_2 I(M_2)$$

9 Explorations of higher-order models suggest that this restriction in fact loses no generality.

The first-order conditions are:

$$\frac{\partial \pi}{\partial X_{11}} = -\frac{1}{2}\left[2\alpha_1(r_S - r_{M1})\right]^{1/2}[X_{11}+X_{21}]^{-1/2} + \frac{1}{2}\left[2\alpha_2(r_S - r_{M2})\right]^{1/2}[(X_1 - X_{11})+(X_2 - X_{21})]^{-1/2} \tag{15'}$$

$$- \frac{1}{2}[2\beta_{11}(r_{M1} - r_{X1})]^{1/2}(X_{11})^{-1/2} + \frac{1}{2}[2\beta_{12}(r_{M2} - r_{X1})]^{1/2}(X_1 - X_{11})^{-1/2}$$

$$= 0$$

$$\frac{\partial \pi}{\partial X_{21}} = -\frac{1}{2}\left[2\alpha_1(r_S - r_{M1})\right]^{1/2}[X_{11}+X_{21}]^{-1/2} + \frac{1}{2}\left[2\alpha_2(r_S - r_{M2})\right]^{1/2}[(X_1 - X_{11})+(X_2 - X_{21})]^{-1/2} \tag{15''}$$

$$- \frac{1}{2}[2\beta_{21}(r_{M1} - r_{X2})]^{1/2}(X_{21})^{-1/2} + \frac{1}{2}[2\beta_{22}(r_{M2} - r_{X2})]^{1/2}(X_2 - X_{21})^{-1/2}$$

$$= 0$$

As mentioned earlier, the household is free to use more than one medium of exchange to buy a given good by using one medium on some shopping trips and the other medium on the remaining trips. Given the compound non-linearities in the first-order conditions, one might expect the general solution to have this characteristic. In fact, however, the solution always is in a corner. The household always uses only one medium to buy a given good. This result emerges from the second-order conditions for the problem, which the reader can verify easily:

$$\frac{\partial^2 \pi}{\partial X_{i1} \partial X_{j1}} > 0 \qquad \text{for } i,j = 1,2 \tag{16}$$

$$\det H > 0$$

where H is the Hessian. These conditions show that the interior extremum is a profit minimum so that the maximum occurs at a corner. In such a situation, the choice facing the representative agent is simple enough. The household merely compares the profit functions associated with each possible pattern of usage and picks the pattern with the highest profit. Assuming the agent finds it profitable to use the savings asset as a cash management store of value, there are four possible solutions:

(1) ($S>0$, $X_{11}=X_1$, $X_{21}=X_2$), i.e., hold S, use M_1 to buy X_1 and X_2

(2) ($S>0$, $X_{11}=X_1$, $X_{22}=X_2$), i.e., hold S, use M_1 to buy X_1 and M_2 to buy X_2

(3) ($S>0$, $X_{11}=X_2$, $X_{21}=X_1$), i.e., hold S, use M_2 to buy X_1 and M_1 to buy X_2

(4) ($S>0$, $X_{11}=X_2$, $X_{21}=X_2$), i.e., hold S, use M_2 to buy X_1 and X_2

If the agents choose not to use the savings asset, there are four additional possibilities:

(5) ($S=0$, $X_{11}=X_1$, $X_{21}=X_2$), i.e., do not use S, use M_1 to buy X_1 and X_2

(6) ($S=0$, $X_{11}=X_1$, $X_{22}=X_2$), i.e., do not use S, use M_1 to buy X_1 and M_2 to buy X_2

(7) ($S=0$, $X_{11}=X_2$, $X_{21}=X_1$), i.e., do not use S, use M_2 to buy X_1 and M_1 to buy X_2

(8) ($S=0$, $X_{11}=X_2$, $X_{21}=X_2$), i.e., do not use S, use M_2 to buy X_1 and X_2

For the bulk of our discussion, we will restrict attention to the first four solutions, where the savings asset is employed.

Given the use of savings as a cash management asset, the choice among these four usage patterns of transactions assets is determined by examining the relevant profit functions given in Table 1. In that Table, the subscripts of the profit functions π_{ijk} take the following values:

i = S if the saving asset is used, 0 otherwise

j = 1 or 2 as M_1 or M_2 is used to buy good 1

k = 1 or 2 as M_1 or M_2 is used to buy good 2

The important question is how the household's choice depends on income, expenditure patterns, interest rates, and both transactions and fixed costs.

Usage versus Average Balances. For some of the possible solutions, only one medium of exchange is used, so its average balance in the second is obviously zero. Non-usage necessarily implies zero average balances. The converse, however, need not be true. Average balances can be zero even if an asset is used. Consider an asset, say M_i, that is used. From equation (5'), average balances of M_i will be zero if all the Z_{gi} equal T_i. The household puts some of its income into M_i but then immediately spends it and lets no wealth reside in it for any duration of time. Consequently, average balances are zero even though the medium is used.

If there is no external requirement that the household use an asset, it will not be optimal for the household to use any asset but hold no balances in it, as a comparison of the profit functions shows. If the household did use the asset but chooses to hold no balances, it would pay the transactions and fixed costs associated with obtaining the asset but derive no interest earnings from it.

IIIC. Comparative Static Results

Given that the optimal behavior is characterized by a selection from the first four profit functions of Table 1, this section will evaluate the behavior of these functions. To do so we examine the effect of various shifts in income, expenditure patterns, interest rates and transaction fees on the solution. Of interest is the effect on the average balances of the chosen medium and how such changes might cause the household to select an alternative payments scheme.

1) Level of Income. The household's level of income affects transaction assets in interesting ways. The effect of household income on the usage pattern of the media of exchange M_1 and M_2 depends on the nature of the two goods. We consider two cases: (i) both goods are superior and respond equiproportionally to an increase in income, and (ii) one good, X_2, is inferior.

In case (i), the media of exchange usage pattern is independent of household income. Compare the difference between any two profit functions; for example, consider π_{S11}-π_{S12}:

$$\begin{aligned}\pi_{S,1,1}-\pi_{S,1,2} = & -[2\alpha_1(r_S-r_{M1})(X_1+X_2)]^{1/2} - [2\beta_{11}(r_{M1}-r_{X1})X_1]^{1/2} - [2\beta_{21}(r_{M1}-r_{X2})X_2]^{1/2} \\ & + [2\alpha_2(r_S-r_{M2})(X_1+X_2)]^{1/2} + [2\beta_{12}(r_{M2}-r_{X1})X_1]^{1/2} + [2\beta_{22}(r_{M2}-r_{X2})X_2]^{1/2} \\ & - F_2\end{aligned} \quad (17)$$

All the expenditure terms X_1+X_2, X_1, and X_2 appear under the square root sign. Consequently, an equiproportional change in income (and therefore X_1 and X_2) has an equiproportional effect on π_{S11}-π_{S12}, leaving its sign unchanged and so not affecting the decision of which usage pattern to choose.

In case (ii), the effect of an income change on the usage pattern is ambiguous. It is clear that the relative desirability of the two media will change. However, the exact effect depends on exactly how much the individual expenditures respond and on the values of initial expenditure, interest rates, transaction costs, and fixed costs. Medium of exchange choice may shift, but nothing more specific can be said.

2) Relative Expenditures. Consider now the effect of changing the composition of expenditures while holding income (i.e., total expenditure) constant. The decision on the use of the saving asset is independent of expenditure composition because all profit differences depend only on the sum X_1+X_2 and not on the division of that total between X_1 and X_2. However, the choice of usage pattern for media of exchange does depend on the expenditure composition. To analyze this dependence, suppose we start with a provisional choice of not using M_1 at all and then consider whether it would be better to make a different choice.

We assume for expository ease that $r_{M1}>r_{M2}$. Because the use of S does not depend on expenditure composition, we can simplify the mathematics by restricting attention to the case where the asset S is not used. We begin by making the two profit comparisons π_{012}-π_{022} and π_{021}-π_{022}, which tell us whether we should start using M_1 to buy at least one good:

$$\pi_{0,1,2}-\pi_{0,2,2} = (r_{M1}-r_{M2})\frac{X_1}{2} - \{[2\beta_{11}(r_{M1}-r_{X1})]^{1/2} - [2\beta_{12}(r_{M2}-r_{X1})]^{1/2}\}[X_1]^{1/2} - F_1 \tag{18}$$

$$\pi_{0,2,1}-\pi_{0,2,2} = (r_{M1}-r_{M2})\frac{X_2}{2} - \{[2\beta_{21}(r_{M1}-r_{X2})]^{1/2} - [2\beta_{22}(r_{M2}-r_{X2})]^{1/2}\}[X_2]^{1/2} - F_1 \tag{19}$$

As X_1 rises and X_2 falls, (18) tends to become positive and (19) tends to become negative, and vice versa if X_1 falls and X_2 rises. Thus, the larger the share of total expenditure that a particular good commands, the more likely it is that the household uses the high-interest medium of exchange to buy it. This relation reflects the opportunity cost associated with holding a medium of exchange. The more one spends on a good, the larger the average balances held. Therefore, the larger the interest foregone compared with holding the savings asset, and so the greater the importance of using a medium paying a high rate of interest.

Suppose (18) is positive and (19) is negative. Then the household will make a new provisional usage decision and begin using M_1 to buy X_1 while continuing to use M_2 to buy X_2. Next, the household must decide whether to use M_1 for both goods, which it does comparing π_{011}

and π_{012}:

$$\pi_{0,1,1} - \pi_{0,1,2} = \frac{(r_{M1} - r_{M2})}{2} X_2 - [2\beta_{21}(r_{M1} - r_{X2})X_2]^{1/2} - [2\beta_{22}(r_{M2} - r_{X2})X_2]^{1/2} + F_2 \qquad (20)$$

This expression can be positive even if (19) is negative because of the different fixed cost terms in (19) and (20). The household thus may choose to use only M_1 in making purchases. However, this outcome is less likely when the share of total expenditure commanded by X_2 is smaller because then (20) is less likely to be positive.

We thus have established two results. The household tends to use the higher-interest medium to buy the larger-share good, and a split use of media of exchange is more likely to occur the more unequal are the expenditures X_1 and X_2. Clearly, households with the same income and facing the same interest rates and transactions costs still may choose different usage patterns of media of exchange solely because their tastes in consumption goods differ. Equations (11) through (13) show that average asset holdings also depend on the allocation of income among different goods.

3) Interest Rates. There are three kinds of interest rates in the model: the rate on savings r_S, the rates on media of exchange r_{Mi}, and the rates on commodity inventories r_{Xi}. Their effects on usage patterns are subtle and not easily discerned.

A change in r_S affects the usage pattern for media of exchange. In the most general case, this effect is of ambiguous sign, but if $\alpha_1 \approx \alpha_2 = \alpha$ and $r_{M1} \approx r_{M2} = r_M$, we can obtain an interesting result. Consider profit differences of the form $\pi_{Sii} - \pi_{Sij}$ or $\pi_{Sii} - \pi_{Sji}$ and see how they are affected by an increase in r_S. For example, consider the derivative of $\pi_{S11} - \pi_{S12}$ when $\alpha_1 \approx \alpha_2 = \alpha$ and

$r_{M1} \approx r_{M2} = r_M$:

$$\frac{\partial(\pi_{S,1,1} - \pi_{S,1,2})}{\partial r_S} = \left[\frac{\alpha}{2(r_S - r_M)}\right]^{1/2} [X_1^{1/2} + X_2^{1/2} - (X_1 + X_2)^{1/2}] \tag{21}$$

which is positive because the square root is a concave function. Thus, an increase in r_s tends to lead to use of just one medium of exchange to buy both goods.

A change in one of the interest rates on money, r_{M1} or r_{M2}, has ambiguous effects on the use of both the savings asset S and the media of exchange M_1 and M_2. The effects of r_{Mi} on the use of media of exchange are governed by profit differences, and the derivatives of these differences with respect to r_{Mi} are always of ambiguous sign. For example, the derivative of π_{S11}-π_{S12} is

$$\frac{\partial(\pi_{S,1,1} - \pi_{S,1,2})}{\partial r_S} = -\left[\frac{\alpha_1}{2(r_S - r_{M1})}\right]^{1/2} [X_1^{1/2} - (X_1 + X_2)^{1/2}] - \left[\frac{\beta_{21} X_2}{2(r_{M1} - r_{X2})}\right]^{1/2} \tag{22}$$

The first term is positive and the second negative, leading to an ambiguous sign for the derivative as a whole.

The rates of return on commodities, r_{X1} and r_{X2}, also affect the usage pattern for the media of exchange. For expository ease, suppose $r_{M1} > r_{M2}$ and $r_{X1} = r_{X2}$. Consider the effect of changing good 1's rate of return r_{X1} on the decision of which medium of exchange to use in purchasing good 1. The derivatives of the relevant profit difference with respect to r_{X1} are of ambiguous sign, but if $\beta_{i1} \approx \beta_{i2} = \beta_i$, then we can obtain definite results. For example, suppose $\beta_{11} \approx \beta_{12} = \beta_1$. Consider the derivative of π_{S11}-π_{S21} with respect to r_{X1}:

$$\frac{\partial(\pi_{S,1,1}-\pi_{S,2,1})}{\partial r_{X1}} = \left(\frac{\beta_1 X_1}{2}\right)^{1/2}\left[(r_{M1}-r_{X1})^{-1/2} - (r_{M2}-r_{X1})^{-1/2}\right] \tag{23}$$

which is negative because $(r_{M2}-r_{X1})^{-½}$ exceeds $(r_{M1}-r_{X1})^{-½}$ when $r_{M1}>r_{M2}$. Thus, an increase in the return on good 1 tends to induce the household to use the medium with the lower return to buy good 1. More generally, the household tends to use the lower-return medium to buy the higher return good.

From (11) through (13), household average balances of each asset are positively related to their own rates of return, provided the household already is using the asset and is not induced by a change in an interest rate to abandon it. However, as we have seen, usage patterns can change in response to interest rate changes. For the media of exchange, changes in own rates of return have ambiguous effects on usage. If usage turns out to be negatively related to own rates, then aggregate demand for average balances of those assets might fall in response to an increase in the own rate. This is because the decline in the number of households using the medium may more than offset the higher balances of those that continue to use it.

4) Transactions and Fixed Costs. The effects of all costs on usage are straightforward for the representative agent. An increase in any transaction or fixed cost reduces the use of the associated asset. These results can be derived easily from the relevant profit differences; the derivation is left to the reader. The effects of costs on average balances also are straightforward and follow from (11) through (13). For the household, given use of an asset, the effects of costs show the adjacency property discovered by Santomero (1974). An increase in the cost of transferring funds into a particular medium, α_i, reduces average holdings of S, raises average holdings of M_i, and does not affect holdings of commodity inventories. An increase in the cost of using that medium to purchase goods, β_{ij}, does not affect holdings of S, but reduces average holdings of M_j, and raises average holdings of X_i. Given use, changes in fixed costs do not affect average holdings of any asset.

For aggregate average balances, we have some ambiguities. An increase in the cost of transfer into a particular medium, α_i, unambiguously reduces aggregate demand for S because it reduces household use and holdings. The effect on aggregate demand for M_i is unclear. Use is reduced, but average balances rise for those households that continue to use M_i. In contrast, an increase in the cost of usage, β_{ij} unambiguously reduces aggregate demand for M_j because it reduces household use and holdings, and it also unambiguously raises aggregate demand for X_i. Finally, an increase in any fixed cost unambiguously reduce aggregate demand for the associated money because it reduces household use and does not affect household holdings.

5) Integrated Media. We have explored models in which media of exchange are more fully integrated than in the foregoing model. In particular, we have examined models where (i) the financial transfer costs, α_i, are all the same and one "trip to the bank" allows transferals among all financial assets, (ii) the shopping costs β_{ij} of all media are all the same, and (iii) the α_i and the β_{ij} are all equal, as might be the case with a "smart card." Although some details change, the conclusions are essentially the same as those reported above, so we do not dwell on such alternative models here.

IV. Conclusions from the Representative Agent Model

The foregoing results have several interesting implications. Here we first examine some of the broad outcomes that spring from the analysis. We have seen that the range of asset use decreases as household income falls. We also have seen that the usage patterns of media of exchange differ among households with the same income but different allocations of income among consumption goods. Given that consumption patterns differ across socio-economic groups, it should not be surprising to find different money usage patterns across different types of consumers.

The dependence of media of exchange usage patterns on income levels and consumption patterns also suggests that we should expect different reactions across economic groups to the newly emerging monetary mechanisms. Various households offered a new medium of exchange with its own implied interest rate, transactions costs, and fixed cost will react differently. A cross-section of consumers will be expected to demand different combinations of media of exchange. To put it another way, demand for medium of exchange will vary across households, just as the demand for automobiles does. Thus, there appears to be room for transfer mechanisms with different combinations of fees and interest rates. In transaction schedules, one size does not fit all. Our theory suggests that a whole set of instruments may be viable with different demanders, depending on the features offered with the instrument.

Consider, in particular, the implications of these results on the emerging stored-value card, or smart card technology, and its competitive position *vis-a-vis* the two most common media of exchange. This transfer medium offers some of the features of demand deposits such as ease of transfer and general acceptability. At the same time, it avoids the fixed costs of a checking account while offering no return on average balances. We have seen that households tend to use the higher-interest medium of exchange to buy the good that constitutes the larger share of its income. Given the relative rates of return on stored value cards and checking account balances, the former is unlikely to dislodge the use of checks for purchases. On the other hand, the stored value card represents a credible threat for cash transactions. Here, rates of return are both zero, although it can be argued that cash may have a negative return due to theft. Given their new ease of use, smart cards could also be competitive from a transfer fee perspective. In fact, given the lower cost of transfer into the card, it may dominate cash in the near future. This might explain why promoters have sought the early successes of such spending vehicles at convenience stores, transit stations and the like.

The foregoing results apply to the individual household. Aggregate results often are more difficult to prove. As we have demonstrated, many variables affect usage decisions and average balances in opposite ways. For example, an increase in the cost of obtaining medium i, α_i, reduces the number of households using M_i but raises average balances of M_i for those households that continue to use it. Thus, the change in aggregate holdings of M_i is ambiguous. Therefore, efforts by banks to lower the costs of using a particular form of medium of exchange may well reduce the average holdings of that medium. Also, interest rates generally have ambiguous effects on money holdings. Determining whether an increase in a particular interest rate will raise or lower holdings of a particular form of money requires a case-by-case analysis, and the conclusions can change from one day to the next with changes in other parameters of concern to the household. For example, simply changing the relative amounts of the kinds of goods bought can change the average holdings of various types of money and also the response of those monies to changes in interest rates and transactions costs.

In general, our results are intriguing in both their complexity and sensitivity. Affecting monetary behavior is no simple matter. As providers of different monies move from experimentation to implementation, these results offer a warning. The choice of money or monies to be used for transactions purposes is a complex decision. It does not lend itself to simple extrapolation from consumer surveys, and, in fact, may result in substantially different outcomes than had been presumed. The innovators would do well to proceed slowly. We would not want to “bet the bank” on any one emerging technology.

TABLE 1

Profit Functions

$$\pi_{S,1,1} = r_S \frac{X_1 + X_2}{2} - [2\alpha_1(r_S - r_{M1})(X_1 + X_2)]^{1/2}$$

$$- [2\beta_{11}(r_{M1} - r_{X1})X_1]^{1/2} - [2\beta_{21}(r_{M1} - r_{X2})X_2]^{1/2}$$

$$- F_S - F_1$$

$$\pi_{S,1,2} = r_S \frac{X_1 + X_2}{2} - [2\alpha_1(r_S - r_{M1})X_1]^{1/2} - [2\alpha_2(r_S - r_{M2})X_2]^{1/2}$$

$$- [2\beta_{11}(r_{M1} - r_{X1})X_1]^{1/2} - [2\beta_{22}(r_{M2} - r_{X2})X_2]^{1/2}$$

$$- F_S - F_1 - F_2$$

$$\pi_{S,2,1} = r_S \frac{X_1 + X_2}{2} - [2\alpha_1(r_S - r_{M1})X_2)]^{1/2} - [2\alpha_2(r_S - r_{M2})X_1]^{1/2}$$

$$- [2\beta_{21}(r_{M1} - r_{X2})X_2]^{1/2} - [2\beta_{12}(r_{M2} - r_{X1})X_1]^{1/2}$$

$$- F_S - F_1 - F_2$$

$$\pi_{S,2,2} = r_S \frac{X_1 + X_2}{2} - [2\alpha_2(r_S - r_{M2})(X_1 + X_2)]^{1/2}$$

$$- [2\beta_{12}(r_{M2} - r_{X1})X_1]^{1/2} - [2\beta_{22}(r_{M2} - r_{X2})X_2]^{1/2}$$

$$- F_S - F_2$$

$$\pi_{0,1,1} = r_{M1}\frac{X_1+X_2}{2} - [2\beta_{11}(r_{M1}-r_{X1})X_1]^{1/2} - [2\beta_{21}(r_{M1}-r_{X2})X_2]^{1/2}$$
$$- F_1$$

$$\pi_{0,1,2} = r_{M1}\frac{X_1}{2} + r_{M2}\frac{X_2}{2} - [2\beta_{11}(r_{M1}-r_{X1})X_1]^{1/2} - [2\beta_{22}(r_{M2}-r_{X2})X_2]^{1/2}$$
$$- F_1 - F_2$$

$$\pi_{0,2,1} = r_{M1}\frac{X_2}{2} + r_{M2}\frac{X_1}{2} - [2\beta_{21}(r_{M1}-r_{X2})X_2]^{1/2} - [2\beta_{12}(r_{M2}-r_{X1})X_1]^{1/2}$$
$$- F_1 - F_2$$

$$\pi_{0,2,2} = r_{M2}\frac{X_1+X_2}{2} - [2\beta_{12}(r_{M2}-r_{X1})X_1]^{1/2} - [2\beta_{22}(r_{M2}-r_{X2})X_2]^{1/2}$$
$$- F_2$$

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NOTE: The subscripts in the profit expression π_{ijk} have the following meanings:

i = S if the saving asset is used, 0 otherwise

j = 1 or 2 as M_1 or M_2 is used to buy good 1

k = 1 or 2 as M_1 or M_2 is used to buy good 2

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