# Controlling Information Premia By Repackaging Asset-Backed Securities.

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**Abstract**

Securities created from a base of underlying receivables are sold to 'individual' and 'institutional' traders. While both classes of traders are uninformed about the payoff characteristics of receivables, institutions are more sophisticated than individuals because they are aware of the information-based transactions costs of all currently open markets. Based on their customers' needs, they carry out any given level of transactions by minimizing transaction costs across all open markets. Closed-form solutions for the optimal correlation between the receivables and the number of securities to be issued are provided as functions of (i) The objective of the security designer — which might be to maximize the profits earned off each kind of trader, or to maximize the consumer surplus of hedgers; (ii) The masses of each kind of trader in the market; and (iii) The elasticities of traders hedging demands with respect to transactions costs. All three parameters can be measured empirically by practitioners. It is found that while profits earned off individuals can be optimized by changing the correlation coefficient between sets of receivables backing different securities, profits earned off institutions are immune to changes in the correlation, and can be controlled only by altering the number of securities created.

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# 1 Introduction

From its first beginnings with the development of the residential mortgage-backed securities business in the 1960's, credit securitization now includes products backed by a range of receivables, including commercial mortgages, auto loans and leases, home equity loans, student loans, credit card receivables, ticket sales, insurance premium loans, and an expanding list of other assets. Receivables are packaged, underwritten and sold in the form of asset-backed securities, for many of which market-makers keep open markets. Buyers of these securities use them for hedging various risks. By the end of 1994, in excess of $1.9 trillion securitization securities were outstanding; in excess of $500 billion of securitization transactions were done in 1994 alone. There are many benefits of securitization; most of all as several authors (such as Schwarcz[1993]), and Hill[1996]) point out, is to increase the value of the receivables by selling them in higher quality, and more liquid financial markets. This paper studies the optimal creation of securities from a pool of receivables, when the goal of the issuer is to enhance the liquidity of these securities – *the* important goal of securitization.

How many securities should be created from a given pool of receivables and what should be the ideal statistical distribution between the receivables underlying the securities issued? In my view the answer partly depends on how insiders with superior information about the receivables underlying the security might affect the liquidity[1] of one asset versus multiple securities markets. I follow Bagehot's[1971] intuition that market makers compensate themselves for bad trades due to the adverse selection of insiders by making markets less liquid. This could shrink the base of customers trading in the markets and lower the profits from the sale. How illiquid these markets must be depends on the information content of the different securities issued, the composition of uninformed traders (individuals vs. institutions) and their understanding of linkages among the markets created.

In financial markets, securities backed by seemingly identical pools of receivables trade at different prices, reflecting the information content of receivables underlying the pools. Becketti and Morris[1991] and Stanton[1994] point out that much of the market commentary on mortgage-backed securities focuses on identifying so-called *fast pay* and *slow pay* pools i.e. pools that consistently pay out faster or slower, respectively, than apparently comparable pools. Stanton studies the pre-payment behavior of five 12 % GNMA pools between January 1983 and December 1989. He finds that the proportion of principal remaining relative to the original principal varies between 10 and 35 percent for the five pools. When the market lacks information on the pre-payment characteristics of home-owners, pools are priced 'generically', that is all comparable GNMA's have roughly the same price at any point in time. In this situation, an investor who can identify fast and slow pay pools can profit from the market's inefficiency.

[1] Ahimud and Mendelson[1986] provide an example of the importance of liquidity in total asset return. Consider a security whose holding period is two years, which is the historical average holding period of NYSE stocks. A trading cost of $.04 in a $1 stock using an 8 % discount rate implies a $.04 + \frac{.04}{1.08^2} + \frac{.04}{1.08^4} + ...$ a 28 % reduction in the net potential market value of the asset. Cutting trading costs to 2 % would raise the market value by about 20 %.

For most receivables securitized, it is possible to vary the correlation of different securities issued, by the separation of micro-characteristics of these receivables. In the case of home mortgages, the prepayment behavior of home buyers is to a large part determined by the path of interest rates and other macroeconomic factors; but it also depends on other characteristics of the buyers like the number of children in the household and duration of marriage and job. Advances in computer and communications technologies have made possible the collection and dissemination of credit information of buyers. This role is often played by mortgage bankers who are often also involved in the underwriting and repackaging of pass-throughs. An agency with detailed information of buyer characteristics can create multiple securities by dividing the pool of buyers into different classes. The correlation between the securities created would depend on how the classes of buyers are chosen.

The optimal security design in the model depends on the nature of the client base the securities are sold to. Securities are sold to two major classes of buyers: individuals and institutions. Both sorts of traders can use the asset-backed securities as part of an overall strategy to hedge endowment risks. We assume that when only one asset is issued, the two sets of traders behave identically. However, when multiple securities are issued, each with the same correlation with the trader's endowment risks — hence offering the trader the same hedging possibilities, apart from differences in transactions costs — we shall assume that individuals use only one of the security markets available. They reduce their use of this security as the transactions costs of participating in this security market increase. Institutional holders will potentially use all available securities, optimizing on the transactions costs across the different markets.

To motivate the assumption regarding the individuals owning one asset-backed security from the set that is available, we use some data from the 1992 *Survey of Consumer Finances*.[2] Of all respondents that owned any stock, 45 percent said they owned only one stock. Of these, roughly half said they owned stock in the company they worked in. The other half owned only one stock, even if they were not working for the company that issued that stock. Therefore, there is some evidence to show that families or 'individuals' would choose only one security from the entire set available. This is just one puzzling aspect of individual behavior in securities markets; other anomalies and a review on the literature are available in Bertaut [1996].

Institutional traders capture the behavior of 'large' institutional traders like insurance companies, investment banks and mutual funds, who buy and sell asset-backed securities to match the hedging needs of their customers. The customers need to trade in asset-backed securities to hedge endowment risks, similar to those described for noise traders. Presented with erratic liquidity needs of their customers, institutional traders optimally choose a portfolio from all open security markets so as to

[2]The Survey of Consumer Finances is a triennial survey sponsored by the Federal Reserve with the cooperation of the Department of Treasury. It is designed to provide detailed information on U.S. families' balance sheets and their use of financial services, as well as on their pension rights, labor force participation, and demographic characteristics at the time of the interview. The 1992 survey was conducted by the National Opinion Research Center at the University of Chicago, in which 3,906 families were interviewed. Data on the 1995 survey are not yet publicly available.

minimize transactions costs. For example, many mutual fund companies offer their customers a GNMA fund. The fund manager chooses the portfolio of GNMA securities on behalf of the customers. Further they understand linkages between markets, in the sense of the impacts of order-flow on one market on the prices in other markets.

The repackager must use different strategies to optimize profits earned from individuals and institutions. Because individuals ignore inter-market linkages, higher profits can be made off them by changing the per-security information content while keeping total information constant. This is done by repackaging the receivables and changing the correlation between the securities information. It is shown that institutional traders costs are immune to such changes in security correlation. The costs of institutional traders depend on the number of securities with which they can form portfolios. With a larger number of securities to choose from, these traders buy and sell smaller quantities, and incur smaller transactions costs while satisfying their hedging needs. Solutions for the optimal number of securities to be issued and the correlation between the receivables underlying these securities are provided as functions of (i) The objective of the security designer; which might be to maximize the profits earned off each kind of trader, or to maximize the consumer surplus of hedgers;[3] (ii) The masses of each kind of trader in the market and (iii) The elasticity of increased hedging volume with respect to transactions costs. All the parameters of the problem can be measured empirically by practitioners.

There has been substantial work done on the role of incomplete markets and the hedging opportunities which are created when existing assets are split up and sold. Allen and Gale[1989] characterize optimal securities which are used for spanning otherwise unhedged risks. Their conclusion is that 'extremal' securities are optimal. They are extremal in the sense that in every state payoffs from the original assets are allocated to the security held by people that value it most. The spanning framework is a very useful one for modeling the splitting up and customizing of securities for specific needs. Investment banks on Wall Street explicitly create securities for very specific needs of large customers like pension funds who, due the uniformity of their clients, might have a very idiosyncratic and large exposure to certain risks. However, the spanning properties of securities must often be sacrificed for liquidity reasons; which may be heavily impeded by the impact of asymmetric information between buyers and sellers. In this paper, I shall assume that securities the receivables underlying the asset-backed securities being marketed have some correlation with the endowments of hedgers. Each of the securities created, can be potentially used to hedge endowment risks, but the sensitivity of the security return to changes in receivables varies across securities, depending on its' information content. The goal here is to design the securities to control the transactions costs that arise as market-makers make up their losses to insiders by charging transaction fees on every order.

[3]Profit-maximization may not be the only objective of the Federal Home Loan Mortgage Corporation. It may be for U.S. Air and Chrysler which have sold several pools backed by ticket-sales receivables and automobile loans and leases respectively.

Duffie and Jackson[1989] and Cuny[1993] study the design of futures contracts to maximize the profits of futures exchanges. Duffie and Jackson maximize $T$ times volume, where $T$ is an exogenous, technologically determined transaction cost per contract. In this paper the level of transactions costs is different for each security created, depending on its information content. In Cuny's paper, the exchange charges investors an endogenously determined one time entry fee for each contract. However there are no per unit costs of transactions thereafter. The assumptions of these papers are better suited for futures markets where informational asymmetries are less of an issue than the asset-backed security market which we study here.

Duffie and DeMarzo[1993] show that a model of security design based on asymmetric information and insider-trade implies a convex objective function (and hence option-like) for the issuer. Based on his superior information relative to the market, the insider has the option to sell the stock to the market or hold on to it, the familiar *lemons phenomenon*. The insider signals his information to outsiders by retaining different amounts of the security. Splitting an asset and giving the insider options on the parts increases the value for him. A survey of other papers on security design within an asymmetric information framework is in Duffie and Rahi[1995]. As in these papers, my model will have a similar options mechanism to create value. However, it is assumed that the fraction of each pool of receivables retained by the insider is not observed by the market, ruling out signaling possibilities by the issuer. For example, it is well known that the three Government Sponsored Entities buying and selling mortgage backed securities, hold a substantial fraction of mortgages that they purchase. While the value of their aggregate holdings are public information, the composition of their portfolio and changes in holdings of different pools are confidential. Additionally, the problem is studied with an endogenous determination of the volume and the composition of the uninformed customer base.

In a similar setting to the one here, Subrahmanyam[1991] provides conditions under which different stocks should be packaged together and sold as an index to lower transactions costs for liquidity traders. This paper provides results for further gains from repackaging and splitting. Boot and Thakor[1993] provide conditions under which splitting assets is optimal to provide incentives for information acquisition for some traders. In this paper, there is no information acquisition, yet splitting is able to lower transactions costs.

The layout of the paper is as follows: In Section 2, I present the major assumptions and the structure of the model. Existence and characterization of equilibrium is studied in Section 3. In Sections 4 and 5, securities are designed to control profits earned from individuals and institutions respectively. The analysis is extended to markets where both sorts of buyers are present, in Section 6. I conclude in Section 7.

## 2 The Model and Discussion of its Components.

I consider an economy with three stylized periods, indexed by the variable $t$. Agents are perfectly patient, so there is no time discount. At $t = 0$, $N$ risky securities are traded. The securities will split up at $t = 1$ the payoff on a risky asset whose payoff is comprised of the sum of returns from a pool of receivables. The joint distribution of the sum of the receivables in the pool is given by:

$$\tilde{S} = 1 + \tilde{v} \tag{1}$$

where $\tilde{v}$ is a Normally distributed random variable with a mean of zero and variance $\sigma_v^2$.[4] The asset is to be sold either as specified, or repackaged into $N$ securities $\tilde{S}_i$, where

$$\tilde{S}_i = \frac{1}{N} + \tilde{v}_i \tag{2}$$

We will require that

$$\sum_{i=1}^{N} \tilde{S}_i = \tilde{S}$$

I shall assume that $\tilde{v}_i$ have mean 0 and variance $\sigma_i^2$ respectively, and for technical convenience, the distributions are Normal. The security structure is summarized in the variance-covariance matrix of signals $\Sigma_v$, which is constrained to satisfy

$$\mathbf{1}' \cdot \Sigma_v \cdot \mathbf{1} = \sigma_v^2 \tag{3}$$

At $t = 1$, settlement of claims on all traded securities are made. Traders, then use their proceeds to purchase 'durable' assets, that provide returns to them at $t = 2$ (this shall be discussed below in the description of the traders).

It is *assumed* that the security designer can divide up the receivables into parts with *any* desired covariance matrix $\Sigma_v$. This may or may not be possible given the correlations among the receivables. Generally, the greater the heterogeneity among the receivables, the larger the set of security correlations (designs) the designer can choose from. The optimal security designs in this paper, will have to be constrained by the available correlation set. Typically, the available set of correlations is a connected set (in the case of two securities, the available set of correlations is an interval) and the objective functions of agents (to be described below) are continuous functions of the correlation parameters; the constrained solution will be the point in the available set closest in distance to the unconstrained choice. The example below, illustrates the possibilities.

[4]It is well known that in Kyle-style insider trading models, insider profits do not depend on the mean of the insider's information. Only unexpected components of information are of value. To make the notation simple, I have set the mean of each security to $\frac{1}{N}$ and separated out the unexpected component.

**Example 1** Receivables are one of two types; receivables in each set are perfectly positively correlated with each other and have a correlation of $\rho$ with each receivable in the other set. Let the sum of receivables in each set be $Y_1$ and $Y_2$ respectively, each distributed Normally with mean 1 and variance 1. Consider the formation of two pools: $X_1 = \delta \cdot Y_1 + \beta \cdot Y_2$ and $X_2 = (1-\delta) \cdot Y_1 + (1-\beta) \cdot Y_2$, where $\delta, \beta \in [0,1]$. Then any correlation coefficient $\rho_{X_1 X_2}$ in the interval $[\rho, 1]$ can be created by choosing appropriate $\delta, \beta \in [0,1]$. It's easily checked that

$$\rho_{X_1 X_2} = \frac{(1-\beta)\ \beta + (1-\delta)\ \delta + (\beta\ (1-\delta) + (1-\beta)\ \delta)\ \rho}{\left((1-\beta)^2 + (1-\delta)^2 + 2\ (1-\beta)\ (1-\delta)\ \rho\right)^{0.5}\ (\beta^2 + \delta^2 + 2\,\beta\,\delta\,\rho)^{0.5}}.$$

In particular, with $\delta = \beta$, $\rho_{X_1 X_2} = 1$, and with $\delta = 1$, $\beta = 0$, $\rho_{X_1 X_2} = \rho$. Because the right-hand side of the above equation is continuous in $\delta$ and $\beta$, it is evident that all values between $\rho$ and 1, can be obtained by appropriately adjusting the weights.

The model is based on the single-period insider-trading model of Kyle[1985] and its extension to the multi-asset case by Caballe and Krishnan[1994]. There are five types of agents: the original issuer / security designer, a monopolistic insider, a risk-neutral market-maker, a continuum of noise traders and a finite number $N$ of liquidity traders.

**Description of the agents:**

- The joint distribution of the $\tilde{v_i}$ is chosen by the security designer. I shall assume that securities are designed to either maximize or minimize (depending on the exact institutional role of the security designer) the profits from trading earned from different classes of uninformed customers. The trades of different uninformed traders are not observed. However, expected profits from each class can be calculated. The insider will take the security design as given and maximize profits. The sharing of the surplus between the insider and the security designer is achieved, at the time of sale of the relevant data set containing information on the receivables. In the case where the security designer wants to maximize profits also, the price of the information is determined by a bargaining process. Assuming that each player is as patient, and a repeated offers game is played before trading starts, each player gets half the total profits as in Rubinstein[1982]. Therefore, both players want to maximize profits. If the profits of issuing $N$ securities are greater than that of issuing just the original asset, then the original asset is not released into the market. In such a case there shall be no organized trading of this asset. If profits from issuing the original asset are greater, then no other securities are sold.

- The monopolistic insider observes the values $\tilde{v_i}$ before trading starts. The securities are designed by the security designer, before the insider observes $\tilde{v_i}$.[5] The insider submits his orders along

[5]Because the securities are designed before the insider observes the information components $\tilde{v_i}$, the profits of the insider will be the same, even if $\sum_{i=1}^{N} \tilde{S}_i \neq \tilde{S}$ as long as the securities created satisfy $\mathbf{1}' \cdot \Sigma_v \cdot \mathbf{1} = \sigma_v^2$. Becketti and Morris[1991] among others, point out that almost all mortgage pass-throughs are so-called modified pass-throughs. Investors are guaranteed to receive interest and principal payments even if mortgages are in default or delinquent. This is done by smoothing payments across different mortgages. This will be possible as long as the sum of the securities issued have the same distribution as the original pool of assets.

with the noise and liquidity traders. He makes profits on the $N$ securities at the expense of noise and liquidity traders. The security designer will maximize or minimize the costs of different classes of uniformed traders (described below), depending on his institutional role. In the case where the objectives of the insider and the security designer are the same, this may be the same person.

• The market maker observes the order flows in all assets. As in Kyle[1985], the market-maker sets the price of each asset to the conditional expected value of the underlying payoff given each asset. He forms his expectation based on knowing the distribution of noise and liquidity trade, the distribution of the underlying asset and the equilibrium strategy of the insider in each security.

• There are two sorts of uninformed players: a continuum of 'noise' traders with mass $\gamma_z$, representative of the behavior of 'small' traders — or individuals, and, a continuum of 'liquidity' traders with mass $\gamma_y$ — representative of the behavior of 'larger' traders — or institutions. Each type of trader behaves identically when there is only one marketed security. At $t = 0$, each trader, $j$, has a unit of an endowment security (security E) that can either be consumed or kept for one period. At $t = 1$, security E provides a random return of $1 + \tilde{\epsilon}_j$. $\tilde{\epsilon}_j$ is identically and normally distributed for each individual with a mean of 0 and variance of $\sigma_v^2$ (the same as the variance of return on the asset-backed security). Further, $\rho_{\epsilon,v} = -1$. Therefore, one unit of the (unsplit) asset-backed security provides a perfect hedge to one unit of security E. Each unit of security E has a price of 1 at $t = 0$. At $t = 1$, the trader needs one unit of the single good in the economy to purchase a 'durable', that will provide a sure return of $1 + \tilde{r}_j$ at time $t = 2$. $\tilde{r}_j$, is known to the trader; across traders, $\tilde{r}_j$ is distributed with c.d.f. $F(r)$.[6] The trader is infinitely risk-averse with respect to his consumption at $t = 2$, and is unable to borrow at $t = 1$. Therefore, to ensure the purchase of the durable, the trader must hedge his endowment risk by selling half a unit of the endowment asset and purchasing half a unit of the asset-backed security at time $t = 0$.

The trader must place the order before observing the price of the marketed security; his decision on the quantity to purchase is made assuming that the security will be purchased at the expected price given the market maker's rule. Given the market maker's rule, the expected price of the asset-backed security equals 1. At $t = 1$, he also pays a transaction cost of $\lambda$ per unit of the asset bought/sold. If the agent buys $z$ units of the asset, his final return at $t = 2$ equals

$$w_j = (1 - z)\,(\,1 \,+\, \epsilon_j\,) + z \cdot (\,1 \,+\, \tilde{v}\,) \;+\; \mathbf{I}_{\{\,\mathrm{var}[w_j]=0,\tilde{r}_j - z\cdot\lambda \geq 0\,\}} \cdot (\tilde{r}_j - z \cdot \lambda),$$

where, $\mathbf{I}$ denotes the indicator function. The last term is the additional return realized at $t = 2$ from purchase of the durable, if trader $j$ is to obtain a riskless return of 1 at $t = 1$, and if his benefit $\tilde{r}_j$ exceeds the transactions costs of purchasing the asset-backed security. Because the trader is infinitely risk averse with respect to his consumption at $t = 2$, he only plans to purchase the durable if he can reduce the variance of his return to zero. Therefore, if transactions costs are lower than her benefit, each

[6]The durable might be for example, a physical asset — such as a house, or investment in education for siblings.

trader will hold half a unit of the asset for every unit of her endowment in period 0 to drop the variance of period 1's return to zero. If transactions costs $\lambda$ exceed benefit of the durable $\tilde{r}_j$, trader $j$ will not enter the security market. Trader $j's$ optimal demand for the asset is therefore

$$\tilde{z}_j = \frac{1}{2} \qquad \text{if} \quad \tilde{r}_j > \frac{1}{2}\cdot\lambda \ \text{and,} \tag{4}$$

$$\tilde{z}_j = 0 \qquad \text{if} \quad \tilde{r}_j \leq \frac{1}{2}\cdot\lambda. \tag{5}$$

The number of agents long in security E at time $t = 0$ is a normally distributed random variable $\tilde{e}_0$, with mean 0 and variance $\gamma_y + \gamma_z$.[7] $\tilde{e}_0$ and $\tilde{r}_j$ are independent for all $j$. Therefore, the aggregate volume of the asset backed security is distributed normally with a mean of 0 and a variance $.25 \cdot ((1 - F(.5\cdot\lambda))^2 \cdot (\gamma_y + \gamma_z)$. Throughout this paper we shall assume that the c.d.f., $F(r)$, is continuous. Since $1 - F(r)$ plays the role of the demand curve of traders with respect to transactions costs, this assumption ensures that there is a unique level of transactions costs that will maximize the profits of the security designer. To get closed-form solutions that will further aid our intuition, explicit solutions will be worked out for the case where $F(r)$ has a uniform c.d.f. on $[0, M]$, where $M$ is an upper bound for $r$.

Since the number of traders with endowment risks is random, the aggregate trade in the distribution has a distribution similar to those of noise traders in Kyle[1985]. The costs of trading in a market are measured by the price sensitivity of demand to order flow.[8] It measures the cost of turning around a position. The aggregate demand exhibits the two important properties:

(a) The demand for the security is proportional to its variance. This follows, because the security is used for hedging.

(b) The variance of the demand is inversely related to transactions costs, because use of the asset-backed security declines when transactions costs increase.

When only the single asset is issued, the behavior of the two sets of uninformed players is identical. When multiple securities are issued, we make a difference in behavioral assumptions of the two kinds of players. Their demands are described below. Let $\Gamma_{ii}$ be the increase in price of the ith asset in the market when the order flow in the ith market increases by 1 unit.[9]

**Noise (Individual) Trader's demand for the securities:** Suppose the security designer creates $N$ securities. Each trader randomly decides to hedge his endowment risk in one of the available security markets. Since each of the securities has the same correlation with the endowment, this is not an

[7] The normal distribution is simply for technical convenience; we make the interpretation that a negative number of agents, implies that in aggregate, agents have sold security E short, and received the price of 1 at $t = 0$ for the short-sale. To hedge their risk, agent with short sales of E must sell the asset backed security.

[8] It is useful to recall that in the Kyle model, there is no bid-ask spread – a more standard measure of transactions costs.

[9] The $\Gamma$ matrix is potentially a function of $\omega$, the information of the market-maker. The results in Caballe and Krishnan[1994] imply that the market-makers strategy can be written as a linear rule $P(\tilde{\omega}) = P_0 + \Gamma\cdot\tilde{\omega}$, where $\tilde{\omega}$ is a $N\times 1$ vector of order flows and $\Gamma$ is a $N\times N$ constant matrix of price sensitivities to the order flow.

irrational assumption from a hedgers perspective. However, the trader does not compare transactions costs on open security markets when completing his trade. Each trader, also has an expectation of the information content, $\sigma_i^{2e}$ of the security the traders in. In equilibrium, his expectation equals the action of the security designer, that is $\sigma_i^2 = \sigma_i^{2e}$. Suppose that $\sigma_i^2 = K \cdot \sigma_v^2$. The aggregate demands of these noise traders are given in Lemma 1 below.

**Lemma 1** *Let the variance of the ith security be given by $\sigma_i^2$, and $r_j$ is distributed with c.d.f. $F(r) = \frac{1}{M}$, for $0 \leq r \leq M$. To hedge his endowment risk, each noise trader demands $z_{ji}$ units of the asset-backed security per unit of endowment. Under the assumption that each security has perfectly negative correlation with the endowment, the trader optimally demands $z_{ji}$ which equals:*

$$\begin{aligned} \tilde{z}_{ji} &= \frac{1}{\frac{1}{N} + \sqrt{K}} \qquad \textit{if} \quad \tilde{r}_j > \tilde{z}_{ji} \cdot \Gamma_{ii} \ \textit{and,} \\ \tilde{z}_{ji} &= 0 \qquad \textit{if} \quad \tilde{r}_j \leq \tilde{z}_{ji} \cdot \Gamma_{ii}. \end{aligned} \tag{6}$$

*for each trader $j$, aggregate noise traders demand in the ith security is normally distributed with a mean of zero and variance equal to $\left(\frac{1}{\frac{1}{N}+\sqrt{K}}\right)^2 \cdot (1 - \frac{z_{ji} \cdot \Gamma_{ii}}{M})^2 \cdot \frac{\gamma_z}{N}$.*

**Proof.** In the appendix. Note that when the single asset is issued, then $N = 1$ and $K = 1$, and therefore, the same demand as in (4) and (5) is obtained.

**Liquidity (Institutional) Trader's demand for the securities:** These traders capture the behavior of institutional traders like investment banks and mutual funds, who frequently buy and sell asset-backed securities to match the hedging needs of their customers. The customers need to trade in asset-backed securities to hedge endowment risks, similar to those described for noise traders. Presented with erratic liquidity needs of their customers, institutional traders optimally choose a portfolio from all open security markets so as to minimize transactions costs. Although these traders are not necessarily better informed about the payoffs of assets than noise traders, they have better information on the depth and transactions costs of the various markets that are open. Since the market-maker sets prices conditional on observing the order flow in all securities, they take into account how order-flow in one security affect the prices of all other securities.

Let the liquidity trader will choose a portfolio $\alpha$ for each dollar of endowment risk. Given his endowment risk, he will seek to minimize $C_y(\Gamma, \alpha) = \alpha^{\mathbf{T}} \cdot \Gamma \cdot \alpha$ subject to the constraint that $\alpha^{\mathbf{T}} \cdot \Sigma_v \cdot \alpha = \sigma_v^2$. The constraint implies that the portfolio will provide the hedger with exactly the same hedging possibilities as when there is a single asset, albeit with the transactions costs $C_y(\Gamma, \alpha)$. Therefore, the liquidity trader optimally demands:

$$\tilde{z}_{j,i} = \alpha_i \cdot \frac{1}{2} \qquad \text{if} \quad \tilde{r} > C_y(\Gamma, \alpha) \ \text{and,} \tag{7}$$

$$\tilde{z}_{j,i} = 0 \qquad \text{if} \quad \tilde{r} \leq C_y(\Gamma, \alpha), \tag{8}$$

for $i = 1, \cdots, N$. In the special case when equally informative securities are issued, and $\Gamma_{11} = \Gamma_{22}$, it is evident that $\alpha_i = 1$ is optimal for each liquidity trader.

Let $\Sigma_y$ and $\Sigma_z$ be the variance-covariance matrices of liquidity traders' and noise traders' demands for the two securities respectively. Then, by Lemma 1,

$$\Sigma_z = \begin{bmatrix} z_{ji}^2 \cdot (1 - F(z_{ji} \cdot \Gamma_{11}))^2 \cdot \frac{\gamma_z}{N} & 0 & 0 & 0 & 0 \\ 0 & z_{ji}^2 \cdot (1 - F(z_{ji} \cdot \Gamma_{22}))^2 \cdot \frac{\gamma_z}{N} & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & z_{ji}^2 \cdot (1 - F(z_{ji} \cdot \Gamma_{NN}))^2 \end{bmatrix}. \tag{9}$$

And, by (7) and (8),

$$\Sigma_y = .25 \cdot \begin{bmatrix} \alpha_1^2 & \alpha_1\alpha_2 & \alpha_1\alpha_3 & \cdots \alpha_1\alpha_N \\ \alpha_2\alpha_1 & \alpha_2^2 & \alpha_2\alpha_3 & \cdots \alpha_2\alpha_N \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_N\alpha_1 & \alpha_N\alpha_2 & \alpha_N\alpha_3 & \cdots \alpha_N^2 \end{bmatrix} \cdot (1 - F(C_y(\Gamma))^2 \cdot \gamma_y \tag{10}$$

Let $\Sigma = \Sigma_y + \Sigma_z$. I shall henceforth call (9) and (10) the "volume equations", because second moments of trade are a measure of the round-trip transactions of noise and liquidity trade in the $N$ securities.

Summarizing, an explicit hedging objective has been made for each trader and hedging volumes have been derived. It is assumed that the traders are not perfectly rational; traders anticipate completing their transactions at the expected price of the security, given the market-maker's pricing rule. It is not possible to have perfectly rational agents and yet obtain endogenously derived transactions costs due to insider trading. This is due to a well known 'No-Trade' theorem (see for example Milgrom and Stokey[1982]) that implies no insider trade in equilibrium when all agents are perfectly rational. The traders do however correctly anticipate the slope of the market-maker's pricing schedule, which determine the transactions costs, and reduce their hedging needs as transactions costs decline.

# 3 Equilibrium.

Let $\tilde{X}_i$, $\tilde{Y}_i$ and $\tilde{Z}_i$ be the trades of the insider, the liquidity traders and the noise traders in security $i$. The same letters without $i$ subscripts refer to the $N \times 1$ vectors of the same variables, representing the above quantities in each security. Let $\tilde{\omega} = \tilde{X} + \tilde{Y} + \tilde{Z}$ be the total order flow vector for the $N$ securities. Profits of the informed trader in security $i$ are given by $\pi_i = (S_i - P_i) \cdot X_i$. Let $\tilde{Y}_i = \alpha_i \cdot \tilde{Y}$ be the liquidity traders trade in the ith security.[10] I analyze equilibrium of the following form :

[10] All liquidity traders make the same portfolio choices and hence I avoid subscripts for each trader.

(i) Profit Maximization: The insiders strategy $X(\tilde{v})$ satisfies

$$\sum_{i=1}^{N} E[\pi_i(X(\tilde{v}), \tilde{Y}, \tilde{Z}, P(\tilde{\omega}))] > \sum_{i=1}^{N} E[\pi_i(\hat{X}(\tilde{v}), \tilde{Y}, \tilde{Z}, P(\tilde{\omega}))] \tag{11}$$

for any other strategy $\hat{X}(\tilde{v})$

(ii) Semi-Strong Market Efficiency: The pricing rule $P(\omega)$ satisfies

$$P(\omega) = E[\tilde{S}|\tilde{\omega}] \tag{12}$$

(iii) Optimizing Portfolio Choices: Noise traders buy securities to completely hedge their endowment risks as given by Lemma 1. The portfolio choice of liquidity traders are given by (7) and (8). $\alpha$ minimizes minimize $\alpha \cdot \Gamma(\tilde{\omega}) \cdot \alpha^T$ subject to the constraint that $\alpha^{\mathbf{T}} \cdot \Sigma_v \cdot \alpha = \sigma_v^2$. Traders have expectations about the information content of the matrix $\Sigma_v^e$. In equilibrium these expectations are correct.

(iv) Optimization of Informed Traders Costs: The security designer maximizes

$$\theta \cdot \Gamma \cdot \Sigma_z + \nu \cdot \Gamma \cdot \Sigma_y$$

where $\theta$ and $\nu$ are both in $\{-1, 1\}$. The first term in the summation are the equilibrium costs of noise traders ('Individuals') and the second term are the costs of liquidity traders ('Institutional Traders'). The weights given to the two classes $\theta$ and $\nu$ vary with the role of the security designer. A negative weight implies that the security designer wants to minimize the costs of that class of uninformed agents.

The equilibrium notion is not quite a game theoretic one because market-makers and noise traders do not explicitly maximize any particular objective. Typically the market-efficiency condition which implies zero expected profits has been justified by invoking Bertrand competition among risk-neutral market makers. Dennert[1989] has shown that the condition obtains if it is assumed that each market-maker can observe the aggregate order flow as opposed to only the order flow directed to him, and that ties are broken by equally splitting the aggregate order flow.

I now provide an explicit characterization of a linear equilibrium. The characterization is an extension of Proposition 3.1 in Caballe and Krishnan[1994]. The existence and characterization has been independently proved by Bhasin[1993].

**Result 1** *There always exists an equilibrium defined as follows. The price function is*

$$P(\tilde{\omega}) = \frac{1}{2} + \Gamma \cdot \tilde{\omega}$$

*where*

$$\Gamma = \frac{1}{2} \cdot \Sigma^{-\frac{1}{2}} \cdot M^{\frac{1}{2}} \cdot \Sigma^{-\frac{1}{2}}$$

*and*

$$M = \Sigma^{\frac{1}{2}} \cdot \Sigma_v \cdot \Sigma^{\frac{1}{2}}$$

*The strategy of the insider is*

$$X(\tilde{v}) = \Theta \cdot \tilde{v}$$

*where* $\Theta = \Gamma^{-1}$.

*This is the unique linear equilibrium.*

**Comment 1**

The unique positive definite square root of the symmetric positive definite matrix $M$ is given by:

$$M^{\frac{1}{2}} = E \cdot \Lambda^{\frac{1}{2}} \cdot E^T$$

where, $\Lambda^{\frac{1}{2}}$ is a diagonal matrix with the positive square roots of the eigenvalues of $M$ along the diagonal, and the corresponding orthogonal eigenvectors are the columns of the matrix $E$. Like the square-root of a number, $M^{\frac{1}{2}} \cdot M^{\frac{1}{2}} = M$.

**Comment 2**

The proof of existence of equilibrium is similar to that of the single period equilibrium in Kyle[1985]. Caballe and Krishnan[1994] proved existence for the multi-asset case. Bhushan[1988] showed that having liquidity traders simply requires an adjustment to the $\Sigma$ matrix to account for the covariance in order flows which liquidity trade induces. The extension I provide here is for the case where the variance of noise and liquidity trade may depend on the price-sensitivity parameters set by the market-maker. To illustrate the point in as simple a framework as possible, I show the necessary steps in the single asset case with only noise traders.

The insiders profits are written as $E\left[[\tilde{S} - P(\tilde{\omega})]\tilde{X}|\tilde{X}(\tilde{v})\right]$, where $\tilde{\omega} = \tilde{X} + \tilde{Y} + \tilde{Z}$. Suppose the insider conjectures that the market makers' rule is linear in the order flow $P(\tilde{\omega}) = 1 + \lambda\omega$. As in Kyle, profits are quadratic in $\tilde{X}$; hence the insiders' strategy is linear in his signal and can be written in the form $X = \beta\tilde{v}$ where $\beta = \frac{1}{2\lambda}$. The market maker sets $P(\tilde{\omega}) = E[\tilde{S}|\tilde{\omega}]$. By the projection theorem and normality of order flow and noise trade, $P(\tilde{\omega}) = 1 + \lambda\omega$, where $\lambda$ satisfies

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \gamma_z\,(1 - F(\lambda))^2} \tag{13}$$

This verifies the insiders' conjecture that the market maker follows a linear strategy. Equilibrium will therefore exist as long as there is a real finite positive solution to (13). This will be true in general if the c.d.f. is continuous. Equilibrium may exist for some parameter values even if there are jumps. In this paper, for convenience, I will choose a uniformly distributed c.d.f.

**Comment 3**

In equilibrium the sum of expected profits of the insider in the two security markets can be written as $\mathrm{Tr}(\Gamma \cdot \Sigma_y) + \mathrm{Tr}(\Gamma \cdot \Sigma_z)$, where the first component is the cost incurred by liquidity traders and the second by noise traders. As is evident from the volume equation (9), the costs to noise traders in each market depend only on own price-sensitivities to order flow, $\Gamma_{ii}$. The cost to liquidity traders as is evident from (10) in addition to those, depends on cross price-sensitivities, $\Gamma_{ij}$. $\Gamma_{ij}$ represents a change in the price of the ith security due to a unit change in order-flow in the jth security market. Therefore, $\alpha \cdot \Gamma(\tilde{\omega}) \cdot \alpha^T$ measures the change in the total costs of hedging, including in addition to the own-price effects on trading, the effects trades in security $i$ have on prices of all other securities $j \neq i$ (measured by the off-diagonal elements $\Gamma_{ij}$).

**Comment 4**

The results in Kyle[1985] can be extended to show that the market-maker's rule can be written as $P(\omega_1) = P_0 + \lambda \cdot \omega_1$, where $\lambda$ is the solution to the equation

$$\lambda = \frac{1}{2}\left(\frac{\sigma_v^2}{\gamma_y \sigma_y^2(\lambda) + \gamma_z \cdot (\,1 - F(\lambda)\,)^2}\right)^{\frac{1}{2}} \tag{14}$$

and expected profits can be written as $\lambda \cdot (\,\gamma_z + \gamma_y\,)(\,1 - F(\lambda)\,)$. The single asset profits put a lower bound on the profits which the security designer can ensure for the insider with appropriate security design. Note, that with a single security, there is no distinction between noise and liquidity traders. The noise traders in Kyle[1985] would have $1 - F(\lambda) \equiv 1$. In this case, $\lambda = .5\left(\frac{\sigma_v^2}{\gamma_y + \gamma_z}\right)^{\frac{1}{2}}$ and expected profits of the insider equal $.5\left(\sigma_v^2(\gamma_y + \gamma_z)\right)^{\frac{1}{2}}$.

**Comment 5**

As noted earlier, the Kyle[1985] model does not yield a bid-ask spread, a more standard measure of transactions costs relative to price-elasticity. Krishnan[1992] has provided an interesting equivalence between the Kyle and the Glosten and Milgrom[1985] model. The latter model has a risk-neutral market maker who sequentially services the trades of customers. Some of the customers might be informed about security payoffs while others are like noise traders in Kyle's model. The equilibrium strategy for the market-maker is to charge a bid-ask spread which compensates him in expected terms for each trade. In a binary version of Kyle's model there is an equivalence in extensive forms for the two games. The slope (price-elasticity) set by the market-maker in Kyle's model is identical to the bid-ask spread in the Glosten and Milgrom model.

## 4 Sale to Individuals.

In this section, I study optimal security design when there are only individuals in the market. It is shown that with $N \geq 2$ securities, globally maximum profits can be attained – by adjusting the correlation of receivables backing the different securities. If securities are designed to maximize the consumer surplus of these traders (recall that these traders have a downward sloping hedging schedule) then the surplus continues to increase as a larger number of perfectly positively correlated securities are created. The globally maximum surplus is reached in the limit, as the number of securities issued approaches infinity.

**Result 2** *Let $1 - F(r) = (1 - \frac{r}{M})$, for $0 \leq r \leq M$, and $\gamma_y = 0$. Then,*

*(i) Informationally unconstrained profits of the insider do not depend on the number of securities issued.*

*(ii) When the single asset is issued, the insiders profits will be generically lower than informationally unconstrained profits.*

*(iii) For any given number $N \geq 2$, the security designer can ensure the informationally unconstrained profits by issuing $N$ equally informative securities, and adjusting the correlation between their payoffs to satisfy:*

$$\left[1 + \frac{(N-1) \cdot (1-\rho)^{.5}}{(1+(N-1) \cdot \rho)^{.5}}\right] = \frac{M}{8} \cdot \frac{\gamma_z^{.5}}{\sigma_v} \cdot N. \tag{15}$$

**Proof.** In the appendix.

Note that the left-hand side in (iii) is a monotonic function in $\rho$ for every $N$, so a unique solution exists for a given set of parameters on the right-hand side.

The main ideas underlying the proof are quite simple; the security designer is to be viewed as a price-setting monopolist facing a downward sloping hedging schedule with respect to transactions costs. Because each noise trader only trades in one market, his cost only depend only on $\Gamma_{ii}$, the own-price elasticity of order flow in security i. To attain informationally unconstrained profits the designer should set transactions costs of a trader in market $i$, — $z_{ji} \cdot \Gamma_{ii}$ in the model — to $\frac{M}{3}$. Given $z_{ji}$ in Lemma 1, one can back out the required depth of the market $\Gamma_{ii}$. The information content of security $i$, $\sigma_i^2 = \frac{1}{N+N\,(N-1)\,\rho}$, which is increasing in $\rho$ for any $N$. Therefore, increasing $\rho$ increases $\Gamma_{ii}$. The demand in Lemma 1 implies that increasing $\rho$ lowers the quantity traded, however the pricing rule of the market maker implies that $z_{ji} \cdot \Gamma_{ii}$ is increasing in $\rho$. Therefore, the transactions costs of noise traders can be *controlled* by changing $\rho$, for any $N \geq 2$. The optimal correlation calculated in Result 2 ensures the optimal level of transactions costs in each market. With the single asset, the security designer does not have this degree of freedom and hence cannot ensure informationally unconstrained profits.

The optimum can be attained for any $N$, although from (2) it is evident that for a larger $N$, a smaller $|\rho|$ is needed. This is because the slope of the left-hand side with respect to $\rho$, is increasing in

$N$. The intuition behind this is as follows: with a larger number of securities, each market is thinner, and this tends to increase $\Gamma_{ii}$. To achieve the optimum level of transactions costs, then the information content must be lowered, by decreasing $\rho$ as explained in the previous paragraph.

The consumer surplus of noise traders is maximized by driving the transactions costs of these traders down to zero. This ensures the largest probability of the hedger being able to adopt his strategy. In the following result, I show that if the original asset is split up repeatedly into parts with uncorrelated receivables, then the aggregate costs of noise traders remain constant. However, if the securities have receivables which are perfectly positively correlated, then the aggregate costs of liquidity traders decrease to zero as the number of securities created increases.

**Result 3** *Let $1-F(r) = (1-\frac{r}{M})$, for $0 \leq r \leq M$, and suppose $\gamma_y = 0$. Suppose the security designer issues $N$ equally informative securities with correlation $\rho$ between them. Then*

*(i) If $\rho = 0$, then the hedging cost in each security market, and hence the consumer surplus of traders, does not depend on the $N$.*

*(ii) If $\rho = 1$, then the transactions costs in each market tend to zero, and $N$ increases to infinity. Thus surplus is maximized in the limit by issuing an arbitrary large number of equally informative and perfectly correlated securities.*

**Proof.** In the appendix.

(i) One provides an important consistency check on the model. With uncorrelated assets, the variance per security $\sigma_i^2 = \frac{\sigma_v^2}{N}$, an Nth of the variance of the single asset. Also the mass of agents per security equals $\frac{\gamma_z}{N}$, an Nth of the total mass. Therefore, the signal-to-noise ratio in each market, measured by the information content of the security divided by the mass of noise trade, does not depend on $N$. Consequently, the market-maker charges the same transactions costs per unit of hedging need irrespective of $N$. When perfectly positively correlated securities are issued, the information content per security equals $\sigma_i^2 = \frac{\sigma_v^2}{N+2\,N\,(N-1)}$. This clearly decreases at a rate faster than $N$, while the mass of noise trade declines at a rate of $N$. Therefore, as $N$ increases, the signal-to-noise ratio in each market declines and the transactions costs decline to zero as $N$ increases to infinity. This explains (ii). If there is a finite cost to issuing a new security then clearly, this will put a limit on the number of securities issued.

# 5 Sale to Institutions.

Institutional traders, as defined in Section 2, observe the transactions costs, and optimize in all open markets. They counter any change in security design, by appropriately adjusting their portfolios, making it extremely difficult to earn profits from. It is shown that the only way to increase profits from these traders, is to provide them with lower per-unit transactions and hence induce a larger volume of trade.

The first result is similar to those in Admati and Pfleiderer[1988] and Bhushan[1991]. It shows that if there is no noise trade then liquidity traders find it beneficial to trade in a single security. Each trader benefits from trading solely in the thickest market available and hence as a group the traders coordinate on one market. Some amount of noise trade in each security market would provide enough thickness for liquidity traders to enter these markets.

**Result 4** *(The Need for Some Noise Trade). Suppose $\gamma_z = 0$, i.e. the volume of noise trade is zero. Then liquidity traders trade in a single security. There is a breakdown of the market in other securities.*

**Proof.** If $\gamma_z = 0$, $\Sigma = \Sigma_y$. An examination of (10), reveals that $\Sigma$ is singular for all $\alpha$, for all $N > 1$. Therefore $\Gamma$ does not exist. For $\alpha_i = 1$, for some $i$, and $\alpha_j = 0$ for $i \neq j$, the usual one security equilibrium as in Kyle[1985] exists. ■

Intuitively, when there is no noise trade, the order flows in different security markets are perfectly correlated, irrespective of the portfolio composition . Each security therefore reveals the same information. Coordination into one security market provides maximum thickness — and hence the lowest transaction costs.

While closed-form solutions are hard to find for the case of unequally informative securities, we show below that (almost) optimum profits can be earned off these traders by issuing the appropriate number of equally informative securities. Since the number of securities must be integer valued, the result provides only a level of profits close to the optimum. For any given number of securities, deviating from the equally-informative strategy will lower profits slightly as liquidity traders will meet their hedging needs in the markets that are cheaper to transact in. Therefore with small perturbations with the number of securities provided in Result 5, the maximum can be achieved. Splitting assets to maximize profits off liquidity traders is profitable because it changes the ratio of noise to liquidity traders in each security market. As the number of markets increases, the mass of noise traders in each market thins out. This changes the depth of trading in the securities and affects the per-unit cost of liquidity traders. The following result characterizes the relationship between the parameters of the problem and the optimal number of securities to be issued.

**Result 5** *(Securities to Maximize the Profits earned of Liquidity Traders ). Let $1 - F(r) = (1 - \frac{r}{M})$, for $0 \leq r \leq M$, and the volume of noise trade be $\gamma_z = 0$, given exogenously. Let $N$ equally informative securities be designed to maximize profits solely off Liquidity Traders. Then,*

*(i) In equilibrium $\sum_{i,j} \Gamma_{ij}$ does not depend on $\rho$.*

*(ii) There exists an equilibrium with an optimal number $N^*$ of identically distributed securities, that if issued, will maximize profits across all security designs. $N^*$, is the integer closest to*

$$N = \sqrt{\frac{\gamma_z \frac{M^2}{81}}{.015\,\sigma_v^2 - \gamma_y \frac{4}{81}}} \tag{16}$$

**Proof.** In the appendix.

The intuition for (i) is as follows: changing $\rho$ affects the correlation of the insider's information, and hence the correlation of his demands for the two securities. The market-maker responds to the change in observed total order-flow, by changing the cross-price sensitivities, $\Gamma_{ij}$. It is shown in the proof that in equilibrium, increasing $\rho$, lowers $\Gamma_{ii}$ and raises $\Gamma_{ij}$ by the *same* amount. Because liquidity traders hold a portfolio with weight $\alpha_i = 1$ in each security, their total costs of hedging per unit of endowment equal $.25 \cdot g_N = .25 \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} \Gamma_{ij}$, which is unaffected by changes in $\rho$. This feature holds true for all continuous c.d.f.'s, $F(r)$.

**Corollary 1** *The optimal number of securities is decreasing in $\sigma_v^2$, and increasing in $M$, $\gamma_y$, and $\gamma_z$.*

**Proof.** The result follows by taking partial derivatives of $N$ with respect to the parameters. ■

The somewhat surprising result is that $N^*$ is decreasing in $\sigma_v^2$, the total information content of the underlying asset. This is because the security designer wants to ensure that per-unit transactions costs of the liquidity trader, $g_N$, equals $\frac{M}{3}$ in equilibrium, which is the level of transactions costs that maximizes profits. Increasing $\sigma_v^2$, increases $g_N$ for a given $N$. To lower this back to $\frac{M}{3}$, the designer lowers the number of securities. It is shown formally in the proof of Result 6, that $g_N$ is increasing in $N$. Intuition for this is provided following the statement of that result. The intuition behind the signs of the other parameters is straightforward. With the increase in the mass of each type of trader, the markets are thicker, resulting in a smaller $\Gamma$ matrix, and a smaller $g_N$. Therefore, the trader increases $N$ to get $g_N$, back up to $\frac{M}{3}$, the optimal choice. With a larger $M$, the optimum $g_N$ itself is higher; a higher $M$ implies that there is a larger probability that hedging benefits exceed any given level of transactions costs, and the designer responds by setting a higher level of transactions costs. Therefore, in equilibrium the designer increases the number of securities.

Securities can also be designed to maximize the consumer surplus of liquidity traders.

**Result 6** *(Securities to Maximize the Consumer Surplus of Liquidity Traders) . Let $1 - F(r) = (1 - \frac{r}{M})$, for $0 \leq r \leq M$, and the volume of noise trade $\gamma_z$ be given exogenously. Then it can be shown that the per-unit costs of liquidity traders are always increasing in the number of securities issued. Therefore to maximize the consumer surplus of these traders, only the original unsplit security should be issued.*

**Proof.** In the appendix.

The intuition behind the result is as follows: noise trades thicken the market and provide a camouflage to both the market-maker and liquidity traders. Increasing the number of securities, thins out the markets (since noise traders each use one securities market) and therefore increases the sensitivities of the market-maker's rule to order-flow, correspondingly increasing the per-unit transactions cost of liquidity traders.

# 6 Sale to Individuals and Institutions.

In Section 4 it is shown that for a given number of securities $N$, profits earned from individual traders could be controlled by repackaging, and hence changing the correlation coefficient between the receivables underlying the securities. In Section 5, it is shown that profits earned from institutional traders are immune to changes in the correlation coefficient, and can be optimized only by changing the number of securities offered. The security designer therefore, has an instrument of control, to maximize profits from each type of trader. Therefore, a combination of strategies used in the previous two sections, will control profits earned from each type.

**Result 7** *(Maximizing Profits off Noise and Liquidity Traders). Let $1 - F(r) = (1 - \frac{r}{M})$, for $0 \leq r \leq M$. Let securities be designed to maximize the sum of the profits made off the two groups of traders. Then, $N^*$ equally informative securities each mutually correlated with the others with correlation coefficient $\rho^*$, will maximize profits across all security designs. $N^*$ is the integer closest to*

$$N = \sqrt{\frac{\gamma_z \cdot \frac{4}{9} \frac{M^2}{81}}{.015\,\sigma_v^2 - .25 \cdot \gamma_y \frac{4}{81}}} \tag{17}$$

*and $\rho^*$ is given by the unique solution to*

$$\frac{(1-\rho)^{.5}}{(1 + (N^* - 1) \cdot \rho)^{.5}} = \frac{\gamma_z^{.5}}{N^{*\,.5}\,(N^* - 1)} \cdot \left( \frac{2}{3} \frac{N^{*\,1.5}}{\sigma_v} M - \frac{1}{\left( \frac{\gamma_z}{N^*} + .25 \cdot N^* \gamma_y \frac{4}{9} \right)^{.5}} \right) \tag{18}$$

**Proof.** In the appendix.

When securities are designed to minimize the consumer surplus of traders, it is again shown that issuing the single unsplit asset-backed security will be optimal.

**Result 8** *(Maximizing The Surplus of Both Noise and Liquidity Traders). Let $1 - F(r) = (1 - \frac{r}{M})$, for $0 \leq r \leq M$. Then the sum of the surplus of noise and liquidity traders will be maximized by issuing the single unsplit asset.*

The result is somewhat surprising given Result 3, where costs of noise traders were lowered by increasing $N$. Intuitively, when both sorts of traders are present, each benefits from the thicker markets created by the other type. Increasing $N$, lowers the volume of noise traders in each market, which due to a higher $\Gamma_{ii}$ leads to a decline in the volume of liquidity traders, which feeds back and lowers the mass of noise traders.

## 7 Conclusion.

Asset securitization has grown rapidly in the last decade to include products backed by a range of receivables by varying agencies. Receivables are packaged, underwritten and sold in the form of asset-backed securities, for many of which market-makers keep open markets. The securities are purchased by individuals, and institutions to hedge various endowments risks. I provide a theory explaining how the information premium earned by the insider with information about the receivables can be controlled by the security designer. Depending on how uninformed agents allocate their resources across markets, and the elasticity of their trading volume, optimal security design differs. It is found that while profits earned off individuals can be optimized by changing the correlation coefficient between sets of receivables backing different securities, profits earned off institutions are immune to changes in the correlation but can be controlled by altering the number of securities created. A combination of the strategies enables the security designer to optimize profits or consumer surplus of each type of trader. Solutions for the optimal correlation between the receivables, and the number of securities to be issued are provided as functions of (i) the objective of the security designer — which might be to maximize the profits earned off each kind of trader, or to maximize the consumer surplus of hedgers; (ii) the masses of each kind of trader in the market; and (iii) the elasticities of traders hedging volumes with respect to transactions costs.

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# 9 Appendix

**Mathematical Fact 1** *Let the information matrix be given by:*

$$\Sigma_v = \begin{bmatrix} \sigma_i^2 & \rho\sigma_i^2 & \cdots & \rho\sigma_i^2 \\ \rho\sigma_i^2 & \sigma_i^2 & \cdots & \sigma_i^2 \\ \rho\sigma_i^2 & \rho\sigma_i^2 & \cdots & \rho\sigma_i^2 \\ \cdots & & & \\ \rho\sigma_i^2 & \rho\sigma_i^2 & \cdots & \sigma_i^2 \end{bmatrix}, \tag{19}$$

*where, $\sigma_i^2$ is constrained by (3) to satisfy: $\sigma_i^2 \cdot (N + N \cdot (N-1)\rho) = \sigma_v^2$. This represents the information matrix $\Sigma_v$, when $N$ equally informative securities are issued. It can be verified that*

$$\Sigma_{v_{ij}}^{.5} = \frac{\sigma_v}{N^{1.5}} \cdot \left[1 - \frac{(1-\rho)^{.5}}{(1+(N-1)\cdot\rho)^{.5}}\right], \qquad \forall i \neq j, \quad \textit{and}, \tag{20}$$

$$\Sigma_{v_{ii}}^{.5} = \frac{\sigma_v}{N^{1.5}} \cdot \left[1 + \frac{(N-1)\cdot(1-\rho)^{.5}}{(1+(N-1)\cdot\rho)^{.5}}\right]. \tag{21}$$

**Proof of Lemma 1.** Suppose the noise trader in the ith security market purchases $z_{ji}$ units of the asset-backed security. The market-maker's rule implies that the expected price of the ith security is $P_i = \frac{1}{N}$, and in equilibrium the trader correctly anticipates the variance of the security, i.e $\sigma_i^{e2} = \sigma_i^2$. His return per unit of endowment equals:

$$w_j = (1 - P_i \cdot z_{ji})\,(1 + \epsilon_j) + z_{ji} \cdot (1 + \tilde{v}_i) + \mathbf{I}_{\{\tilde{r}_j - z_{ji}\cdot\Gamma_{ii} \geq 0\}} \cdot (\tilde{r}_j - z_{ji} \cdot \Gamma_{ii}).$$

Therefore, $E[w_j] = 1 + \mathbf{I}_{\{\tilde{r}_j - \Gamma_{ii} \geq 0\}} \cdot (\tilde{r}_j - \Gamma_{ii})$, and,

$$\text{Var}[w_j] = (1 - \frac{z_{ji}}{N})^2 \cdot \sigma_v^2 + z_{ji}^2 \cdot \sigma_i^2 - 2 \cdot (1 - \frac{z_{ji}}{N}) \cdot z_{ji} \cdot \sigma_i \cdot \sigma_v.$$

Because the trader is infinitely risk averse with respect to his consumption at $t = 2$, he only plans to purchase the durable if he can reduce the variance of his return to zero. Since $\sigma_i^2 = K \cdot \sigma_v^2$, to reduce the variance of the return to zero when $\tilde{r}_j - \Gamma_{ii} \geq 0$, the trader must choose $z_{ji}$ satisfying

$$1 - z_{ji} \cdot (\frac{2}{N} + 2 \cdot \sqrt{K}) + z_{ji}^2 \cdot (\frac{1}{N^2} + K + 2 \cdot \frac{\sqrt{K}}{N}).$$

This quadratic equation has a unique solution, given in the statement of the lemma. Now integrating over the set of traders with independent risks implies that the aggregate uninformed trade is distributed normally with a mean of zero and a variance of $z_{ji}^2 \cdot (1 - F(z_{ji} \cdot \Gamma_{ii})^2)$, which completes the proof. ■

**Proof of Result 2.** Let the security designer issue $N$ securities, Let each noise trader in security market $i$ purchase $z_{ji}$ units of the security he chooses, profits of the insider when $N$ securities are issued are given by:

$$\pi_N^U = \sum_{i=1}^{N} \frac{\gamma_z}{N} \cdot z_{ji}\,\Gamma_{ii} \cdot (1 - \frac{z_{ji}\,\Gamma_{ii}}{M})^2. \tag{22}$$

Suppose, that $z_{ji}\,\Gamma_{ii}$ could be chosen by the security designer, unconstrained by any information criterion. Then, the unique optimal (global) choice would be to choose $z_{ji}\,\Gamma_{ii} = \frac{M}{3}$. Therefore, maximum unconstrained profits in the ith market are: $\pi_i = \frac{4}{27}\,M\,\frac{\gamma_z}{N}$. Since, there is an equal mass $\frac{\gamma_z}{N}$, in each security market, maximum unconstrained profits with $N$ securities are: $\pi_N^U = \frac{4}{27}\,M$, clearly independent of $N$. This proves (i).

For $N = 1$, using (14), the market depth parameter $\lambda$ satisfies: $\lambda \cdot (1 - \frac{\lambda}{M}) = \frac{\sigma_v}{\gamma_z^{.5}}$. The solutions of this equation are:

$$\hat{\lambda} = \frac{M}{2} \cdot (1 \pm \sqrt{1 - 4\frac{\sigma_v}{\gamma^{.5}}\frac{1}{M}}) \tag{23}$$

that are not generically equal to $\frac{M}{3}$, the unique unconstrained optimum. This proves (ii).

I now show that with $N \geq 2$ equally informative securities, the two-security unconstrained maximum can be attained. By Result 1, $\Gamma = .5 \cdot \Sigma^{-\frac{1}{4}} \cdot \Sigma_v^{\frac{1}{2}} \cdot \Sigma^{-\frac{1}{4}}$. The noise matrix $\Sigma$ is given by: $\text{Diag}(\frac{\gamma_z}{N}\,z_{ji}^2\,(1 - \frac{z_{ji}\,\Gamma_{NN}}{M})^2, \cdots, \frac{\gamma_z}{N}\,z_{ji}^2\,(1 - \frac{z_{ji}\,\Gamma_{11}}{M})^2)$, where by Lemma 1, $z_{ji} = \frac{1}{\frac{1}{N} + \sqrt{K}}$, for $i = 1, \cdots N$. The elements of the matrix $\Sigma_v$ are given by (20), and (21). Result 1 therefore implies that in equilibrium,

$$z_{ji} \cdot \Gamma_{ii} \cdot (1 - \frac{z_{ji} \cdot \Gamma_{ii}}{M}) = .5 \cdot \frac{1}{N} \cdot \frac{\sigma_v}{\gamma_z^{.5}} \left[1 \; + \; \frac{(N-1) \cdot (1-\rho)^{.5}}{(1 + (N-1) \cdot \rho)^{.5}}\right], \tag{24}$$

for $i = 1, \cdots, N$. Now, setting $z_{ji} \cdot \Gamma_{ii} = \frac{M}{2}$ into (24), implies that if the security designer issues $N$ equally informative securities with correlation $\rho$ between them satisfying:

$$\frac{4}{9} \cdot M \; = \; .5 \cdot \frac{1}{N} \cdot \frac{\sigma_v}{\gamma_z^{.5}} \left[1 \; + \; \frac{(N-1) \cdot (1-\rho)^{.5}}{(1 + (N-1) \cdot \rho)^{.5}}\right]$$

then, in equilibrium the unconstrained maximum with $N$ securities will be attained. Now simplifying completes the proof of (iii). ∎

**Proof of Result 3.** The demands given in Lemma 1, completely hedge the risk of each trader, at a cost of $z_{ji} \cdot \Gamma_{ii}$. To maximize the total consumer surplus of these traders we need to find the security design that reduces the hedging cost to zero. In equilibrium (24) is satisfied. If $\rho = 0$, then the right-hand side of (24) equals $\frac{\sigma_v}{\gamma_z^{.5}}$, which is independent of $N$. Therefore, the roots of the equation, which determine $z_{ji} \cdot \Gamma_{ii}$ in equilibrium, do not depend on $N$; consequently the transactions costs incurred by traders and their consumer surplus is invariant to $N$. If the designer creates $N$ equally informative and perfectly positively correlated securities, then $z_{ji} \cdot \Gamma_{ii} \cdot (1 - \frac{z_{ji} \cdot \Gamma_{ii}}{M}) = .5 \cdot \frac{\sigma_v}{\gamma_z^{.5}} \cdot \frac{1}{N}$. Now, letting $N$ tend to infinity implies that there are two equilibria; in the first the hedging cost tends to zero and in the second to $M$. In the second equilibrium, volume tends to zero therefore, consumer surplus is minimized. Letting the security designer choose the equilibrium completes the proof of (ii). ■

**Proof of Result 5.** The first part of the proof shows that costs of liquidity traders do not depend on the correlation in the information content of different securities, as long as these securities are equally informative. Let the information matrix be given by (19). By Result 1, $\Gamma_N = .5 \cdot \Sigma_N^{-\frac{1}{4}} \cdot \Sigma_v^{\frac{1}{2}} \cdot \Sigma_N^{-\frac{1}{4}}$. Using the expression for $\Sigma_v^{.5}$ from Mathematical Fact 1 and multiplying through implies that $\Gamma_N$ is of the form:

$$\Gamma_N = .5 \cdot \frac{\sigma_v}{N^{1.5}} \cdot \Sigma_N^{-.25} \cdot \tag{25}$$

$$(E_N + \begin{bmatrix} (N-1)\cdot g(\rho) & -g(\rho) & \cdots & -g(\rho) \\ -g(\rho) & (N-1)\cdot g(\rho) & \cdots & -g(\rho) \\ -g(\rho) & -g(\rho) & \cdots & -g(\rho) \\ \cdots & & & \\ -g(\rho) & -g(\rho) & \cdots & (N-1)\cdot g(\rho) \end{bmatrix}) \cdot \Sigma_N^{-.25}$$

where, $g(\rho) = \frac{(1-\rho)^{.5}}{(1+(N-1)\cdot\rho)^{.5}}$, and $E_N$ is an $N \times N$ matrix of ones. To simplify notation, let $\sum_{i=1}^N \sum_{i=1}^N \Gamma_{N_{ij}} = g_N$. Lets conjecture that each liquidity trader buys a unit of each security per unit of endowment security, i.e. $\alpha_i = \frac{1}{2}$, for $i = 1, \cdots, N$. We shall verify this guess below. Under this assumption, $C_y(\Gamma) = .25 \cdot g_N$, and $\Sigma_y = .25 \cdot E_N \cdot \gamma_y \cdot (1 - \frac{1}{M} g_N)^2$, and $\Sigma_z = I_N \cdot \frac{\gamma_z}{N}$. Therefore,

$$\Sigma_{N_{ii}}^{-.25} = \frac{N-1}{N} \cdot \frac{1}{\left(\frac{\gamma_z}{N}\right)^{.25}} + \frac{1}{N \cdot \left(\frac{\gamma_z}{N} + .25 \cdot N \cdot \gamma_y \cdot (1 - .25 \cdot \frac{1}{M} g_N)^2\right)^{.25}}, \qquad \forall i,$$

$$\Sigma_{N_{ij}}^{-.25} = -\frac{1}{N} \cdot \frac{1}{\left(\frac{\gamma_z}{N}\right)^{.25}} + \frac{1}{N \cdot \left(\frac{\gamma_z}{N} + .25 \cdot N \cdot \gamma_y \cdot (1 - .25 \cdot \frac{1}{M} g_N)^2\right)^{.25}}, \qquad \forall i \neq j.$$

Both terms do not depend explicitly on $\rho$, but only implicitly through $g_N$. Now substituting for $\Sigma_{N_{ii}}$ and $\Sigma_{N_{ij}}$ into (25) implies that,

$$\frac{N^{1.5}}{.5\,\sigma_v} \cdot \Gamma_{ii} = (N-1) \cdot g(\rho) \cdot \frac{1}{\left(\frac{\gamma_z}{N}\right)^{.5}} + \frac{1}{\left(\frac{\gamma_z}{N} + .25 \cdot N \cdot \gamma_y \cdot (1 - .25 \cdot \frac{1}{M} g_N)^2\right)^{.5}} \qquad \forall i, \tag{26}$$

$$\frac{N^{1.5}}{.5\,\sigma_v} \cdot \Gamma_{ij} = -g(\rho) \cdot \frac{1}{\left(\frac{\gamma_z}{N}\right)^{.5}} + \frac{1}{\left(\frac{\gamma_z}{N} + .25 \cdot N \cdot \gamma_y \cdot (1 - .25 \cdot \frac{1}{M} g_N)^2\right)^{.5}} \qquad \forall i \neq j. \tag{27}$$

Because, $\Gamma_{ii} = \Gamma_{jj} \;\; \forall i, j$ and, $\Gamma_{ij} = \Gamma_{kl} \;\; \forall i \neq j$ and $k \neq l$, and therefore the equal portfolio weight in each asset is optimal. Now, summing the elements of (26) and (27), implies that $C_y(\Gamma_N) = .25 \cdot g_N$ is the solution of:

$$.25 \cdot g_N = .125 \cdot \frac{\sigma_v}{N^{1.5}} \cdot \frac{1}{\left(\gamma_z + .25 \cdot N^2 \cdot \gamma_y \cdot (1 - .25 \cdot \frac{1}{M} g_N)^2\right)^{.5}}, \tag{28}$$

which is independent of $\rho$ (terms containing $\rho$ exactly cancel out). This proves (i).

(28) provides the 'information constraint' for the security designer on per-unit transaction costs. Simplifying (28) implies that

$$g_N^2 \;=\; \frac{.016 \cdot \sigma_v^2 \cdot N^2}{\left(\gamma_z \;+\; .25 \cdot N^2 \cdot \gamma_y \cdot (\,1 - .25 \cdot \frac{1}{M}\, g_N)^2\,\right)} \tag{29}$$

Suppose the security designer was to choose the per-unit transaction costs $.25 \cdot \hat{g}$, of liquidity traders to maximize profits off these traders. Given their $\Sigma_y$ matrix, profits would be $\pi_L = .25 \cdot \hat{g} \cdot (1 - .25 \cdot \frac{1}{M} \cdot \hat{g})^2 \cdot \gamma_y$. Given the objective, profits will be maximized by choosing $.25 \cdot \hat{g} = \frac{M}{3}$. These would be the global maximum of profits across all security designs. Maximum profits can be achieved by issuing $N$ uncorrelated, equally informative securities, by setting $.25 \cdot g_N = \frac{M}{3}$, in (29). Substituting $.25 \cdot g_N = \frac{M}{3}$ into (29) then provides the expression in the statement of the result. ■

**Proof of Result 6.** With $N$ equally informative securities, irrespective of the correlation between them, the costs of liquidity traders are given by (29). Simplifying further we can write, $\Omega(g_N^2, N) \;=\; 0$, where

$$\Omega(g_N^2, N) \;=\; g_N^2\, \gamma_z \;+\; .25 \cdot N^2\, \gamma_y\, g_N^2\, (1 - .25 \cdot \frac{g_N}{M})^2 - .25\, \sigma_v^2\, N^2.$$

Now by the Implicit Function Theorem,

$$\frac{\partial g_N}{\partial N} \;=\; -\frac{\frac{\partial \Omega}{\partial N}}{\frac{\partial \Omega}{\partial g_N}} \;=\; \frac{2\, N[\, .25 \cdot \gamma_y\, g_N^2\, (1 - .25 \cdot \frac{g_N}{M})^2 - .25\, \sigma_v^2\,]}{2\, g_N\, \gamma_z \;+\; 2 \cdot .25 \cdot\, N^2\, g_N\, \gamma_y\, (1 - .25 \cdot \frac{g_N}{M})\, (2 - .25 \cdot \frac{g_N}{M})}.$$

Because $g_N \geq 0$, and $(1 - \frac{g_N}{M}) \geq 0$ (since it is a probability), the denominator is positive; therefore, $\text{sign}(\frac{\partial g_N}{\partial N}) \;=\; -\text{sign}(\,\gamma_y\, g_N^2\, (1 - \frac{g_N}{M})^2 - .25\, \sigma_v^2\,)$. However, because (29) holds, this equals $\text{sign}(\,-\frac{g_N^2\, \gamma_z}{N^2}\,)$, which is clearly negative. Therefore, we have shown that $g_N$ is always increasing in $N$, and to minimize the transactions costs of liquidity traders, it is optimal to issue the smallest $N$ possible, that is $N = 1$. Since the lowest transactions costs also induce the highest benefit from hedging, this maximizes consumer surplus. ■

**Proof of Result 7.** The proof dichotomizes into parts similar to those of Results 2 and 5. By Result 1, we can write the equilibrium conditions similar to (26) and (27). To maximize profits off each kind of trader, the security designer must adjust $\rho$ and $N$ to ensure that $z_{ji} \cdot \Gamma_{ii} = \frac{M}{3}$ and $.25 \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} \Gamma_{ij} = \frac{M}{3}$. Equilibrium conditions can now be obtained by using these conditions in (26) and (27), replacing $\gamma_z$ by $\gamma_z \cdot \frac{4}{9}$. Summing the depths implies that profits off liquidity traders again are independent of $\rho$. Completely analogously to Result 5, the optimal number of securities $N^*$ is given by

(17) in the statement of the result. Similarly, substituting these optimal quantities in (26), and using $N^*$ above provides the expression for $\rho$ in (18) in the statement of the result. ■

**Proof of Result 8.** (26) implies that $\Gamma_{ii}$ is lowest when each security has the lowest information content, i.e. $\rho = 1$; in this case $g(\rho)$ equals zero and the first term on the right-hand side disappears. Comparing with (27) implies that $\Gamma_{ii} = \Gamma_{ij}$. Therefore, $g_N = N^2 \cdot \Gamma_{ii}$. Substituting $(1 - \frac{z_{ji}}{M})^2 \cdot \gamma_z$ into (29) in place of $\gamma_z$, and taking the implicit derivative as in the proof of Result 6, again implies that $g_N$ is increasing in $N$. Therefore, $\Gamma_{ii}$ is increasing in $N$, and so is $z_{ji} \cdot \Gamma_{ii}$, because $z_{ji}$ is increasing in $N$. Therefore, each traders costs are increasing in $N$, and thus lowest costs will be attained when the single asset is issued. ■