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# *Profits and Productivity*

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97-18

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*The Working Paper Series is made possible by a generous grant from the Alfred P. Sloan Foundation*

Profits and Productivity [1]

October 1996

Abstract: In this study we consider the linkage between productivity change and profit change. We develop an analytical framework in which profit change between one period and the next is decomposed into three sources: (i) a productivity change effect (which includes a technical change effect and an operating efficiency effect), (ii) an activity effect (which includes a product mix effect, a resource mix effect and a scale effect), and (iii) a price effect. We then show how to quantify the contribution of each effect, using only observed prices and quantities of products and resources in the two periods. We illustrate our analytical decomposition of profit change with an empirical application to Spanish banking during the period 1987 - 1994.

This paper was presented at the Wharton Financial Institutions Center's conference on *The Performance of Financial Institutions,* May 8-10, 1997.

Keywords: Profits, Productivity

JEL codes: D2, G2, O3

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***PROFITS AND PRODUCTIVITY****

## 1. Background

The objective of this study is to analyze the linkage between business productivity and business profit. The analysis begins at the theoretical level, continues with a discussion of the analytical techniques required to implement the analysis, and concludes with an empirical application. Although the empirical application is to a collection of independent businesses observed over a number of years, the analysis is equally suitable to other applications, such as a collection of regional offices, branches, or profit centers within a particular business, each observed over a sequence of possibly shorter (e.g., quarterly) accounting periods. The techniques can also be applied to a single business, provided only that a sufficient number of accounting periods are available to implement the analysis, or to a sufficiently large collection of businesses in a single accounting period. In each of these applications the analysis can provide a sort of benchmarking technique, in which a business or one of its constituent parts benchmarks against the remaining observations. The great virtue of these techniques, when applied in a benchmarking exercise, is that they utilize *causes* of profit change (such as productivity change), rather than *consequences* of profit change (such as a change in return on assets). This makes them amenable for use by management in an effort to improve future performance, which would then lead to future profit gains. Finally, although we analyze the linkage between productivity and profit, it is not necessary that profit be the sole objective of the units being analyzed. The analysis simply provides a linkage between productivity and the bottom line.

It is clear that productivity gains have the potential to contribute to an increase in business profit, but it is equally clear that other factors (e.g., a more favorable price structure) can also contribute to an increase in business profit. It is of interest, therefore, to develop an analytical model of the determinants of business profit change, among them being productivity change. The development and implementation of such an analytical framework is the objective of this study.

The linkage between productivity change and profit change has been explored previously in the business literature (see Genescà and Grifell (1992) for citations). However none of the previous models of this linkage is entirely convincing. Gold (1973, 1985) and others specify several partial productivity measures (such as total output or sales revenue per worker). But partial productivity measures can vary in opposite directions, and so no single partial productivity measure can be unambiguously linked to a business performance measure such as profit. Eilon, Gold and Soesan (1975) and Ishikawa and Sudit (1981) define productivity with a single, more inclusive, total productivity measure ("output," an index of all products, per unit of "input," an index of all resources). The advantage of a total productivity measure is that it can be linked unambiguously to profit. In several studies business profit change is decomposed into three sources: a price effect, including changes in resource prices paid and product prices received; a productivity effect, typically attributed solely to an improvement in technology; and an activity effect, capturing the effect of changes in the size and, less frequently, the scope, of the business. Kurosawa (1975), Eldor and Sudit (1981), Chaudry, Burnside and Eldor (1985), Miller (1984, 1987) and Miller and Rao (1989) each propose variants of this three-way decomposition. However the three components of profit change vary from study to study, because different studies employ different accounting relations. More significantly, these latter studies suffer from the lack of a firm economic foundation. They fail to exploit the economic theory of production and, through a duality relationship, the economic theory of profit. It is our contention that exploiting this relationship enables one to extend, in a theoretically and empirically useful way, the profit/productivity relationships which have been developed to date.

One branch of the economics literature on production and profit is based on a duality relationship between the structure of production technology and the structure of maximum profit, the latter also depending on the structure of product and resource prices. In this framework change in actual profit between two periods is attributed to changes in product and resource prices (similar to the price effect in the business literature), the structure of production technology (similar to the activity effect in the business literature), and to changes in the structure of production technology and changes in operating efficiency between two periods (similar to the productivity effect in the business literature). Diewert (1973) and Lau (1976) provide extensive treatments of this literature. A second branch of the economics literature focuses on the sources of productivity change, and so complements the business literature on the profit/productivity relationship. Recent efforts (e.g., Grifell and Lovell (1995) and Färe et al. (1996)) have been directed toward a decomposition of the quantity effect (the sum of a productivity effect and an activity effect) into components capturing the separate effects of the magnitude and biases of technical change, the magnitude of efficiency change, and scale economies. They have not, however, sought to relate these components to profit change. What remains is to merge the two branches of the economics literature with the business literature. The result will be a deeper insight into the determinants of business profit change between one period and the next, one based as much on the economic theory of production as on business accounting relationships.

It should be apparent from this brief review that the business and economics literatures, while having different institutional structures, different motivations and different objectives, have been discussing different aspects of the same problem: how business profit change can be allocated to its constituent sources. In this study we continue the tradition of those who have sought to establish a linkage between business profit change and productivity change. We pull together salient contributions from the two literatures, and we extend them to develop a new model of the linkage. Our strategy is to imbed a productivity change decomposition similar to that developed in the economics literature within the profit/productivity linkage developed in the recent business literature. The new model highlights the contribution of productivity change and its components, while at the same time not neglecting the contribution of other determinants of profit change. Our analysis sheds new light on four aspects of the linkage.

*First,* we provide a three-stage decomposition of profit change. In the first stage we decompose profit change into a price effect and a quantity effect. In the second stage we decompose the quantity effect into a productivity effect. The combination of these two decompositions is in the spirit of the relevant business literature developed by Kurosawa (1975) and Eldor and Sudit (1981). In the third stage we extend the previous business literature by exploiting both branches of the economics literature on production and profit. This enables us to decompose the productivity effect into a technical change effect and an operating efficiency effect. Eilon (1985) properly stressed the impact of the efficiency of resource use on business performance, and although the contribution of efficiency has been largely ignored in the subsequent literature, it plays a key role in our decomposition of profit change. We also decompose the activity effect into a product mix effect, a resource mix effect, and a scale effect. These six components of profit change are mutually exclusive and exhaustive. *Second*, we achieve this three-stage decomposition without imposing restrictive assumptions on the behavioral objective of the business or on the environment in which it operates. We do not assume profit maximizing behavior, we do allow the business to set some or all of its prices, we do not require that the business operate efficiently, and we do allow the business to produce a variety of products. *Third*, we provide computational guidance for implementing the three-stage profit decomposition. The computational technique consists of a sequence of linear programs. These programs modify and extend a technique known as data envelopment analysis (DEA), which is widely used in the management science and economics literatures to analyze business performance. *Fourth* and finally, we believe that the *ex ante* determinants of business profit change we identify are better suited to a useful benchmarking exercise than are the more frequently used *ex post* financial ratios, primarily because they are forward-looking rather than backward-looking.[1]

The study is organized as follows. In Section 2 we develop our analytical framework for the decomposition of business profit change. The decomposition involves unobserved as well as observed quantities of the products a business produces and the resources it consumes in their production, and so in Section 3 we show how to express all unobserved quantities as scalar multiples of observed quantities. We employ distance functions widely used in production economics to provide a theoretical expression for these scalar multiples, and we show how to calculate these distance functions empirically. In Section 4 we illustrate the working of our analytical decomposition with an empirical application to the recent performance of Spanish banking. We measure profit change among a sample of Spanish commercial banks during the period 1987-1994, and we obtain for each bank an empirical decomposition of its measured profit change into six determinants, for each pair of adjacent years and through the entire period. Section 5 contains a summary and our conclusions.

## 2. The Analytical Framework

### 2.1 The Production Technology

We consider a business using N resources represented by the nonnegative input quantity vector $x = (x_1,...,x_N)$ to produce M products represented by the nonnegative output quantity vector $y = (y_1,...,y_M)$. The business pays resource prices represented by the strictly positive input price vector $w = (w_1,...,w_N)$, and receives product prices represented by the strictly positive output price vector $p = (p_1,...,p_M)$. These prices may be exogenously determined by the forces of market competition, or by a regulatory agency, or they may be endogenously determined by the business itself. Business profit in period t, $\pi^t$, is defined as the difference between total revenue and total cost, and so $\pi^t = p^t \times y^t - w^t$

$\times x^t = \Sigma_i p_i^t y_i^t - \Sigma_j w_j^t x_j^t$, t = 1,...,T. We are interested in identifying the determinants of the change in profit from one period to the next, $(\pi^{t+1} - \pi^t)$, which can be positive, zero or negative. We require that the determinants satisfy two criteria: (i) their selection must be grounded in the economic theory of production; and (ii) they must be observable in conventional data sources, such as business financial statements.

We begin with a presentation of some basic concepts from production economics, which provide the necessary grounding for our decomposition. The concepts to be developed in equations (1) - (7) below represent a fairly conventional way of modeling the structure of production technology, and of describing the efficiency of observed resource use and observed output provision in light of the constraints imposed on managerial choice by the structure of production technology.

The *production set* in period t is the set of output quantity vectors and input quantity vectors that is feasible with technology in place in period t, and so

$$S^t = \{(y^t,x^t): y^t \text{ is producible with } x^t\}, \qquad t = 1,...,T. \qquad (1)$$

An *output set* is the set of all output quantity vectors which are producible with a given input quantity vector and with the technology in place in period t, and is defined in terms of $S^t$ as

$$P^t(x^t) = \{y^t: (y^t,x^t) \in S^t\}, \qquad t = 1,...,T. \qquad (2)$$

Output sets are assumed to be closed, bounded, convex, and to satisfy strong disposability of outputs $[y^t \in P^t(x^t) \Rightarrow y'^t \in P^t(x^t), 0 \leq y'^t \leq y^t]$. The outer boundary of an output set is its *output isoquant*

$$\text{Isoq}P^t(x^t) = \{y^t: y^t \in P^t(x^t), \lambda y^t \notin P^t(x^t), \lambda > 1\}, \qquad t = 1,...,T. \qquad (3)$$

The output sets $P^t(x^t)$ $[P^{t+1}(x^{t+1})]$ are within-period output sets containing the set of output quantity vectors which could be produced by an input quantity vector with technology prevailing in that same period. We also make use of the mixed-period output sets $P^{t+1}(x^t)$, which contains the hypothetical but analytically useful set of output quantity vectors which input quantity vector $x^t$ could have produced with the help of technology prevailing in the subsequent period t+1; and $P^t(x^{t+1})$, which contains the equally hypothetical and equally useful set of output quantity vectors which input quantity vector $x^{t+1}$ could have produced had it been forced to use technology prevailing in the previous period t. Mixed-period output isoquants $\text{Isoq}P^{t+1}(x^t)$ and $\text{Isoq}P^t(x^{t+1})$ are the outer boundaries of these mixed-period output sets, and are interpreted similarly.

An output quantity vector $y^t$ must belong to its contemporaneous output set $P^t(x^t)$, but it need not be located on its outer boundary $\text{Isoq}P^t(x^t)$. We need a measure of the distance from an output quantity vector $y^t$ to $\text{Isoq}P^t(x^t)$. Shephard's (1970) *output distance function*, which is the Debreu (1951) - Farrell (1957) output-oriented measure of operating efficiency, provides a radial measure of this distance. A within-period output distance function is defined in terms of a within-period output set as

$$D_o^t(x^t,y^t) = \min\{\phi: \ y^t/\phi \in P^t(x^t)\}. \tag{4}$$

$D_o^t(x^t,y^t) \leqq 1$ because $y^t \in P^t(x^t)$, and $D_o^t(x^t,y^t) = 1 \iff y^t \in \text{Isoq}P^t(x^t)$. Thus $D_o^t(x^t,y^t) = 1$ signals that $x^t$ is producing maximum feasible output with technology prevailing in period t, and $D_o^t(x^t,y^t) < 1$ suggests that $x^t$ is producing only $[100\times D_o^t(x^t,y^t)]\%$ of maximum feasible output with technology prevailing in period t. Finally, $D_o^t(x^t,y^t)$ is homogeneous of degree +1 in $y^t$, so that $D_o^t(x^t,\lambda y^t) = \lambda D_o^t(x^t,y^t)$, $\lambda > 0$. Mixed-period output distance functions $D_o^{t+1}(x^t,y^t)$ and $D_o^t(x^{t+1},y^{t+1})$ are obtained by replacing $P^t(x^t)$ with $P^{t+1}(x^t)$, and by replacing $P^t(x^t)$ with $P^t(x^{t+1})$, respectively, and they are interpreted in a similar manner. However since quantity data from one period may not be feasible with technology prevailing in another period, it follows that $D_o^{t+1}(x^t,y^t) \gtreqqless 1$ and $D_o^t(x^{t+1},y^{t+1}) \gtreqqless 1$.

Although our orientation is one of maximizing the production of outputs from given inputs and with given technology, we shall also have occasion to adopt the opposite orientation of minimizing input use in the production of given outputs with given technology. This requires the introduction of input sets, input isoquants and input distance functions. An *input set* is the set of all input quantity vectors capable of producing a given output quantity vector with technology prevailing at the time, and it is defined in terms of $S^t$ by means of

$$L^t(y^t) = \{x^t: (x^t,y^t) \in S^t\}, \quad t = 1,...,T. \tag{5}$$

Input sets are assumed to be closed, bounded, convex and to satisfy strong disposability of inputs $[x^t \in L^t(y^t) \Rightarrow x'^t \in L^t(y^t),\ x'^t \geq x^t]$. The inner boundary of the input set is its *input isoquant*

$$\mathrm{Isoq}L^t(y^t) = \{x^t: x^t \in L^t(y^t), \lambda x^t \notin L^t(y^t), \lambda < 1\}, \quad t = 1,...,T. \tag{6}$$

Mixed-period input sets $L^{t+1}(y^t)$ and $L^t(y^{t+1})$, and mixed-period input isoquants $\mathrm{Isoq}L^{t+1}(y^t)$ and $\mathrm{Isoq}L^t(y^{t+1})$, are defined exactly as mixed-period output sets and mixed-period output isoquants are. Finally, although an input quantity vector $x^t$ must belong to its contemporaneous input set $L^t(y^t)$, it need not necessarily belong to its inner boundary $\mathrm{Isoq}L^t(y^t)$. The Malmquist (1953) - Shephard (1953) *input distance function*, which is the reciprocal of the Debreu - Farrell input-oriented measure of operating efficiency, provides a radial measure of the distance from an input quantity vector to an input isoquant. It is defined by

$$D_i^t(y^t,x^t) = \max\{\theta : x^t/\theta \in L^t(y^t)\}. \tag{7}$$

$D_i^t(y^t,x^t) \geqq 1$ since $x^t \in L^t(y^t)$, and $D_i^t(y^t,x^t) = 1 \Leftrightarrow x^t \in \mathrm{Isoq}L^t(y^t)$. Also, input distance functions are homogeneous of degree +1 in inputs, and so $D_i^t(y^t,\lambda x^t) = \lambda D_i^t(y^t,x^t)$, $\lambda > 0$. Mixed-period input distance functions $D_i^{t+1}(y^t,x^t)$ and $D_i^t(y^{t+1},x^{t+1})$ are defined in the same manner as mixed-period output distance functions are, and $D_i^{t+1}(y^t,x^t) \gtreqless 1$ and $D_i^t(y^{t+1},x^{t+1}) \gtreqless 1$.

## 2.2 The Profit Change Decomposition

We are now prepared to decompose business profit change between periods t and t+1. Our strategy is to proceed in three stages. In the first stage we decompose the profit change resulting from a movement from $(x^t,y^t)$ to $(x^{t+1},y^{t+1})$ into a pure quantity effect which holds prices constant, and a pure price effect which holds quantities constant. This decomposition appears in Proposition 1, and is similar to decompositions previously obtained by Kurosawa (1975) and illustrated by Eldor and Sudit (1981). In the second stage we decompose the quantity effect into its two basic components: a productivity effect and an activity effect. This decomposition appears in Proposition 2, and is illustrated in Figure 1. It is similar to decompositions previously obtained by Kurosawa (1975) and Miller (1984, 1987). In the third stage we decompose the productivity effect into its two components: a technical change effect and an operating efficiency effect. We also decompose the activity effect into its three components: a product mix effect, a resource mix effect, and a scale effect. We decompose the productivity effect in Proposition 3, and we decompose the activity effect in Proposition 4. Both third stage decompositions are illustrated in Figures 1 - 3. The third stage decompositions appear to have no precedent in the business literature, although they do have precedents in the economics literature.

**Proposition 1:** The **profit change** between period t and period t+1 decomposes as

$$[\pi^{t+1} - \pi^t] =$$

$$[(y^{t+1} - y^t)\times p^t - (x^{t+1} - x^t)\times w^t] \qquad \textbf{\textit{quantity effect}}$$

$$+ [(p^{t+1} - p^t)\times y^{t+1} - (w^{t+1} - w^t)\times x^{t+1}] \qquad \textbf{\textit{price effect}}$$

The quantity effect shows the impact on profit of an expansion or contraction of the business, holding prices fixed. The quantity effect uses base period prices to weight the quantity changes, and so it can be interpreted as the difference between a Laspeyres type of output quantity index and a Laspeyres type of input quantity index, both being expressed in difference form rather than in the conventional ratio form. The price effect shows the impact on profit of changes in the price structure of the business, holding quantities fixed. The price effect uses comparison period quantities to weight the price changes, and so it can be interpreted as the difference between a Paasche type of output price index and a Paasche type of input price index, both being expressed in difference form rather than in ratio form.[2]

The following three propositions show how the quantity effect decomposes into a total of five determinants of profit change. All five determinants use base period prices to weight quantity changes, and so all five determinants are Laspeyres type quantity indexes, or differences between Laspeyres type quantity indexes.

**Proposition 2**: The **quantity effect** between period t and period t+1 decomposes as

$$[(y^{t+1} - y^t)\times p^t - (x^{t+1} - x^t)\times w^t] =$$

$$[(p^t)\times(y^B - y^t) - (p^t)\times(y^C - y^{t+1})] \qquad \textbf{\textit{productivity effect}}$$

$$+ [(p^t)\times(y^C - y^B) - (w^t)\times(x^{t+1} - x^t)] \qquad \textbf{\textit{activity effect}}$$

Figure 1 provides a partial illustration of the decomposition of the quantity effect, partial because it is assumed that M=N=1. In period t $(x^t,y^t) \in S^t$, and in period t+1 $(x^{t+1},y^{t+1}) \in S^{t+1}$. Since $S^t \subset S^{t+1}$, technical progress has occurred between periods t and t+1, although this assumption is unnecessary for the analysis. The path from $(x^t,y^t)$ to $(x^{t+1},y^{t+1})$ can be decomposed into three components, each of which exerts an influence on profit change, with magnitude and direction depending on prevailing prices and how they change from the base period to the comparison period.[3]

The *productivity effect* compares the path from $y^B$ to $y^t$ in period t with the path from $y^C$ to $y^{t+1}$ in period t+1. Part of the productivity effect is measured along the path from $y^B$ to $y^A$ in period t, representing the additional output which can be produced with no increase in input usage as a result of an improvement in technology, which expands the production set from $S^t$ to $S^{t+1}$. Thus technical progress necessarily contributes positively to profit change (and technical regress would contribute negatively to profit change). The remainder of the productivity effect is measured along the paths from $y^A$ to $y^t$ in period t, and from $y^C$ to $y^{t+1}$ in period t+1. These two paths represent a pair of deductions from productivity, due to a failure to produce maximum output in period t, and a failure to produce maximum output in period t+1. If operating efficiency improves during the period (i.e., if $(y^C - y^{t+1}) < (y^A - y^t)$), then efficiency gains contribute positively to profit change. If operating efficiency declines during the period, then deteriorating efficiency detracts from productivity change.[4]

The *activity effect* is typically conceived as reflecting the consequence of changes in the scale and scope of the organization. Changes in scale are adequately characterized in Figure 1; changes in scope are not, and are discussed in Proposition 4. With this qualification in mind, the activity effect is measured along the path from $y^B$ to $y^C$, and represents the change in output $(y^C - y^B)$ generated by the change in resource usage $(x^{t+1} - x^t)$. It appears that if output production increases proportionately more (less) than resource usage does, then scale economies (diseconomies) cause the activity effect to contribute positively (negatively) to profit change. However there is more to the activity effect than just economies or diseconomies of scale, since the scope dimension of the activity effect has yet to be introduced. Moreover, both the output change and the input change are weighted by base period prices, and so the price structure matters as well.

Notice that the decomposition of the quantity effect involves unobserved as well as observed quantity data. Although $(x^t,y^t)$ and $(x^{t+1},y^{t+1})$ are observed, the output quantity vectors $y^B$ and $y^C$ are not observed. Consequently in order to render this decomposition procedure empirically useful, it is necessary to be able to recover the unobserved output quantity vectors from the observed quantity data. We will consider this problem after we present the third stage decompositions, which introduce additional unobserved quantity vectors, and to which we now turn.

**Proposition 3**: The **productivity effect** between period t and period t+1 decomposes as

$$[(p^t)\times(y^B - y^t) - (p^t)\times(y^C - y^{t+1})] =$$

$$[(p^t)\times(y^B - y^A)] \qquad \textit{technical change effect}$$

$$- [(p^t)\times(y^C - y^{t+1}) - (p^t)\times(y^A - y^t)] \qquad \textit{operating efficiency effect}$$

The decomposition of the productivity effect is adequately illustrated in Figure 1. Both the technical change effect and the operating efficiency effect influence the revenue side of profit change. The technical change effect is measured as the increase in output quantity ($y^B - y^A$) allowed by the improvement in technology, the output quantity increase being evaluated at base period output prices.[5] The operating efficiency effect is measured as the difference between comparison period productive inefficiency ($y^C - y^{t+1}$), and base period productive inefficiency ($y^A - y^t$), both evaluated at base period output prices.[6] The decomposition of the productivity effect is also illustrated in Figure 2, which provides a different partial illustration, partial in this case because the input quantity vector is fixed, either at $x^t$ or at $x^{t+1}$. In Figure 2, which allows M=2, the technical change effect is measured as the equiproportionate increase in output quantites ($y^B - y^A$) allowed by the improvement in technology [$P^{t+1}(x^t) \supset P^t(x^t)$], evaluated at base period output prices. The operating efficiency effect is measured as the difference between comparison period productive inefficiency ($y^C - y^{t+1}$) and base period productive inefficiency ($y^A - y^t$), both evaluated at base period output prices

**Proposition 4:** The **activity effect** from period t to period t+1 decomposes as

$$[(p^t)\times(y^C - y^B) \; - \; (w^t)\times(x^{t+1} - x^t)] =$$

$$[(p^t)\times(y^C - y^D)] \qquad \textbf{\textit{product mix effect}}$$

$$- \; [(w^t)\times(x^{t+1} - x^E)] \qquad \textbf{\textit{resource mix effect}}$$

$$+ \; [(p^t)\times(y^D - y^B) \; - \; (w^t)\times(x^E - x^t)] \qquad \textbf{\textit{scale effect}}$$

The decomposition of the activity effect is not adequately illustrated in Figure 1, as we noted above. There the movement from $y^B$ to $y^C$ comingles the scale effect with the product mix effect, and the corresponding movement from $x^t$ to $x^{t+1}$ comingles the scale effect with the resource mix effect. The scale and product mix effects are disentangled in Figure 2, which provides a third partial illustration, partial because the input quantity vector is fixed, either at $x^t$ or at $x^{t+1}$. In Figure 2, which allows M=2, the output side of the scale effect holds the product mix fixed along the path from $y^B$ to $y^D$, using base period output prices to evaluate the adjustment, as input use increases from $x^t$ to $x^{t+1}$. The product mix effect is measured along the path from $y^D$ to $y^C$, using base period output prices to evaluate the adjustment. The sum of the scale effect and the product mix effect produces a movement from $y^B$ to $y^C$, which corresponds to the output side of the activity effect introduced in Proposition 2 and illustrated in Figure 1.

The final component of the activity effect, the input mix effect, is illustrated in Figure 3, which provides a third partial illustration, partial because the output quantity vector is fixed, either at $y^t$ or at $y^{t+1}$. In Figure 3, which allows N=2, the input side of the scale effect holds the resource mix fixed along the path from $x^t$ to $x^E$, using base period input prices to evaluate the adjustment. The resource mix effect is measured along the path from $x^E$ to $x^{t+1}$, also using base period input prices to evaluate the adjustment. The sum of the scale effect and the resource mix effect produces a movement from $x^t$ to $x^{t+1}$, which corresponds to the input side of the activity effect introduced in Proposition 2 and illustrated in Figure 1.[7,8]

The product mix effect and the input mix effect can be due to any number of factors. Perhaps the most significant factor is the ability of a business to react to product price changes by adjusting its product mix, and to react to resource price changes by adjusting its resource mix. The ability to substitute toward products whose prices are rising and resources whose prices are falling, and away from products whose prices are falling and resources whose prices are rising, contributes to profit gain through the product mix effect and the resource mix effect. Thus the two mix effects capture the substitution possibilities permitted by the structure of production technology, as well as the ability of management to exploit these possibilities. A second factor is a changing regulatory environment which allows business more or less freedom to optimize its product and resource mixes. This factor is particularly significant in the empirical example we use to illustrate our profit decomposition. A third factor is an consequence of the way we have measured technical change and scale economies. Both the technical change effect and the scale effect involve equiproportionate changes in variables, as Figures 2 and 3 make clear. If technical change is not neutral with respect to outputs, any unmeasured bias shows up in the product mix effect. If technical change is not neutral with respect to resources (for example, if it is labor-saving), any unmeasured bias shows up in the resource mix effect. Similarly, if efficient expansion or contraction of the business involves nonproportionate expansion or contraction of outputs of inputs, the disproportionate features of scale economies shows up in the product mix effect and the resource mix effect.[9]

The intent of Propositions 1-4 and Figures 1-3 is to demonstrate that even in the multiple input, multiple output case, it is in principle possible to decompose the profit change resulting from a producer's movement from $(x^t,y^t)$ to $(x^{t+1},y^{t+1})$ into several sources. The first source is an improvement or a deterioration in the price structure of the business, which may have both external and internal causes. A second source is an increase in output not requiring any increase in resource use, due to technical change. A third source is an improvement or a deterioration in operating efficiency. These two sources make up the productivity effect. A fourth source is a change in output that can be proportionately greater than or less than the change in input, due to the presence of economies or diseconomies of scale which characterize the production technology. A fifth source is a change in the product mix, and a sixth is a change in the resource mix. The product mix effect and the resource mix effect encompass a number of phenomena, as we noted above. The last three sources comprise the activity effect. The six sources are mutually exclusive and exhaustive sources of profit change between period t and period t+1.

## 3. Implementing the Profit Decomposition

The price effect in Proposition 1 is expressed in terms of observed base period and comparison period price vectors, and observed comparison period quantity vectors. The technical change effect in Proposition 3 is expressed in terms of an observed base period output price vector and two unobserved output quantity vectors $y^B$ and $y^A$. The operating efficiency effect in Proposition 3 is expressed in terms of an observed base period output price vector, observed base period and comparison output quantity vectors, and two unobserved output quantity vectors $y^A$ and $y^C$. The scale effect in Proposition 4 is expressed in terms of observed base period output and input price vectors, the observed base period input quantity vector, and two unobserved output quantity vectors $y^B$ and $y^D$ and an unobserved input quantity vector $x^E$. The product mix effect in Proposition 4 is expressed in terms of an observed base period output price vector and two unobserved output quantity vectors $y^C$ and $y^D$. Finally, the resource mix effect in Proposition 4 is expressed in terms of an observed base period input price vector, an observed comparison period input quantity vector, and an unobserved input quantity vector $x^E$. It is necessary to recover the unobserved quantity vectors $(y^A,y^B,y^C,y^D,x^E)$ in order to make the profit change decomposition analysis empirically useful. We now show how to recover each of these unobserved quantity vectors from the observed quantity vectors $(x^t,y^t)$ and $(x^{t+1},y^{t+1})$. Our strategy should be apparent from an inspection of Figures 2 and 3, where it is clear that each unobserved quantity vector appears as either a radial expansion or a radial contraction of an observed quantity vector. The distance functions introduced in equations (4) and (7), being radial distance measures, provide the tools with which to recover the unobserved quantity vectors.

**Proposition 5:** The unobserved quantity vectors ($y^A$,$y^B$, $y^C$,$y^D$,$x^E$) can be recovered from the observed quantity vectors ($x^t$,$y^t$) and ($x^{t+1}$,$y^{t+1}$) by means of

**(i)** $y^A = y^t/D_o^t(x^t,y^t)$;

**(ii)** $y^B = y^t/D_o^{t+1}(x^t,y^t)$;

**(iii)** $y^C = y^{t+1}/D_o^{t+1}(x^{t+1},y^{t+1})$;

**(iv)** $y^D = y^t/D_o^{t+1}(x^{t+1},y^t)$;

**(v)** $x^E = x^t/D_i^{t+1}((y^t/D_o^{t+1}(x^{t+1},y^t),x^t))$.

Substituting the equalities in Proposition 5 into the six components of profit change identified in Propositions 2 - 4 enables one to recover each of the unobserved quantity vectors, and thus to conduct an empirical analysis of the sources of profit change from one period to the next. Even though the profit change decomposition involves five unobserved quantity vectors, it can nonetheless be undertaken. The key element in the decomposition is the distance functions. Since these distance functions must be calculated from observed data, we now show how to calculate them. All that is required is input and output quantity data for a sample of producers over a period of time. Price data are not required to obtain the unobserved quantity vectors; price data are used in the price effect, and as weights in each of the five quantity effects.

Proposition 5 provides the theoretical foundation required to recover the unobserved quantity vectors ($y^A$,$y^B$, $y^C$,$y^D$,$x^E$) from the observed quantity vectors ($x^t$, $y^t$, $x^{t+1}$, $y^{t+1}$). The distance functions employed in Proposition 5 can be calculated empirically using a linear programming technique which provides a modification of data envelopment analysis (DEA). Originally developed by Charnes, Cooper and Rhodes (1978, 1981) and by Banker, Charnes and Cooper (1984), DEA is a widely used operations research technique for measuring business performance. Although DEA was originally intended for use in public sector and other not-for-profit environments, it is ideally suited to the problem at hand. Nonetheless, we believe this is the first application of DEA to the problem of decomposing profit change.

In conventional DEA period t technology is constructed from input and output quantity data describing the operations of all producers in period t. In this approach technologies in place in previous periods are "forgotten" in period t, since period t technology is constructed from period t activities only. Our modification of DEA allows period t technology to be constructed from input and output quantity data of all producers *in all periods prior to and including* period t. In our modified approach, technologies in place in previous periods are "remembered," and remain available for adoption in the current period. This modification influences the way we set up the linear programming problems.[10]

Let $Y^t = [y^{1s},...,y^{os},...,y^{Is}]$ be an $M \times \Sigma_{s=1}^{t} I_s$ matrix of M outputs produced by $I_s$ producers in periods s = 1,...,t, and let $X^t = [x^{1s},...,x^{os},...,x^{Is}]$ be an $N \times \Sigma_{s=1}^{t} I_s$ matrix of N inputs used by $I_s$ producers in periods s = 1,...,t. Thus the data matrices $Y^t$ and $X^t$ are "sequential," since they include output and input quantity data for all producers from the beginning of the sample through the current period. Also let $(y^{ot},x^{ot})$ be the "contemporaneous" $M \times 1$ output quantity vector and the $N \times 1$ input quantity vector of the producer whose period t profit change is being decomposed. Then the five unobserved quantity vectors $(y^A,y^B,y^C,y^D,x^E)$ can be recovered for each producer by solving each of the following linear programming problems $I_t$ times, once for each producer in the sample in period t.

Unobserved output quantity vector $y^{oA}$ can be recovered from the solution to the linear programming problem

$$[D_o^t(x^{ot},y^{ot})]^{-1} = \max \theta^A \tag{8}$$

subject to

$$\theta^A y^{ot} \leq Y^t \lambda^t$$

$$X^t \lambda^t \leq x^{ot}$$

$$\lambda^t \geq 0$$

$$\Sigma_i \lambda_i^t = 1, \quad i = 1, .... , \Sigma_{s=1}^{t} I_s,$$

where $\lambda^t$ is a $\Sigma_{s=1}^{t} I_s \times 1$ activity vector. From Proposition 5 (i), $y^{oA} = \theta^{oA} y^{ot}$.

Unobserved output quantity vector $y^{oB}$ can be recovered from the solution to the linear programming problem

$$[D_o^{t+1}(x^{ot},y^{ot})]^{-1} = \max \theta^B \tag{9}$$

subject to

$$\theta^B y^{ot} \le Y^{t+1} \lambda^{t+1}$$

$$X^{t+1} \lambda^{t+1} \le x^{ot}$$

$$\lambda^{t+1} \ge 0$$

$$\Sigma_i \lambda_i^{t+1} = 1, \qquad i = 1, \ldots, \Sigma_{s=1}^{t} I_{s+1},$$

where $\lambda^{t+1}$ is a $\Sigma_{s=1}^{t} I_{s+1} \times 1$ activity vector. From Proposition 5 (ii), $y^{oB} = \theta^{oB} y^{ot}$.

Unobserved output quantity vector $y^{oC}$ can be recovered from the solution to the linear programming problem

$$[D_o^{t+1}(x^{ot+1},y^{ot+1})]^{-1} = \max \theta^C \tag{10}$$

subject to

$$\theta^C y^{ot+1} \le Y^{t+1} \lambda^{t+1}$$

$$X^{t+1} \lambda^{t+1} \le x^{ot+1}$$

$$\lambda^{t+1} \ge 0$$

$$\Sigma_i \lambda_i^{t+1} = 1, \qquad i = 1, \ldots, \Sigma_{s=1}^{t} I_{s+1},$$

where $\lambda^{t+1}$ is a $\Sigma_{s=1}^{t} I_{s+1} \times 1$ activity vector. From Proposition 5 (iii), $y^{oC} = \theta^{oC} y^{ot+1}$.

Unobserved output quantity vector $y^{oD}$ can be recovered from the solution to the linear programming problem

$$[D_o^{t+1}(x^{ot+1},y^{ot})]^{-1} = \max \theta^D \tag{11}$$

subject to

$$\theta^D y^{ot} \leq Y^{t+1}\lambda^{t+1}$$

$$X^{t+1}\lambda^{t+1} \leq x^{ot+1}$$

$$\lambda^{t+1} \geq 0$$

$$\Sigma_i \lambda_i^{t+1} = 1, \qquad i = 1, \ldots, \Sigma_{s=1}^{t} I_{s+1},$$

where $\lambda^{t+1}$ is a $\Sigma_{s=1}^{t} I_{s+1} \times 1$ activity vector. From Proposition 5 (iv), $y^{oD} = \theta^{oD} y^{ot}$.

Finally, unobserved input quantity vector $x^{oE}$ can be recovered by inserting the solution to the linear programming problem (11) into the solution to the linear programming problem

$$[D_i^{t+1}(\hat{y}^{ot},x^{ot})]^{-1} = \min \phi^E \tag{12}$$

subject to

$$\hat{y}^{ot} \leq Y^{t+1}\lambda^{t+1}$$

$$X^{t+1}\lambda^{t+1} \leq \phi^E x^{ot}$$

$$\lambda^{t+1} \geq 0$$

$$\Sigma_i \lambda_i^{t+1} = 1, \qquad i = 1, \ldots, \Sigma_{s=1}^{t} I_{s+1},$$

where $\lambda^{t+1}$ is a $\Sigma_{s=1}^{t} I_{s+1} \times 1$ activity vector, $\hat{y}^{ot} = y^{ot} / D_o^{t+1}(x^{ot+1},y^{ot})$ and $Y^{t+1} = [y_1^{t+1}, \ldots, y_I^{t+1}]$. From Proposition 5 (v), $x^{oE} = \phi^{oE} x^{ot}$.

To summarize, recovery of the five unobserved quantity vectors requires the solution of a series of five linear programming problems for each producer. For any reasonable number of producers and time periods, and for any reasonable number of inputs and outputs, this task can easily be handled on any personal computer. Once the five unobserved quantity vectors have been recovered, empirical implementation of the profit decomposition presented in Propositions 1 - 4 is straightforward.

## 4. An Application to Spanish Banking

In this Section we report results of an empirical investigation into the sources of profit change within the Spanish commercial bank sector during the period 1987 - 1994. Annual data for commercial banks are reported in *Anuario Estadístico de la Banca Española*. The commercial bank sample consists of roughly two-thirds of all commercial banks in existence during the period 1987-1994. However the sample does contain 92% of all commercial bank assets in 1993, so the missing banks are very small. The sample size varies from 59 in 1987 to a high of 61 in 1990 and a low of 56 in 1993. A detailed discussion of the data describing the recent history of Spanish banking is available in Grifell and Torrent (1995).[11]

Annual profit consists of operating profit, or profit from intermediation activities, and is defined as gross profit less gains and losses from trading in stocks and public debt instruments, and less extraordinary profit. Extraordinary profit typically comes from sales of fixed assets, but during the 1987-1990 period extraordinary losses also arose from the legally mandated establishment of employee pension plans. On balance, operating profit has accounted for 76% of gross profit among the banks in our sample.

Well over 90% of revenue consists of net loan and investment income, defined as gross loan and investment income less provision for bad debt. We decompose this income into quantity and price components by specifying the quantity component as the average of the beginning-of-period and the end-of-period value of all loans and investments, and by specifying the price component as the ratio of net loan and investment income to the average value of all loans and investments. Thus $y_1^t$ is expressed in pesetas, and $p_1^t$ is expressed as a per cent.

The remaining source of revenue consists of net commission income, the difference between commission income generated and commission expenses incurred. On the assumption that net commission income is a function of the number of deposit accounts, we proxy the quantity component of net commission income by the average of the beginning-of-period and end-of-period number of deposit accounts. The price component is then the ratio of net commission income to the average number of deposit accounts. Thus $y_2^t$ is expressed as a pure number, and $p_2^t$ is expressed in pesetas.

Approximately two-thirds of cost is financial expense, consisting of interest paid on deposit accounts and other liabilities. The quantity component of financial expense is defined as the average of the beginning-of-period and end-of-period value of all deposits and other liabilities which generate financial expense. The price component is the ratio of financial expense to the average value of all deposits and other liabilities. Thus $x_1^t$ is expressed in pesetas, and $w_1^t$ is expressed as a per cent.

Labor expense accounts for approximately 20% of cost. The quantity component of labor expense is defined as the average of the beginning-of-period and end-of-period number of employees. The price component is the ratio of labor expense to the average number of employees. Thus $x_2^t$ is expressed as a pure number, and $w_2^t$ is expressed in pesetas.

The remaining source of cost is non-financial, non-labor expense, consisting of non-labor operating expense, direct expenditure on buildings, and amortization expense. The quantity component of this expense category is proxied by the average of the beginning-of-period and end-of-period value of all fixed assets. The price component is calculated as the ratio of non-financial, non-labor expense to the average value of all fixed assets. Thus $x_3^t$ is expressed in pesetas, and $w_3^t$ as a per cent.

Summary statistics for all variables are collected in Table 1. Average operating profit doubled from 1987 to 1990, and declined dramatically thereafter, despite the fact that margins ($p_1$ - $w_1$) remained above 4% until 1991.

Results of implementing the profit change decomposition are summarized in Tables 2 - 4. Table 2 provides a summary of the initial decomposition of profit change into a productivity effect, an activity effect, and a price effect, averaged over the number of banks indicated in the final column.[12] Bank profit increased during the first three years of the sample, declined during the next three years, and increased again in 1994. Over the entire period, profit increased by an average of 2.4% per year. The productivity effect made a positive contribution on average, and in six of seven years. The activity effect made a larger positive contribution on average, and in all seven years. However these two positive contributions to profit change were nearly offset by the price effect, which was large and negative, on average and in six of seven years.

The sources of the negative price effect are indicated in Table 1. Although returns on loans and other financial investments held fairly steady through 1993, and commission income increased throughout the period, deposit rates increased through 1991 and prices of the remaining resources increased more than proportionately. Gradual deregulation of the Spanish banking system and the consequent increase in competition took its toll on the price structure and the profitability of commercial banks.

We now turn to an analysis of the productivity effect, the decomposition of which appears in Table 3. The technical change component was large and positive on average, and was positive in every year. We attribute a positive technical change component to an improvement in the productivity of the best practice banks. Perhaps because their price structure was deteriorating, banks responded by increasing service provision more rapidly than they increased resource usage. The operating efficiency component was very small and negative on average, and was negative in all seven years. We attribute a negative operating efficiency component to a failure of the remaining banks to keep pace with the improved performance of best practice banks.

A decomposition of the activity effect appears in Table 4. The large positive activity effect is primarily attributable to a very large product mix effect, which was positive on average, and was positive in every year. The favorable product mix effect is attributable to a doubling of loans and other financial investments, the return to which held steady through 1993, and also to a decline in deposits which was proportionately smaller than the rapid increase in deposit rates. A large positive scale effect also contributed to the positive activity effect, although as we pointed out in note 8 a positive scale effect is not necessarily evidence of increasing returns to scale. Indeed Grifell and Lovell (1996) have found a wide range of service provision over which Spanish commercial banks experience roughly constant returns to scale. The resource mix effect was large and negative on average, and was negative in all seven years. The source of the large negative resource mix effect is apparent from Table 1; deposits and other liabilities nearly doubled as their price increased through 1993, and the value of fixed assets more than doubled as their price increased throughout the period.

The profit decline among Spanish commercial banks during the period was thus the consequence of a number of factors, three positive and three negative. The positive contributions came from an improvement in the performance of best practice banks (the technical change effect), a continuing emphasis on loans and other financial investments having high and relatively stable rates of return (the product mix effect), and a general expansion in average bank size, which either exploited scale economies or at least was not offset by diseconomies of scale (the scale effect). However these positive contributions to profit change were nearly offset by the negative contributions of a deterioration in the banks' price structure brought on by deregulation (the price effect), a failure of the remaining banks to keep pace with the improving performance of the best practice banks (the operating efficiency effect), and a rapid growth in deposits and other liabilities and in fixed assets (the resource mix effect).

## 5. Summary and Conclusions

Business profit changes from year to year, increasing in some years and declining in others. The business and economics literatures have adopted somewhat different approaches to an analysis of the sources of profit change, although we have found evidence of both considerable overlap and substantial divergence. This has motivated us to develop a three stage decomposition of profit change which draws from, and extends, both literatures. In the initial stage profit change is decomposed into a quantity effect and a price effect, as expressed in Proposition 1. In the second stage the quantity effect is decomposed into a productivity effect and an activity effect, as expressed in Proposition 2. The first two stages are broadly consistent with some decompositions appearing in the business literature. In the third stage the productivity effect is further decomposed into a technical change effect and an operating efficiency effect, and the activity effect is further decomposed into a product mix effect, a resource mix effect, and a scale effect. The third stage is broadly consistent with the traditional focus in the economics literature on technical change, efficiency change, and the structure of technology as characterized by the nature of scale and scope economies. The third stage decompositions obtained in Propositions 3 and 4 appear to be new, however.

The profit decompositions we have derived are based on observed prices, but they are based on both observed and unobserved quantities, of the resources the business employs and the services it provides. This makes it necessary to obtain expressions for the unobserved quantities in terms of observed quantities. This we achieve in Proposition 5, in which the distance functions introduced in equations (4) and (7) are used to express unobserved quantities as radial expansions or contractions of observed quantities. Once these relationships have been analytically derived, we show in equations (8) - (12) how to use linear programming techniques to empirically calculate the requisite distance functions, and hence to obtain solutions for the five unobserved quantities. This enables us to obtain the desired profit decomposition.

We have illustrated the profit decompositions using data describing the operations of a sample of Spanish commercial banks. The data cover a difficult period of adjustment to a changing regulatory environment in which competition was increasing. The raw data show a slight improvement in profitability among commercial banks during the sample period, despite two very difficult years in 1992 and 1993. We have attributed the observed profit change to six effects, three positive and three negative. When the various effects are grouped into endogenous and exogenous influences, the picture brightens somewhat. The combined productivity and activity effect, which arguably reflects factors largely under the control of bank management, made a positive contribution to profit change in all seven years. However the price effect, which presumably captures the impact of macroeconomic and other influences beyond the control of bank management, made a negative contribution to profit change in the first six years. The negative price effect almost offset the positive combined productivity and activity effect during the entire sample period, and swamped it during the 1991-1993 subperiod.

## Footnotes

* We are grateful to “Fundación Fondo para la Investigación Económica y Social” (FIES) for generous financial support, to Carmen Matutes and J. L. Raymond for helpful discussions concerning the data and our findings, and to Hugo Fuentes for his computer support.

1. Our distinction between *ex post* financial ratios and *ex ante* operational determinants of profit change is reminiscent of the “balanced scorecard” approach of Kaplan and Norton (1992).

2. Alternatively, it is possible to decompose profit change using Paasche types of quantity indexes and Laspeyres types of price indexes. It is also possible to use arithmetic means of the base period and comparison period weights, which would produce Fisher types of quantity and price indexes, expressed in difference form rather than ratio form. We employ Laspeyres types of quantity indexes and Paasche types of price indexes because these are consistent with the approach adopted in much of the business literature on profit change decomposition.

3. In Figure 1, the path from $(y^t,x^t)$ to $(y^{t+1},x^{t+1})$ goes through $y^A$, $y^B$ and $y^C$. This path measures the technical change effect at $x^t$, and measures the activity effect along period t+1 technology. It is also possible to create a path from $(y^t,x^t)$ to $(y^{t+1},x^{t+1})$ which goes through $y^A$, $y^F$ and $y^C$. This path measures the technical change effect at $x^{t+1}$, and measures the activity effect along period t technology. Details of the alternative decomposition are provided in the Appendix.

4. The productivity effect in Proposition 2 corresponds to an output-oriented period t+1 Malmquist (1953) productivity index. Our productivity effect is positive, zero or negative according as the period t+1 Malmquist productivity index indicates productivity growth, stagnation or decline. The productivity effect in Appendix Proposition A1 corresponds to an output-oriented period t Malmquist productivity index. See Førsund (1990) for details on Malmquist productivity indexes.

5. A Malmquist productivity index decomposes into a technical change component and an efficiency change component. Our technical change effect is isomorphic to the magnitude index of technical change $\Delta T(x^t,y^t)$ of Färe et al. (1996). That is, our technical change index is positive, zero or negative according as $\Delta T(x^t,y^t) \begin{smallmatrix}>\\=\\<\end{smallmatrix} 1$. Consequently technical progress (regress) contributes positively (negatively) to productivity change, and hence contributes positively (negatively) to profit change.

6. Our operating efficiency effect is isomorphic to the technical efficiency change index $\Delta TE(x^t,y^t,x^{t+1},y^{t+1})$ of Färe et al. (1996). That is, our operating efficiency effect is positive, zero or negative according as $\Delta TE(x^t,y^t,x^{t+1},y^{t+1}) \begin{smallmatrix}>\\=\\<\end{smallmatrix} 1$. Consequently an improvement (decline) in operating efficiency contributes positively (negatively) to productivity change, and hence contributes positively (negatively) to profit change.

7. Our product mix effect is isomorphic to the output bias index $OB(y^t,x^{t+1},y^{t+1})$ of technical change proposed by Färe et al. (1996). That is, our product mix effect is positive, zero or negative according as $OB(y^t,x^{t+1},y^{t+1}) \begin{smallmatrix}>\\=\\<\end{smallmatrix} 1$. The only difference between the two concepts is one of interpretation. Färe et al. interpret their index as a characteristic of technical change, while we interpret our effect more liberally, allowing biased technical change to be one of several potential components of the product mix effect. Similar remarks apply to the relationship between our resource mix effect and their input bias index.

8. Our scale effect is related to, but is not isomorphic to, the the notion of returns to scale in production. This is because our scale effect is not positive, zero or negative according returns to scale are increasing, constant or decreasing. This is easily seen by assuming constant returns to scale, so that $y^D = \lambda y^B$ and $x^E = \lambda x^t$, $\lambda > 0$. In this case the scale effect is equal to $(\lambda - 1)[p^t \times y^B - w^t \times x^t]$, which does not collapse to zero unless $\lambda = 1$. The condition $\lambda = 1$ is not a condition for constant returns to scale; constant returns to scale holds for all $\lambda > 0$. Increasing or decreasing returns to scale would magnify or dampen any scale effect generated by constant returns to scale. The condition $\lambda = 1$ states that the scale of operations of the producer remains unchanged. It is in this sense, rather than in the narrow constant returns to scale sense, that we refer to this term as a scale effect.

9. Somewhat more formally, if production technology is not *jointly homothetic*, then nonproportionate scale effects show up in the product mix effect and the resource mix effect. See Färe and Primont (1995) on joint homotheticity.

10. Tulkens and Vanden Eeckaut (1995) refer to such an approach as "sequential" DEA, since period t technology is constructed sequentially. Although we find sequential DEA more plausible than conventional DEA (which Tulkens and Vanden Eeckaut refer to as "contemporaneous" DEA), it is straightforward to conduct the following analysis using conventional DEA. All that is required is to redimension the output and input matrices, making them contemporaneous rather than sequential.

11. Other studies have examined either profitability or productivity in Spanish banking, but none has examined the linkage between the two. Pastor (1995) and Grifell and Lovell (1996) have examined productivity change in Spanish banking, without attempting to assess the contribution of productivity change to profit change.

12. The number of commercial banks listed in the final column of Table 2 is smaller than the number listed in the final row of Table 1. This is due to three factors. Mergers occuring in period t eliminate two or more banks which existed at the end of period t-1, and create a new bank at the end of period t. For these merging banks none of the linear programs can be solved for both periods, and the profit decomposition cannot be implemented. In addition, banks occasionally appear in only one of a pair of adjacent years, due to data problems. In this case adjacent-period linear programming problems cannot be solved for these banks, and the profit decomposition cannot be implemented for them. Finally, the mixed period linear programs (9) and (11) are not guaranteed to have solutions for the smallest banks. Consequently the profit change indicated in the first column of Table 2 does not correspond to the annual change in average operating profit calculated from the entries in the first row of Table 1.

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## Appendix

In this Appendix we provide an alternative profit change decomposition. The decomposition given in Propositions 1 - 4 measures the technical change effect with the shift in production technology at the input vector $x^t$, and then measures the activity effect (and each of its three components) along period t+1 technology. The alternative profit change decomposition measures the activity effect (and each of its three components) along period t technology, and then measures the technical change effect with the shift in production technology at the input vector $x^{t+1}$. Proposition A1 below merges Propositions 1 - 4 in the text, using the new decomposition path. All unobserved quantity vectors are indicated in Figures 1 - 3. The price effect and the operating efficiency effect are unchanged by the new path. Proposition A2 below parallels Proposition 5 in the text, and shows how to calculate the unobserved quantity vectors ($y^A$, $y^C$, $y^F$, $y^G$, $x^H$).

**Proposition A1**: The **profit change** between period t and period t+1 decomposes as

$$[\pi^{t+1} - \pi^t] =$$

$$[(p^{t+1} - p^t)\times(y^{t+1}) - (w^{t+1} - w^t)\times(x^{t+1})] \qquad \textit{price effect}$$

$$+ [(p^t)\times(y^C - y^F)] \qquad \textit{technical change effect}$$

$$- [(p^t)\times(y^C - y^{t+1}) - (p^t)\times(y^A - y^t)] \qquad \textit{operating efficiency effect}$$

$$+ [(p^t)\times(y^F - y^G)] \qquad \textit{product mix effect}$$

$$- [(w^t)\times(x^{t+1} - x^H)] \qquad \textit{resource mix effect}$$

$$+ [(p^t)\times(y^G - y^A) - (w^t)\times(x^H - x^t)] \qquad \textit{scale effect}$$

**Proposition A2:** The unobserved quantity vectors ($y^A$, $y^C$, $y^F$, $y^G$, $x^H$) can be recovered from the observed quantity vectors ($x^t$,$y^t$) and ($x^{t+1}$,$y^{t+1}$) by means of

**(i)** $y^A = y^t/D_o^t(x^t,y^t)$

**(ii)** $y^C = y^{t+1}/D_o^{t+1}(x^{t+1},y^{t+1})$

**(iii)** $y^F = y^{t+1}/D_o^t(x^{t+1},y^{t+1})$

**(iv)** $y^G = y^t/D_o^t(x^{t+1},y^t)$

**(v)** $x^H = x^t/D_i^t(y^{t+1}/D_o^t(x^{t+1},y^t),x^t)$

TABLE 1. Summary Statistics for Spanish Commercial Banks, 1987 - 1994

| | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
|---|---|---|---|---|---|---|---|---|
| Average Operating Profit | 4178 | 4368 | 8574 | 8633 | 6347 | 4632 | 1896 | 3142 |
| (Y1) Average Value of Loans & Other Financial Investments (millions of pts) | 363554 | 340824 | 464403 | 531899 | 510950 | 674317 | 738764 | 703157 |
| P1 (%) | 12.00% | 11.90% | 12.73% | 13.20% | 12.94% | 12.16% | 11.43% | 8.86% |
| (Y2) Average Number of Deposits (thousands) | 532 | 442 | 519 | 494 | 383 | 475 | 409 | 337 |
| P2 (millions of pts) | 7.726 | 10.651 | 11.600 | 14.409 | 14.125 | 18.458 | 18.183 | 19.788 |
| (x1) Average Value of Deposits & Other Liabilities (millions of pts) | 380929 | 359024 | 474684 | 522017 | 489106 | 654492 | 716153 | 685618 |
| w1 (%) | 7.42% | 6.96% | 7.94% | 8.91% | 9.13% | 8.88% | 8.64% | 6.10% |
| (x2) Average Number of Employees | 2504 | 2144 | 2554 | 2476 | 2024 | 2460 | 2214 | 1848 |
| w2 (millions of pts) | 3.404 | 3.715 | 4.104 | 4.550 | 4.914 | 5.540 | 5.902 | 6.112 |
| (x3) Average Fixed Assets (millions of pts) | 8656 | 7653 | 11288 | 12377 | 13129 | 20314 | 19866 | 17785 |
| w3 (%) | 53.96% | 57.19% | 64.85% | 65.82% | 62.61% | 68.18% | 68.30% | 61.42% |
| Number of Commercial Banks | 59 | 58 | 57 | 61 | 59 | 60 | 56 | 50 |

## TABLE 2.
## Profit Change Decomposition for Spanish Commercial Banks, 1987 - 1994
### Average Results

| YEAR | Profit Change = | Productivity Effect + | Activity Effect + | Price Effect | Number |
|---|---|---|---|---|---|
| **1988 - 1987** | 1189 | 211 | 1217 | -238 | 53 |
| **1989 - 1988** | 2688 | 671 | 2021 | -4 | 50 |
| **1990 - 1989** | 279 | 1647 | 3357 | -4725 | 54 |
| **1991 - 1990** | -731 | 896 | 1699 | -3326 | 55 |
| **1992 - 1991** | -2386 | -317 | 831 | -2900 | 56 |
| **1993 - 1992** | -3147 | 1753 | 1108 | -6008 | 52 |
| **1994 - 1993** | 2998 | 103 | 1381 | 1514 | 45 |
| Average **1987 - 1994** | **127** | **709** | **1659** | **-2241** | **52** |

## TABLE 3.
## Productivity Effect Decomposition for Spanish Commercial Banks, 1987 - 1994
### Average Results

| YEAR | Productivity Effect = | Technical Change+ | Operating Efficiency |
|---|---|---|---|
| **1988 - 1987** | 211 | 318 | -107 |
| **1989 - 1988** | 671 | 1176 | -505 |
| **1990 - 1989** | 1647 | 1849 | -202 |
| **1991 - 1990** | 896 | 1054 | -158 |
| **1992 - 1991** | -317 | 116 | -433 |
| **1993 - 1992** | 1753 | 2021 | -268 |
| **1994 - 1993** | 103 | 223 | -120 |
| Average **1988 - 1994** | **709** | **965** | **-256** |

**TABLE 4.**
**Activity Effect Decomposition for Spanish Commercial Banks, 1987 - 1994**
**Average Results**

| YEAR | Activity Effect = | Product Mix Effect + | Resource Mix Effect + | Scale Effect |
|---|---|---|---|---|
| **1988 - 1987** | 1217 | 2600 | -3280 | 1897 |
| **1989 - 1988** | 2021 | 4374 | -4611 | 2258 |
| **1990 - 1989** | 3357 | 7203 | -6809 | 2962 |
| **1991 - 1990** | 1699 | 5951 | -8648 | 4395 |
| **1992 - 1991** | 831 | 5628 | -7549 | 2752 |
| **1993 - 1992** | 1108 | 10444 | -13189 | 3853 |
| **1994 - 1993** | 1381 | 5274 | -5795 | 1903 |
| Average **1987 - 1994** | **1659** | **5925** | **-7126** | **2860** |