CAN INSURERS PAY FOR THE "BIG ONE"? MEASURING THE CAPACITY OF THE INSURANCE MARKET TO RESPOND TO CATASTROPHIC LOSSES

By

J. David Cummins, Neil Doherty, and Anita Lo
Wharton School
University of Pennsylvania

June 24, 1999

Address Correspondence to: J. David Cummins
Wharton School
3641 Locust Walk
Philadelphia, PA 19104-6218

J. David Cummins	Neil Doherty	Anita Lo
Phone: 215-898-5644	Phone: 215-898-7652	Phone: 215-898-3589
Fax: 215-898-0310	Fax: 215-898-0310	Fax: 215-898-0310

e-mail: **cummins@wharton.upenn.edu** **doherty@wharton.upenn.edu** **anita77@wharton.upenn.edu**

Can Insurers Pay For The "Big One"?
Measuring The Capacity of the Insurance Market to Respond to Catastrophic Losses

J. David Cummins, Neil Doherty, and Anita Lo
June 24, 1999

Abstract

This paper presents a theoretical and empirical analysis of the capacity of the U.S. property-liability insurance industry to finance major catastrophic property losses. The topic is important because catastrophic events such as the Northridge earthquake and Hurricane Andrew have raised questions about the ability of the insurance industry to respond to the "Big One," usually defined as a hurricane or earthquake in the $100 billion range. At first glance, the U.S. property-liability insurance industry, with equity capital of more than $300 billion, should be able to sustain a loss of this magnitude. However, the reality could be different; depending on the distribution of damage and the spread of coverage as well as the correlations between insurer losses and industry losses. Thus, the prospect of a mega catastrophe brings the real threat of widespread insurance failures and unpaid insurance claims.

Our theoretical analysis takes as its starting point the well-known article by Borch (1962), which shows that the Pareto optimal result in a market characterized by risk averse insurers is for each insurer to hold a proportion of the "market portfolio" of insurance contracts. Each insurer pays a proportion of total industry losses; and the industry behaves as a single firm, paying 100 percent of losses up to the point where industry net premiums and equity are exhausted. Borch's theorem gives rise to a natural definition of industry capacity as the amount of industry resources that are deliverable conditional on an industry loss of a given size. In our theoretical analysis, we show that the necessary condition for industry capacity to be maximized is that all insurers hold a proportionate share of the industry underwriting portfolio. The sufficient condition for capacity maximization, given a level of total resources in the industry, is for all insurers to hold a net of reinsurance underwriting portfolio which is perfectly correlated with aggregate industry losses. Based on these theoretical results, we derive an option-like model of insurer responses to catastrophes, leading to an insurer response-function where the total payout, conditional on total industry losses, is a function of the industry and company expected losses, industry and company standard deviations of losses, company net worth, and the correlation between industry and company losses. The industry response function is obtained by summing the company response functions, giving the capacity of the industry to respond to losses of various magnitudes.

We utilize 1997 insurer financial statement data to estimate the capacity of the industry to respond to catastrophic losses. Two samples of insurers are utilized – a national sample, to measure the capacity of the industry as a whole to respond to a national event, and a Florida sample, to measure the capacity of the industry to respond to a Florida hurricane. The empirical analysis estimates the capacity of the industry to bear losses ranging from the expected value of loss up to a loss equal to total company resources. We develop a measure of industry *efficiency* equal to the difference between the loss that would be paid if the industry acts as a single firm and the actual estimated payment based on our option model.

The results indicate that national industry efficiency ranges from about 78 to 85 percent, based on catastrophe losses ranging from zero to $300 billion, and from 70 to 77 percent, based on catastrophe losses ranging from $200 to $300 billion. The industry has more than adequate capacity to pay for catastrophes of moderate size. E.g., based on both the national and Florida samples, the industry could pay at least 98.6 percent of a $20 billion catastrophe. For a catastrophe of $100 billion, the industry could pay at least 92.8 percent. However, even if most losses would be paid for an event of this magnitude, a significant number of insolvencies would occur, disrupting the normal functioning of the insurance market, not only for property insurance but also for other coverages.

We also compare the capacity of the industry to respond to catastrophic losses based on 1997 capitalization levels with its capacity based on 1991 capitalization levels. The comparison is motivated by the sharp increase in

capitalization following Hurricane Andrew and the Northridge earthquake. In 1991, the industry had $0.88 in equity capital per dollar of incurred losses, whereas in 1997 this ratio had increased to $1.56. Capacity results based on our model indicate a dramatic increase in capacity between 1991 and 1997. For a catastrophe of $100 billion, our lower bound estimate of industry capacity in 1991 is only 79.6 percent, based on the national sample, compared to 92.8 percent in 1997. For the Florida sample, we estimate that insurers could have paid at least 72.2 percent of a $100 billion catastrophe in 1991 and 89.7 percent in 1997. Thus, the industry is clearly much better capitalized now than it was prior to Andrew.

The results suggest that the gaps in catastrophic risk financing are presently not sufficient to justify Federal government intervention in private insurance markets in the form of Federally sponsored catastrophe reinsurance. However, even though the industry could adequately fund the “Big One,”doing so would disrupt the functioning of insurance markets and cause price increases for all types of property-liability insurance. Thus, it appears that there is still a gap in capacity that provides a role for privately and publicly traded catastrophic loss derivative contracts.

Can Insurers Pay For The "Big One"?
Measuring The Capacity of the Insurance Market to Respond to Catastrophic Losses

J. David Cummins, Neil Doherty, and Anita Lo

June 14, 1999

1. Introduction

Catastrophic events such as the Northridge earthquake and Hurricane Andrew each cost the insurance industry in excess of $10 billion. While most insured losses were paid, each event resulted in insurer insolvencies and illustrate the potential stress facing insurance markets. Andrew, which cost the insurance industry about $17 billion,[1] would have been much more severe had its path veered slightly to hit Miami. Moreover, scenarios constructed by catastrophe modelers suggest the feasibility of a $76 billion hurricane in Florida, a $21 billion Northeast hurricane, a $72 billion California earthquake and a $101 billion New Madrid earthquake.[2] At first glance, it might appear that the insurance industry would be available to pay for such mega catastrophes. The U.S. property liability insurance industry′s equity capital, is somewhat over $300 billion. This capital is potentially available to pay for losses which exceed the reserves (established for their payment from premiums). However, the reality would be different; depending on the distribution of damage and the spread of coverage, many insurers would become insolvent.[3]

[1]Source Property Claims Services of the American Insurance Services Group.

[2]These figures relate only to the insured damage. The total damage would be higher. For hurricane losses a substantial portion of total losses is likely to be insured. However, for earthquake losses. many properties are not insured and others carry high deductibles. Thus, for earthquake losses the total societal loss could be multiples of this estimate. These figures were produced in a study by Risk Management Solutions (2.5.95) though similar "ballpark" figures are being produced in other studies.

[3]Insurers can spread their liabilities to other insurers through reinsurance. In principle, the effects of catastrophes can be spread through the worldwide reinsurance market. In practice the available capacity of reinsurers is limited even though it has increased significantly since Hurricane Andrew. Although estimates vary, it seems clear that a substantial gap exists between existing reinsurance coverage and a catastrophic loss exceeding the $15-20 billion range. For example, SwissRe (1997) estimated that reinsurers would pay 39 percent of a $53 billion U.S. catastrophe loss, based on reinsurance cover in effect in 1997. The reinsurance brokerage firm Guy Carpenter estimates that the capacity of the reinsurance market to reinsure any one primary insurer was about $500 million in both 1997 and 1998 (Guy Carpenter, 1998). Renaissance Reinsurance Company estimates that reinsurers would pay about $14 billion from existing excess of loss reinsurance programs in each of four U.S. regions – the Northeast, the Southeast, the New Madrid region, and

Technically, this problem should be solved by the state operated insurance guaranty funds which re-allocate defaulted liabilities among solvent insurers. But these only operate within small limits,[4] and even this burden would stretch the already strained resources of surviving insurers. Thus, the prospect of a mega catastrophe brings the real threat of widespread insurance failures and unpaid insurance claims. Moreover, surviving insurers would be so depleted of surplus, and thus over-levered, they would have to reduce the future sale of all types of property-liability insurance causing price increases and severe availability problems.[5]

These scenarios have led both state and federal governments to contemplate legislative solutions involving the government as a reinsurer and directly enlisting capital markets as providers of catastrophe capital (Lewis and Murdock, 1996, Cummins, Lewis and Phillips, 1999). Both Florida and California have such proposals and the Natural Disaster Protection Act was introduced in Congress in 1993 with similar provisions. Moreover, the vulnerability of insurance markets had led to financial market innovations such as the catastrophe options traded on the Chicago Board of Trade. New instruments have appeared such as CAT Bonds in which borrowers contract for some degree of debt forgiveness in the event of a predefined catastrophe. Another innovative instrument is the CatEPut in which re-capitalization can be achieved after a catastrophe by the firm's exercising a put option on its own stock. Also, in the absence of adequate reinsurance, insurers have sometimes swapped their catastrophe exposures.

In this paper, we conduct a theoretical and empirical analysis of the capacity of the insurance industry to respond to catastrophic events. Given appropriate technical (weather, seismic, etc) data, plus descriptions of insured properties for each insurer, one can estimate an insurer response for any given event such as a force 5 hurricane hitting Miami or an 8.2. earthquake in San Francisco. Such scenario analysis is carried out by modeling firms such as Applied Insurance Research and Risk Management Solutions. However, there is a very large number (approaching infinite) of potential catastrophe scenarios, and the data demands for conducting such an analysis for the entire

---

California (letter from William Riker, President and COO, Renaissance Reinsurance Ltd., June 11, 1999).

[4] Guaranty funds limit the amount paid for any given loss, typically to $300,000, as well as limiting the annual assessment against solvent insurers to 2 to 3 percent of premiums.

[5] See Froot, Scharfstein and Stein (1993) and Cummins and Danzon (1997).

insurance industry are enormous. Moreover, while such scenarios are valuable for planning at the firm level, they provide too much detail for assessing the efficiency of the insurance market in spreading risk. Rather, we seek a more general response function. We estimate the distributional characteristics of catastrophic losses and allocate such losses to individual insurers by use of correlations and financial data. The result is a function that defines the estimated deliverable insurance payments conditional on any given size of aggregate catastrophic loss. By default, it also estimates the capital that will be lost through insurer insolvencies.

Such a measure of capacity rests on two broad components; size and diversification – how much equity or "surplus" is available and how effectively the riskiness of insurance losses is spread though the insurance market. The traditional instrument to spread risk between insurers is reinsurance. By buying and selling "options" on their portfolios with each other, and to specialized reinsurers, insurers can change the risk characteristics of their portfolios. In a paper that anticipated the capital asset pricing model, Borch (1962), showed that the value maximizing trades would leave all insurers holding net of reinsurance portfolios defined solely on the market aggregate loss and that insurance would be priced solely on the correlation with this aggregate portfolio. We show that the distribution of insurance liabilities which minimizes insolvencies, and thereby maximizes payments to policyholders, is similar to Borch′s equilibrium. However, this structure also provides a framework for measuring the available capacity of the industry to respond to major catastrophes.

The paper is organized as follows: Section 2 sets forth our theory of industry capacity and derives the option-like model used in our capacity estimation. Section 3 discusses sample selection and our empirical approach to estimating capacity. Section 4 presents the results, and section 5 concludes.

2. Diversification And The Mutuality Principle

In this section, we develop a theoretical model of capacity in an insurance market. We begin by examining a baseline case in which the liabilities in an insurance market are distributed amongst insurers so as to maximize payouts to policyholders for any given loss scenario. The baseline case establishes a basic relationship between the capacity of the insurance industry to respond to catastrophic loss experience and the correlation structure of its liabilities. We then derive a measure of capacity which is parameterized by these correlations together with other firm and market features.

A Definition of Insurance Capacity

We examine a baseline case in which the liabilities in an insurance market are distributed amongst insurers so as to maximize payouts to policyholders for any loss scenario. This base case is useful for defining industry capacity and also provides a yardstick for measuring capacity. In the baseline case, insolvencies will be minimized for any given level of industry losses and thus actual payments to policyholders will be maximized.

It is well known that in a market in which risk bearing is costly to firms but where transacting between firms is costless, the Pareto optimal risk sharing arrangement is one in which the industry "mutualizes" its risk in the sense that all insurers hold the same net (after reinsurance) liability portfolio. This result, due to Borch (1962), is identical to (and preceded) the capital asset pricing model. According to Borch, the Pareto optimal reinsurance arrangement is one in which each insurer holds a net (after reinsurance) portfolio which is a proportionate claim on total insured losses, L. This result is equivalent to the CAPM proposition that each investor will hold the market portfolio. The implication is that all insurers′ portfolios are perfectly correlated after reinsurance transactions have been exploited. After all possibilities for diversification through reinsurance are exhausted, insurers will hold the same loss portfolio though the scale may differ. The aggregate loss for the market is $\sum L_i \equiv L$, where L_i = the loss sustained by insurer i. The riskiness of the aggregate portfolio will depend on the total number of individual policies insured, "n", and on their correlations. If the number of policyholders is very large and the policy correlations are low then, by the law of large numbers, L will have little risk ($\sigma(L/n) \rightarrow 0$ as "n"$\rightarrow \infty$, where $\sigma(L/n)$ = the standard deviation of average losses per policy). But with small "n" and/or high correlation among insured losses, L will have higher risk.

To address the implications of limited liability, first consider the terminal value of equity, T_i, of an insurer, i, in a simple one period model:

(1) $T_i = MAX\{(P_i + Q^o_i)(1+r) - \alpha_i L; 0\}$

where Q^0_i is opening equity or "surplus" for insurer i, P_i is premium income net of expenses, and r is the rate of return on investments. Insurer i is assumed to hold a proportionate share α_i of the market insurance portfolio so that its losses $L_i = \alpha_i L$. For simplicity, assume that the market is competitive, thus $P_i = E(L_i)/(1+r)$. Denoting Q_i

$=Q^o_i(1+r)$, terminal equity is re-stated as:

(1') $T_i = MAX\{E(L_i) - L_i + Q_i;\ 0\}$

Now consider the implications of limited liability for policyholders. The amount which insurer "i" can pay to policyholders, L_i^p, is the minimum of the face value of its liability or its financial resources which, in this model are the sum of equity and net premiums $Q_i + E(L)$, i.e.,

(2) $L_i^p = MIN\{L_i\,;\ Q_i + E(L_i)\}$

If there is a bad draw from the loss distribution, i.e., a catastrophic loss, the ability of the insurer to pay the unexpected loss L_i-$E(L_i)$ depends on the surplus Q. If we scale up this problem, then the ability of the market to respond to unexpected losses depends on the total industry surplus, but also on how the liabilities and surplus are distributed across insurers. We will use this concept to define and measure market capacity.

If we compare this limited liability world with Borch's equilibrium, there is an apparently stark contrast. In Borch's world, insurers are risk averse and will gain from risk sharing through reinsurance transactions. In our limited liability model, insurers own a put option on the value $E(L_i)+Q_i-L_i$ where the striking price is $E(L_i)+Q_i$ and the value of this option will increase as variance of the underlying asset (in this case the loss portfolio) increases. Thus, apparently, insurers would not engage in risk reducing reinsurance transactions. We can add more structure to resolve this difference by allowing premium rates to depend on insurer risk.[6] This additional structure is not necessary for our present task, but it does focus our attention on what the payouts to policyholders would be when insurers are perfectly diversified as shown by Borch.

Consider a Borch equilibrium in which each insurer, "i" holds a share α_i of L and assume that each insurer's surplus is scaled to its share of aggregate loss. The first implication is that the aggregate terminal equity of insurers will be the difference between the unexpected industry loss E(L)-L, and the industry equity $\sum Q_i$ as shown in equation (3a) below. The second implication is that the industry′s whole surplus will be available to meet unexpected losses. Thus, the amount of aggregate losses that will be paid to policyholders, L^p, will be the minimum of the face value of

[6]See Doherty and Tinic (1982) and Cummins and Danzon (1997).

losses L and the industry's total resources E(L)+Q , as shown in equation (3b).

$$\text{(3a)} \qquad \sum_{i=1}^{N} T_i = MAX\{ \sum_{i=1}^{N} [\alpha_i E(L) + Q_i - \alpha_i L]; 0\} = MAX\{ E(L) + \sum_{i=1}^{N} Q_i - L ; 0\}$$

$$\text{(3b)} \qquad \sum_{i=1}^{N} L_i^p = MIN\{ L ; E(L) + \sum_{i=1}^{N} Q_i \}$$

where N = the total number of insurers in the market. Currently, the U.S. property-liability insurance industry′s equity capital is about $300 billion. If our model applied to the industry, the entire amount of the equity capital would be available to pay unexpected losses. In effect, with perfect diversification, the industry acts as a single firm. No one firm would become insolvent until the entire industry capital is exhausted and, at this point, all firms would simultaneously become insolvent. This equilibrium distributes industry liabilities and resources in a way that maximizes payouts to policyholders.

> *Definition: For any configuration of losses for which insurers are liable, the capacity of the insurance market is the proportion of those liabilities that is deliverable given the financial resources of firms on whom the losses fall and given all arrangements (such as reinsurance, guarantee funds, etc) for re-allocating those losses among insurers.*

In the equilibrium considered, all industry surplus would be accessible by policyholders.

Conditions for Capacity Maximization

Consider each insurer′s aggregate loss as the sum of its catastrophe exposure and its idiosyncratic risk. Part of the individual insurer loss, d_i, is idiosyncratic and diversifiable; i.e., COV(d_i, d_j = 0 for all $i \neq j$). The remaining part of the insurer′s loss is catastrophe risk in the sense that all insurers are exposed to highly correlated losses, L_U, from events such as hurricanes and earthquakes. The proportion of the total pool of catastrophe losses written by insurer "i" is c_i. Thus, the loss of insurer i is:

$$\text{(4)} \qquad L_i = c_i L_U + d_i$$

Given that $\sum L_i$ must equal the aggregate industry losses, $L \equiv L_U + D$; (where $D \equiv \sum_i d_i$ is the total industry diversifiable losses), then $\sum c_i = 1$. The essential characteristic of diversifiable risk is that it will tend to zero if a large enough number of policies is insured. To provide a rationale for a reinsurance market, we assume that any individual insurer holding n_i policies is insufficiently diversified to secure this risk elimination, but the total insurance market having $\sum_i n_i \equiv n$ policies does effectively eliminate risk, i.e.,

$$(5) \qquad \sigma\left(\frac{D}{n}\right) = 0\,; \quad \sigma\left(\frac{d_i}{n_i}\right) \neq 0\,; \quad \sigma\left(\frac{c_i L_U}{n_i}\right) \neq 0\,; \quad \sigma\left(\frac{L_U}{n}\right) \neq 0$$

The first expression in (5) says that diversifiable risk can be substantially eliminated by diversification across the marketplace. The second expression says that each individual insurer′s endowment of potentially diversifiable exposures is not sufficient to eliminate this risk (i.e., it does not have sufficient policies to exploit the law of large numbers). The third and fourth expressions in (5) assert that the risk of L_U is not diversifiable (i.e., losses are positively correlated). The third expression is particularly important in providing a rationale for insurance. By definition of d_i and c_iL_u, the former can be reduced through further risk spreading whereas the latter cannot.

We now develop the following necessary condition for optimal risk sharing behavior:

PROPOSITION: A necessary condition for the average industry capacity per policyholder, $\sum_i E(L_i^p/n)$, *to be maximized is that all firms hold a net of reinsurance portfolio which is proportional to* L_U *and D.*

The proposition requires that all insurers hold portfolios of the form $\alpha_i L = c_i L_U + k_i D$ where α_i, c_i, and k_i are firm specific constants.[7] Suppose that this were not true, then at least one insurer would hold a portfolio containing some idiosyncratic risk; i.e., $\alpha_i L_U + d_i$, where $d_i \neq k_i D$. Since $D \equiv \sum_i d_i$, the existence of one insurer holding $c_i L_U + d_i$ implies that all other insurers must hold in total

$$(1 - c_i) L_U + D - d_i = \sum_{j \neq i}^{N} c_j L_U + D - d_i$$

[7]It will be noticed that the reinsurance structure that maximizes industry capacity ($\alpha_i L = c_i L_U + k_i D$ for all i) is of similar structure to the Pareto optimal reinsurance market identified by Borch (1962). The similarity is more pronounced when it is noticed that, since D is diversifiable, the value of k_i makes little difference to the availability of surplus to pay catastrophic losses. Thus, one can consider the special case in which $\alpha_i = k_i$. However, even for this special case, our result and that of Borch are not necessary identical. While, in both results, insurers′ loss portfolios are defined solely on L, we rely on a maximization of aggregate dollar surplus whereas Borch relied on expected utility maximizing trade between risk averse insurers. The non-linearity in our results comes from the truncating effects of insolvency whereas non-linearity in Borch′s reinsurance structure comes from the parameters of the various insurers utility functions.

which cannot be of the form

$$\sum_{j \neq i}^{N} c_j L_U + \sum_{j \neq i}^{N} k_j D$$

since $d_i \neq k_i D$ and $D \equiv \sum_i d_i$.[8] Thus, at least one other insurer must hold a portfolio of the form $c_j L_U + d_j$ where $d_j \neq k_j D$. Of the universe of insurers "M" we define a subset "m_1" having such "undiversified" portfolios $\alpha_i L_U + d_i$ and subset "m_2" having "diversified" portfolios of the form $\alpha_j L_U + k_j D$. Since

$$(1 - \sum_{j \in m_2} k_j) D = \sum_{i \in m_1} d_i$$

then the following mutual exchange is possible. All type m_1 insurers pool their diversifiable risk which leads to an aggregate m_1 diversifiable liability of $(1-\sum_{j \in m2} k_j)D$. Now define a set of weights k_i' and apportion this aggregate liability over m_1 insurers such that each assumes a liability of :

$$k_i' (1 - \sum_{j \in m_2} k_j) D = k_i D \qquad since\ k_i' \equiv k_i \left(\frac{1}{1 - \sum_{j \in m_2} k_j}\right) \quad and \quad \sum_{j \in m_1} k_i' = 1$$

These conditions ensure that $\sum k_i = 1$ (i.e. that diversifiable risk D is fully allocated over all insurers). Since the only requirement placed on k_i' is that it sum to unity, these weights can be chosen such that the $E(d_i) = E(k_i D)$. Thus, these transactions will leave all m_2 insurers unaffected and will leave the expected face value of liability of all m_1 unchanged. However, since $\sigma(d_i/n_i) > 0$; and $\sigma(k_i D/n_i) \Rightarrow 0$, these transactions are mean preserving, and risk reducing, for all m_1 insurers. Now since the payable loss on any insurer is a short position in a put option, its value will increase as its standard deviation is reduced. Consequently, these transactions will leave $E(L_i^p/n)$, where L_i^p is defined by (3b), unchanged for all m_2 insurers but increased for all m_1 insurers. As a result, aggregate available

---

[8]To see this, consider that all other insurers did hold portfolios of the form $\sum_{j \neq i} \alpha_j L_U + \sum_{j \neq i} k_j D$. Thus the total of the diversifiable risk portfolios of all insurers would be:

(a). $D = \sum_{j \neq i} k_j D + d_i$

This can be re-stated as

(b). $D = \sum_{j \neq i} k_j D + k_i D - k_i D + d_i = \sum_j k_j D + (d_i - k_i D)$

which is equal to

(c). $D = D + (d_i - k_i D)$

since $\sum_j k_j = 1$. However, since $d_i \neq k_i D$; then (c), and therefore (a), is contradicted.

industry capacity $\sum_i E(L_i^p /n)$ will be increased. Q.E.D.

The proposition shows the necessary conditions for capacity maximization. The sufficient conditions concern the relationship between the liability allocation, α_i, and the and the distribution of surplus, Q_i, across insurers. The effect of surplus will become important in the capacity measures derived in the next section.

COROLLARY: *When the necessary conditions for maximization of capacity per policyholder $\sum_i E(L_i^p /n)$ are satisfied, all insurers will hold net of reinsurance portfolios L_i that are perfectly correlated with aggregate industry losses, L.*

Note that COV $(L_i ; L) = E\{[c_i(L_U - E(L_U)) + (d_i - E(d_i))][L - E(L)]\}$ which can be simplified to $E\{[c_i(L_U - E(L_U))][L - E(L)]\}$ since d_i is independent of L by assumption. Using COV(D,L) = 0 and $L = L_U+D$, we can write; $\sigma^2(L) = E\{[L_U - E(L_U)][L - E(L)]\}$. Thus, COV $(L_i ; L) = c_i \sigma^2(L)$. Proof of the corollary follows immediately from the proposition noting that COV $(L_i ; L) = c_i \sigma^2(L)$ and that c_i and k_i are constants. Q.E.D.

The corollary shows that each insurer must hold a net portfolio which is perfectly correlated with the aggregate insurable loss L to maximize capacity. This will provide a yardstick for measuring capacity. Since $\alpha_i L = c_i L_U + k_i D$ maximizes capacity for a given initial industry surplus Q, and since this result is characterized by perfect correlation between all L_i and L, then it seems that the actual correlations will provide a measure of capacity utilization.

Various frictions can frustrate the conditions described in the proposition and corollary. In addition to factors that limit firm size, reinsurance and other insurer hedges are costly. Froot and O'Connell 1996, recently estimated the cost of catastrophe reinsurance from the complete set of contracts brokered by the largest reinsurance broker. The transaction cost, (Price-Expected Loss)/Expected loss), ranges between about 10% and 140% from 1970-1995. In the last decade of the series, the average transaction cost is about 65%. Several explanations can be given for this high cost including diverging estimates of expected losses, moral hazard and excessive rent taking. Another explanation for incomplete diversification lies in the prospect that shareholders may seek to expropriate wealth from policyholders by choosing a high risk financial structure (Myers, 1977, and Doherty and Tinic, 1982). This expropriation will be mitigated by reputation effects and where the policyholders and/or their agents can monitor the financial condition and reinsurance purchases of their insurers. We now examine the relationship between capacity, correlations between insurer loss distributions, and the financial structure of insurers.

Correlations And Capacity Utilization For a Given Catastrophic Loss

Our task is to estimate the ability of the industry to respond to an abnormal loss experience defined by equation (3b). This is the industry response conditional on industry losses of any given size “L”.

The response function is illustrated in Figure 1. The horizontal axis measures possible values for aggregate insurance industry losses, and the vertical axis measures the *expected* payout of all firms combined. Consider just two possible loss scenarios: first, a California earthquake that causes an industry loss of $30 billion over and above the expected loss E(L) and, second, a combination of a Florida hurricane of $20 billion and automobile losses that are $10 billion above expected. Both scenarios lead to industry losses that are $30 billion above expected value (denoted E(L)+30). But the scenarios would impact different insurers and could lead to different numbers of insolvencies depending on the distribution of coverage across insurers. For example, the expected payout in the first scenario might be “W” which is very low because much of the California earthquake coverage is from local insurers that are poorly diversified and poorly capitalized. However, the second scenario might be spread more evenly over firms and the payout is shown as “Y”. Points W and Y are the conditional responses which are described below in equations (7) through (9). These are only two of many potential configurations that could result in industry losses of $30 billion above expected value. The average of all possible payouts for all feasible scenarios which sum to $30 billion above expected loss is denoted “X”. This value, X, is the conditional response, i.e., the expected payout of the industry conditional on an industry loss of E(L) + $30 billion. The locus of all such conditional payouts is the response function which is shown as OZ. Notice that OZ lies at or below the 45° line and, we postulate, will diverge from the 45° line as loss realizations increase. The divergence implies that insolvencies will increase dis-proportionately with losses as more and more insurers are stressed and that failures are passed through the market via reinsurance thus causing “knock on” insolvencies.

It is useful to start with the average surplus per policy available to pay unexpected claims of insurer “i”:

(6) $$E(\frac{T_i}{n_i}) = (\frac{1}{n_i})\int_0^{Z_i}[E(L_i) + Q_i - L_i]f(L_i)\,dL_i$$

$\int Z_i$

where $Z_i = E(L_i)+Q_i$. To derive the conditional response function note that the aggregate industry terminal equity, conditional on, industry losses being L, is:

$$(7) \qquad \sum_{i=1}^{N} E(T_i|L) = \sum_{i=1}^{N} \int_0^{Z_i} [E(L_i) + Q_i - L_i] f(L_i|L)\, dL_i$$

This value is shown in Figure 1 as the distance between $E(L)+\sum Q_i$ and the response function OZ. Thus, the response function can be defined as $R|L \equiv E(L)+\sum Q_i - \sum_i E(T_i|L)$.

To estimate the response function, it is necessary to make distributional assumptions about L. Using the normal distribution and using the properties of conditional moments, the response function becomes:

$$(8) \qquad E(T_i\,|\,Q_{i0}, L) = (P_i + Q_{i0} - \mu_{L_i|L})\, N[\frac{P_i + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}] + \sigma_{L_i|L}\, \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{P_i + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}})^2}$$

$$where \quad \mu_{L_i|L} = \mu_i + \frac{\rho_i\, \sigma_i}{\sigma_L}(L - \mu_L) \quad , and \quad \sigma^2_{L_i|L} = \sigma_i(1 - \rho_i^2)$$

where ρ_i is the correlation coefficient between L_i and L and μ is used to denote expectation. Not surprisingly, this formulation resembles option pricing models. The response function is:

$$(9) \qquad R_i|L = P_i + Q_{i0} - E(T_i\,|\,Q_{i0},L) = (P_i + Q_{i0})\, N(-C_i) + \mu_{L_i|L}\, N(C_i) - \sigma_{L_i|L}\, n(C_i)$$

$$where \quad C_i = \frac{P_i + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}} \quad ,$$

$N(\cdot)$ = the standard normal distribution function, and $n(\cdot)$ = the standard normal density function. Note that $R_i|L = f(E(L_i), E(L), \sigma(L_i), \sigma(L), r_i, Q_i : L)$. Thus, we can measure the capacity utilization of the industry for any industry loss L, as a function of two industry variables $\{E(L), \sigma(L)\}$ and four firm variables $\{E(L_i), \sigma(L_i), r_i, Q_i\}$.

Alternatively, the response function can be estimated under the assumption that firm and industry losses are jointly lognormal, leading to the following formulation:[9]

$$(10) \qquad E(T_i \mid Q_{i0}, L) = (P_i + Q_{i0})\, N(D_{1i}) - e^{D_{2i}} N\left(D_{1i} - \xi_i \sqrt{1-\gamma_i^2}\right)$$

$$where \quad D_{1i} = \frac{\ln(P + Q_{i0}) - \nu_i - \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L)}{\xi_i \sqrt{1-\gamma_i^2}} \quad , \ and \ D_{2i} = \nu_i + \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L) + \frac{\xi_i^2 (1-\gamma_i^2)}{2}$$

The parameters ν_i and ν_L are lognormal drift parameters and ξ_i and ξ_L are lognormal risk parameters, in both cases for insurer i and the industry (L), respectively. The parameter γ_i is the correlation coefficient between ln(L) and ln(L_i). In the lognormal case the response function is:

$$(11) \qquad R_i|L = P_i + Q_{i0} - E(T_i \mid Q_{i0}, L) = (P_i + Q_{i0})\, N(-D_{1i}) + e^{D_{2i}} N\left(D_{1i} - \xi_i \sqrt{1-\gamma_i^2}\right)$$

Comparative statics analysis of the response functions (9) and (11) reveals that both are decreasing in σ_i and increasing in ρ_i , i.e., industry capacity is inversely related to σ_i and directly related to ρ_i .

3. Measuring the Capacity of the U.S. Insurance Industry

In this section we develop estimates of response functions for the U.S. property-liability insurance industry. We do this by selecting samples of insurers and estimating the parameters of equations (9) and (11). The response functions are then calculated for various values of L, the total industry loss. The overall objective of the analysis is to determine the ability of the U.S. insurance industry to respond to catastrophic losses and to measure the efficiency of the industry in spreading risk across the market. This section discusses the technique we use to measure industry efficiency as well as sample selection and parameter estimation. The results are presented in section 4.

Measuring Industry Efficiency

Recall from the preceding discussion that in a fully efficient insurance market, the industry responds to losses

[9] Equations (8) and (9) are derived in the Appendix.

as if it were a single firm. The response function for an efficient market is thus given by the line 0AC in Figure 1, i.e., an efficient insurance industry would pay 100 percent of all losses up to the point when all industry resources are exhausted. Thus, one measure of market inefficiency is the magnitude of the wedge between the fully efficient response function and the actual industry response function represented by the line 0Z in Figure 1. We measure the area of the wedge bordered by line segment 0A, the response curve 0Z, and the dotted vertical line segment originating at the point $V = E(L)+\sum Q_i$ on the horizontal axis. The ratio of the area under the response curve to the area of the triangle 0AV is our primary measure of market efficiency. This area is equal to the area of the triangle minus the area of the wedge divided by the area of the triangle. For a fully efficient industry, market efficiency would equal 1; and for an inefficient industry, market efficiency is between 1 and 0. We also consider other measures of industry performance, including the percentage of the total losses that would be paid by the industry for catastrophes of various magnitudes and the number of insolvencies that result.

Sample Selection and Modeling Approach

The data for the study are taken from the regulatory annual statements filed by insurers with the National Association of Insurance Commissioners (NAIC). Our efficiency estimates are for the most recent report year currently available, 1997. To estimate parameters, we use data from the period, 1983-1997, providing fifteen annual observations on the companies in the sample. Insurance prices and profits have been shown to be cyclical, with a cycle period between six and seven years (Cummins and Outreville, 1987). The fifteen year sample period thus gives us approximately two complete underwriting cycles. We decided not to extend the sample prior to 1983 because the number of insurers for which we have complete time series would have been reduced significantly by including earlier years. Although companies present in the data base for 1997 but not for earlier years are included in our capacity estimation, the companies present in the sample for the entire sample period (the *full-time series (FTS) companies)* are important because they are used to estimate regression models to obtain the parameters of the companies that are not present for the entire sample period (see below). To obtain reliable parameter estimates, it is important to include as many companies as possible in the FTS regressions.

Two samples of insurers were selected – a national sample and a Florida sample — to represent the capacity of the industry to respond to national catastrophes and to Florida catastrophes, respectively. Use of the national sample assumes that the total reserves and equity capital of the industry are potentially available to pay catastrophic losses, while the use of the Florida sample assumes that the total resources of companies operating in Florida are potentially available to pay the costs of a Florida catastrophe. Both the Florida and national estimates represent upper bound estimates of the industry's ability to respond to catastrophe claims. In both cases, the total resources of all insurers in the respective samples are assumed to be available to pay catastrophic loss claims, even though some insurers do not write policies likely to be triggered by a catastrophe. E.g., a company specializing in commercial liability insurance is not exposed to the property losses caused by a catastrophe and hence its resources would not be called upon to fund catastrophic loss claims. Moreover, the analysis based on the national sample assumes that all of the reserves and equity capital of the industry would be available to fund catastrophic losses, even though most catastrophes that are currently being projected by insurers and modeling firms are localized in one or a few states and only a subset of insurer are licensed in any given state. Thus, the actual amount of money that would be forthcoming from the insurance industry to fund any given catastrophic loss is expected to be smaller than projected in our analysis.

In selecting both the national and Florida samples, our objective was to maximize the number of companies that could meaningfully be included in the analysis. Thus, the screening criteria applied in selecting the sample focused primarily on eliminating insurers that were not viable operating entities in 1997. Thus, we excluded companies from the sample that were experiencing severe financial difficulties or were in runoff mode, i.e., not actively participating in the market.[10] To be included in the Florida sample, insurers also were required to have positive losses in Florida in 1997.[11]

---

[10]Insurers are required to report financial data to the NAIC even if they are undergoing severe financial difficulties, are inactive, or are in "runoff mode." An insurer in runoff mode is engaged in settling existing claims but not writing or renewing policies currently. Such an insurer would not be "on the risk" for projected catastrophes.

[11]Losses rather than premiums were used for the Florida screen because an insurer can remain liable for loss payments in a given year even if it writes no premiums in that year because coverage is provided by policies written in the preceding year (and on which premiums had been paid in the preceding year) that had

Ownership structure in the insurance industry also is likely to have an effect on market capacity. Many insurance firms are organized as *insurance groups*, consisting of several individual companies under common ownership. Under U.S. corporation law, the owners of the group hold a valuable option — the option to allow a financially troubled subsidiary to fail. The claimants against the insolvent subsidiary cannot reach the assets of other insurers in the group unless they succeed in "piercing the corporate veil," which usually requires showing that the owners engaged in fraud or some other abnormal activity (Easterbrook and Fischel, 1985). Although the owners may decide to rescue a failing subsidiary to protect reputational or franchise value, they are under no legal obligation to do so. The default option may have particularly severe implications for the financing of catastrophic risk because the probability that franchise value will exceed the costs of a bailout is likely to be inversely proportional to the magnitude of the event. Thus, parents may choose to bail out failing subsidiaries for small catastrophes but not for large ones.

To allow for the potential impact of ownership structure on capacity, we conduct the analysis separately on the basis of two alternative assumptions about the effects of insurance groups on capacity. The first analysis is based on the assumption that the full resources of the group are available to support losses arising from any subsidiary of the group, i.e., the group is considered to act as a single firm. This is equivalent to assuming that groups *always* rescue failing subsidiaries. The second analysis ignores group affiliations entirely and conducts the analysis as if the members of groups are freestanding, unaffiliated companies. This analysis implicitly assumes that groups *never* bail our failing subsidiaries. The two analyses can be viewed as giving upper and lower bounds on the capacity of the industry to pay claims.

The losses used in estimating capacity are net losses incurred, defined as direct losses incurred plus losses due to reinsurance assumed minus losses due to reinsurance ceded. Direct losses incurred are defined as losses paid or owed directly to policyholders, while net losses incurred reflect the netting out of reinsurance transactions. Because reinsurance is expected to reduce the standard deviations of individual insurer losses and increase the loss correlations

---

not yet expired at the beginning of the current year. The results would be nearly identical based on a premium screen.

among insurers, our analysis implicitly takes into account the effects of reinsurance on capacity.

Parameter Estimation

To estimate capacity for the industry in 1997, we included in the sample all of the companies reported on by the NAIC in 1997 that met our screening criteria for operating viability. However, only a subset of these companies are in the NAIC data base for the full time period covered by the study (1983-1997). Accordingly, we adopt a three-stage procedure for estimating parameters. At the first stage, we estimate parameters for the companies that have data for the full time period 1983-1997. We refer to this set of companies as the *full-time series (FTS)* sample. Two sets of parameters are estimated – *raw* parameter estimates calculated directly from the FTS data, and *detrended* estimates based on the residuals from time trend regressions. The reason for computing the detrended estimators is that property-liability insurance losses are subject to a strong positive time trend. The raw estimates of the standard deviation of losses, for example, will capture trend-related growth in losses across years as volatility. However, the trend is highly predictable and insurers can easily plan for it by increasing premiums each year. Differences in losses across years due to this trend effect thus are not unanticipated loss fluctuations and probably should not be included when measuring the effect of catastrophes and other types of random shocks on insurance market capacity.[12] By measuring capacity using both the raw and detrended parameters, we avoid any possible biases that may arise from relying on only one set of parameter estimates that is subject to time trend bias..

To define the raw and detrended parameters more precisely, let L_{it} = the observed losses of company i in year t and let $L_t = \sum_i L_{it}$ = total industry losses in year t. The raw standard deviations are obtained using the following formulas:

(12) $$\hat{\sigma}_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(L_{it} - \bar{L}_i)^2 \qquad and \qquad \hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(L_t - \bar{L})^2$$

where $\hat{\sigma}_i^2$ = the estimator of the standard deviation of losses for company i, $\hat{\sigma}^2$ = the estimator of the standard

[12]It might be argued that insurers also could set aside reserves sufficient to pay for large catastrophic losses on the grounds that the probable maximum loss amounts due to catastrophes are reasonably predictable even though the time of these losses is unpredictable. However, as Jaffee and Russell (1997) point out, accumulation of large catastrophe reserves is not possible in practice due to legal, regulatory, accounting, and tax rules that constrain the ability of insurers to accumulate reserves for events that have not yet taken place.

deviation of losses for the industry , $\bar{L}_i = \frac{1}{T} \Sigma_t L_{it}$ and $\bar{L} = \frac{1}{T} \Sigma_t L_t$. The correlation coefficient between company i's losses and the industry losses is estimated using the following formula:

$$\hat{\rho}_i = \frac{(\frac{1}{T-1}) \sum_{t=1}^{T} (L_{it} - \bar{L}_i)(L_t - \bar{L})}{\hat{\sigma}_i \hat{\sigma}} \tag{13}$$

For the lognormal case, estimates of ξ_i and ξ_L are obtained from (12) and the estimate of γ_i is obtained from (13), with the quantities L_{it} and L_t replaced by $\ln(L_{it})$ and $\ln(L_t)$, respectively.

To obtain the detrended parameter estimates, we first conduct the following regressions for the normal and lognormal cases, respectively:

$$\begin{aligned} L_{it} &= \alpha_{0i} + \alpha_{1i} t + \epsilon_{it} \\ L_t &= \alpha_0 + \alpha_1 t + \epsilon_t \end{aligned} \tag{14}$$

$$\begin{aligned} ln(L_{it}) &= \beta_{0i} + \beta_{1i} t + \omega_{it} \\ ln(L_t) &= \beta_0 + \beta_1 t + \omega_t \end{aligned} \tag{15}$$

Detrended estimates of $\hat{\sigma}_i^2$ and $\hat{\sigma}^2$ are obtained by applying the formulas in (12) to the estimated values of residuals ϵ_{it} and ϵ_t, respectively, from (14); and the detrended estimate of ρ_i by applying (13) to the estimated residual series ϵ_{it} and ϵ_t from (14). Detrended parameter estimates for the lognormal case are obtained similarly, using the estimated residuals from (15).

The final parameters to be estimated are the means, μ_i and μ_L , for the normal case, and the location parameters, ν_i and ν_L , for the lognormal case. Because we are estimating capacity for 1997, we set μ_i and μ_L equal, respectively, to company i's and the total industry losses incurred for 1997. This implicitly assumes that the companies' net premiums for the year are equal to incurred losses. Although this is not precisely correct, it is a good approximation, especially in view of the fact that there were no major catastrophes during the year. As a robustness check, we also used fitted values from the regression time trend lines for 1997. The results were virtually the same using the actual and fitted losses to represent μ_i and μ_L. The lognormal parameters were estimated so that application of the formula for the mean of the lognormal distribution would reproduce actual losses for 1997. E.g., for company i, $\hat{\nu}_i = \ln(L_i) - \hat{\sigma}_i^2/2$. Again, using the trended loss value for 1997 produced virtually identical results.

In the <u>second stage</u> of our parameter estimation procedure we estimate regression models using the FTS sample with the estimated parameters as dependent variables and company financial characteristics as independent variables. The rationale for the regression analysis is twofold: First, the estimated parameter values for some insurers are likely to reflect non-recurring financial shocks leading to unusually high or low estimated parameters. Such parameters are likely to be non-representative of the actual parameters affecting these insurers in future periods. By using the fitted values of parameters from the regression models in place of the parameter values estimated in stage one of the analysis, we are thus able to smooth the parameter series by tampering the extreme observations. The second, and equally important, reason for conducting the regression analysis is that the regression models can be used to provide parameter estimates for firms that are not in the data base for a sufficiently long period to permit the reliable estimation of stage one parameter values. Parameters for these companies can be estimated by inserting their 1997 financial data into the regression models. Estimation of parameters for this set of companies (called *non-full time series (NFTS) companies*), by obtaining fitted values from the regression models, is the <u>third stage</u> in the parameter estimation process. By conducting the third stage of the analysis we are able to include the maximum number of insurers in our 1997 sample and thus to obtain a comprehensive estimate of industry capacity.

4. Empirical Results

The capacity estimation process was conducted both for national companies and Florida companies. The estimation proceeds by calculating the response function using equation (9) for the normal case and equation (11) for the lognormal case. The response functions give the expected payout for each insurer as a function of its parameters and the total industry loss L. By varying L, we generate expected payments for each company for a range of industry losses, starting with a value of L approximately equal to industry expected losses (nationally and in Florida, respectively) and increasing to the point where L is equal to total industry resources. In addition to the expected loss payments by company for each value of L, the analysis also determines whether a company becomes insolvent (if its loss payment, conditional on L, exceeds its premiums and equity capital). For insolvent firms the total payment is capped at the sum of premiums plus equity capital. A number of supporting statistics can be computed from the response function output, including industry efficiency, the percentage of total catastrophe losses paid for events of various magnitudes, and the number of insolvencies.

As mentioned above, we conduct the estimation for two definitions of the industry in terms of recognizing the presence of insurance groups. The first definition is based on the assumption that each insurance groups acts as if it were a single firm. Based on this definition, the industry is defined as consisting of groups and unaffiliated single insurers. We refer to this version of the industry as the or ***group*** sample. The second definition of the industry is based on the assumption that companies that are members of insurance groups operate independently. This version of the industry is referred to as the ***company*** sample.

Summary Statistics

Summary statistics on losses and equity capital, the two most important determinants of industry capacity, are shown in Table 1. Losses and equity are shown for both the Florida sample and the national sample. The Florida figures are the countrywide totals of losses and equity for insurers doing business in Florida, rather than the Florida business of these companies, based on the rationale that the total resources of the company are potentially available to pay losses from Florida catastrophes rather than just the resources generated from Florida operations. The national sample captures xxd percent of industry losses and 96.6 percent of industry equity, and thus provides an excellent representation of the industry as a whole. Companies doing business in Florida account for xxd percent of total industry losses and 79.9 percent of industry equity.

Average values of the raw and detrended parameter estimates are shown in Table 2. As expected, detrending significantly reduces the magnitudes of loss standard deviations, correlations between company and industry losses, and correlations between the logs of company losses and the logs of industry losses (the rho parameters). Recall that the response functions are decreasing in the insurer's loss standard deviation and increasing in the correlation between the insurer loss and the industry loss. Because detrending leads to larger reductions in the correlations than in the standard deviations, we expect the estimated loss payments to be lower for the detrended parameter estimates than for the raw parameter estimates.

The standard deviation estimates tend to be larger for the company sample than for the group sample in both the raw and detrended cases, reflecting the smoothing effect of intragroup reinsurance transactions. The raw estimates of the correlation coefficients and rho statistics are somewhat larger for the group sample than for the company sample samples (in both the national and Florida cases), as expected if intra-group reinsurance transactions tend to increase the covariability of the loss series. However, the detrended correlation coefficients and rho statistics are lower for the group sample than for all company sample. Thus, after removal of the time trend, there is less covariability among firms in the group sample than in the company sample.

Industry Capacity: The National Sample

The response functions for the national sample are shown in Figure 2. The figure shows the estimated amounts that would be paid for industry losses ranging from $200 billion to $500 billion. These limits were chosen because total losses and loss adjustment expenses for the U.S. property-liability insurance industry in 1997 were approximately $200 billion and the total equity capital was approximately $300 billion. Thus, the response curve ranges from the industry's actual loss up to the industry's total resources. Four response curves are shown in the figure, based on (a) raw parameters for the group sample, (b) detrended parameters for the group sample, (c) raw parameters for the company sample, and (d) detrended parameters for the company sample.

The estimated response curves are expected to follow certain ordering relationships, based on option pricing theory and the assumption regarding the exercise of the default option for failing insurers that are members of groups. Larger payments are expected when raw parameter estimates are used rather than detrended parameter estimates. The rationale is that removal of the time trend leads to a larger reduction in the correlation coefficients than in the standard deviations (see Table 2). Because the response function values are positively related to the correlation coefficient and negatively related to the standard deviation, the detrended parameters give lower estimated expected payments. Secondly, expected payments for the group and unaffiliated company sample are expected to be higher than payments using the all company sample, holding constant the parameters used in the estimation. That is, for the raw parameter estimates, the estimated payments for the group sample should be larger than the payment for the company sample, and likewise for the detrended parameter estimates. In the group sample, if a company that is a group member exhausts its resources, payments continue to be made from the resources of other insurers that are members of the group until the group's resources are exhausted. However, in the company sample, the failure of a company does not trigger additional payments from members of the same group, because group relationships are ignored in this estimation.

The expected relationships are borne out in the estimated response curves shown in Figure 2. The largest estimated payments are obtained using the raw parameter estimates for groups, followed by the estimates based on the raw parameters estimates for companies, the detrended parameter estimates for groups, and the detrended parameter estimates for companies. Generally, a high proportion of total losses are paid for industry losses near to the expected value of $200 billion and ranging up to about $300 billion. Above that level, noticeable gaps begin to appear between the industry loss and the estimated amounts paid.

The estimated efficiencies for the national sample are shown in Figure 3. Recall that efficiency is defined as the ratio of the area below the curved lines in Figure 2 to the total area represented by the triangle bordered by the 100 percent payment line and the horizontal axis in Figure 2. Obviously, the efficiencies will differ depending upon the value of industry loss used as the starting or attachment point. Accordingly, Figure 3 shows capacity for various starting (attachment) points ranging from $200 to $500 billion. The efficiencies are inversely related to the attachment points. For an attachment point of $200 billion, the efficiency estimates range from 91 percent based on raw parameters for the group sample to about 78 percent based on detrended parameter estimates for the company sample. For the highest attachment points, efficiencies range from about 80 percent based on raw parameters for the group sample to about 65 percent based on detrended parameters for the company sample.

The response function analysis also produces estimates of the percentage of losses that would be paid for catastrophes of different sizes. These percentages are shown in Figure 4. For relatively small catastrophes, the industry would be able to pay very high percentages of the loss. E.g., for a $20 billion catastrophe, our estimates indicate that the industry would be able to pay at least 98.6 percent of the loss. The percentages that would be paid for larger losses decline at an increasing rate. For example, using the detrended parameter estimates, for a catastrophic loss of $100 billion the industry would be able to pay about 96.4 percent of the loss based on the group sample and 92.8 percent based on the company sample. For a $200 billion catastrophe, the industry would pay between 84.0 percent based on the group sample and 78.6 percent based on the company sample.

The significant capacity of the industry to respond to catastrophes in the range of losses represented by Hurricane Andrew and the Northridge earthquake is primarily due to an increase in the relative capitalization of the industry over the past few years. The ratio of premiums to surplus, a commonly used leverage ratio in the insurance industry, was 1.4 in 1991, prior to Andrew, but had declined to 0.9 in 1997. The increase in capitalization reflects in part the strong investment performance of recent years but also reflects increasing concern about catastrophes in the industry as well as the introduction of risk-based capital regulations in 1995. We conduct an additional analysis to determine the impact on capacity of the increase in capitalization levels in the industry since 1991. A gauge of the capitalization increase that is more consistent with our model than the premiums to surplus ratio is the ratio of equity capital to losses and loss adjustment expenses incurred (hereafter referred to as losses incurred). In 1991, the ratio of equity capital to losses incurred was $0.88, while in 1997 the ratio was $1.56. We recalculated the 1997 capacity of the industry after reducing equity capital proportionately for the firms included in our sample so that the ratio of 1997 capital to losses incurred was the same as in 1991, i.e., $0.88. The results are presented in Figure 5, which plots the expected company and group payments for catastrophes of various sizes, based on 1991 and 1997 capitalization levels. In order to reduce the number of curves on the chart and focus on what we consider the most realistic results, Figure 5 includes only the capacity estimates based on detrended parameter values.

Figure 5 reveals a dramatic increase in industry capacity to bear catastrophic risk between 1991 and 1997. Focusing on the detrended company loss estimates, the industry would be able to pay 98.6 percent of a $20 billion catastrophe based on 1997 capitalization levels but would be able to pay only 94.5 percent of a $20 billion catastrophe based on 1991 capitalization. The results are even more dramatic for larger catastrophes. For a $100 billion catastrophe, again based on detrended company loss estimates, the industry could pay 92.8 percent based on 1997 capitalization but only 79.6 percent based on 1991 capitalization. For a $200 billion catastrophe, the industry could pay 78.6 percent based on 1997 capitalization but only 56.4 percent based on 1991 capitalization. The capacity of the industry, even for catastrophes in the $100 billion range, is clearly much larger in 1997 than it was prior to Andrew and Northridge.

However, even at 1997 capitalization levels, catastrophes in the $100 billion range would disrupt the market by causing a significant number of insolvencies. For example, a $100 billion catastrophe is projected to cause 30 insolvencies based on the detrended parameter estimates for the group sample and 136 insolvencies based on the detrended parameters for the company sample. The comparable numbers of insolvencies at 1991 capitalization levels would have been 108 groups and 216 companies.

Industry Capacity: The Florida Sample

The response function for the Florida sample is shown in Figure 6. Because the number of insurers operating in Florida is smaller than the number operating nationally, the amount of resources available to pay claims in Florida is commensurately reduced. This can be seen more clearly from the efficiencies plotted in Figure 7. At an attachment point of $200 billion, the Florida efficiency based on raw parameter estimates for the group sample is about 85 percent, compared to 91 percent at the same attachment point for the national case. Based on detrended parameter estimates for the company sample, the efficiency at the $200 billion attachment point is 72 percent in Florida, compared to 78 percent nationally.

The estimated Florida payments for catastrophic losses of various sizes are shown in Figure 8 for the detrended parameter estimates at both 1997 and 1991 capitalization levels. As with the national sample, the capacity of the industry to respond to moderate catastrophes appears to be adequate both at 1997 and 1991 capitalizations. For a catastrophe of $20 billion, the expected payment for the group sample at 1997 capitalization levels would be 99.4 percent, compared to 97.9 percent at 1991 capitalization. The comparable figures for companies are 98.6 percent and 94.4 percent, respectively. Capacity in 1997 also appears to be reasonably adequate for a catastrophe of $100 billion. The expected payment for groups would be 94.2 percent and the payment for companies would be 89.7 percent. At 1991 capitalization levels, on the other hand, the capacity of the industry to finance a $100 billion catastrophe was much lower – 77.5 percent payment by groups and only 72.2 percent by companies. The principal finding is that the capacity of the industry increased dramatically between 1991 and 1997 and now is adequate to bear catastrophes in the range of the projected "Big One" (e.g., $100 billion). However, such a catastrophe would still be disruptive to the insurance market because it would be projected to cause the failure of 34 companies and 10 groups.

Regression Models For Parameter Estimation

Finally, we provide examples of the regression models used in estimating the parameters for the companies in the market in 1997 that did not have data for the full time series covered by the study (the *non-full time series (NFTS) companies)*. Recall that the procedure for parameter estimation is to estimate regression models with the parameters of the FTS companies as dependent variables and various financial characteristics of the companies as independent variables. The parameters for the NFTS companies are estimated by inserting the financial characteristics of these firms into the equation to obtain fitted values to be used in the capacity estimation model. For consistency and to smooth out unusually large or small values of the FTS parameters obtained directly from the data, fitted values of the parameters of the FTS companies are also obtained and used in the capacity estimation. The fitted values of both the NFTS and the FTS parameters are based on 1997 financial data.

The regression models are based on the underlying theoretical principle that insurers seek to maximize returns for a given level of risk, i.e., are attempting to achieve an efficient rate of return on equity in relationship to the risks they bear (see Cummins and Sommer, 1996). Insurers may have a target level of overall risk for a number of reasons. However, there are two primary explanations for having a risk target: (1) For financial firms, the firms creditors are also its customers. E.g., the primary liabilities (debt capital) of property-liability insurers consist of policy reserves, which constitute funds held to pay policyholder claims (Merton and Perrold, 1993). Because the purpose of insurance is likely to be subverted if the debt claims are overly risky, insurers are likely to incur a product market penalty from taking excessive risk, providing one important rationale for having a risk target. (2) Insurers are subject to rigorous solvency regulation that has included risk-based capital requirements since 1994. The risk-based capital rules subject insurers to increasingly restrictive regulations if capital declines below a specified level. This regulatory restriction imposes costs on insurers that most companies try to avoid by holding capital significantly in excess of the amounts required by the risk-based capital standards. Thus, even if there were no product market penalty for being too risky, insurers would still have an incentive to maintain adequate financial safety to avoid regulatory costs (Cummins, Harrington, and Niehaus, 1994).

We also argue that there is a range of insurers in the market in terms of overall firm risk. Some insurers seek to be high cost, low risk providers to serve the needs of relatively risk averse buyers, whereas other insurers offer a lower cost, higher risk product to appeal to buyers with lower risk aversion. In effect, insurance is priced like risky corporate debt, with the price of coverage inversely related to the degree of default risk. Thus, we may observe insurers that take low (high) underwriting risk and low (high) financial risk to appeal to different segments of the product market.

Because both the penalty for risky debt and the regulatory risk-based capital levels are primarily driven by the standard deviations of losses and investment returns, the regression examples we provide are those with the standard deviation of losses as the dependent variable. The regression models with the raw standard deviations of losses as dependent variables are presented in Table 3. Consistent with the hypothesis that insurers with low (high) underwriting risk also may take less (more) financial risk, the coefficient of the equity capital to assets ratio is inversely related to the standard deviation of losses. In addition, we find that the standard deviation is directly related to the return on assets, suggesting that insurers that are willing to take higher risk in their asset portfolios also take more underwriting risk. The ratio of net income to net premiums written is inversely related to the loss standard deviation in three of the four equations shown in Table 3. This result is consistent with the hypothesis that insurance is priced like risky debt, accounting for the inverse relationship between net income and underwriting risk. The regressions also show a positive relationship between the ratio of reinsurance receivables to assets and underwriting risk. This is closely monitored by regulators because higher levels of reinsurance receivables may indicate that the firm is placing excessive reliance on reinsurance from financially vulnerable reinsurers that are relatively slow in paying claims. Insurers willing to deal with risky reinsurers also may have a tendency to take more underwriting risk.

Two variables are included to measure the effects of size on risk taking. The log of net losses incurred is inversely related to the standard deviation of losses. This finding is consistent with the well known result that insurers with larger underwriting portfolios achieve better diversification and thus have relatively less idiosyncratic risk than smaller insurers. The log of equity capital has a positive coefficient in the regression equations. The coefficient on this variable is consistent with the argument that larger firms tend to take more risk, because of the better diversification achieved in their underwriting portfolios.

The proportion of an insurer's portfolio held in stocks is inversely related to underwriting risk. However, this variable is significant in only two of the four equations presented in Table 3, for national companies and groups. This variable would seem to suggest that insurers trade off investment and underwriting risk, contrary to the interpretation of some of the other variables. A possible interpretation of this result is that insurers invest in stocks in part because of their relatively favorable tax treatment relative to taxable bonds. Insurers that are more concerned about tax management are likely to be operating in the convex segment of the income tax schedule and also have an incentive to reduce underwriting risk to lower expected tax payments (Cummins, Phillips, and Smith, 1996).

5. Conclusions

In this article, we conduct a theoretical and empirical analysis of the capacity of the U.S. property-liability insurance industry to finance major catastrophic property losses. The topic is important because catastrophic events such as the Northridge earthquake and Hurricane Andrew have raised questions about the ability of the insurance industry to respond to the "Big One,"usually defined as a hurricane or earthquake in the $100 billion range. At first glance, the U.S. property-liability insurance industry, with equity capital of more than $300 billion, should be able to sustain a loss of this magnitude. However, the reality could be different; depending on the distribution of damage and the spread of coverage as well as the correlations between insurer losses and industry losses. Thus, the prospect of a mega catastrophe brings the real threat of widespread insurance failures and unpaid insurance claims.

Our theoretical analysis takes as its starting point the well-known article by Borch (1962), which develops a theorem similar to the capital asset pricing model for reinsurance markets. Borch's theorem shows that the Pareto optimal result in a market characterized by risk averse insurers is for each insurer to hold a proportion of the "market portfolio" of insurance contracts. Each insurer pays a proportion of total industry losses; and the industry behaves as a single firm, paying 100 percent of losses up to the point where industry net premiums and equity are exhausted. Borch's theorem gives rise to a natural definition of industry capacity as the amount of industry resources that are deliverable conditional on an industry loss of a given size.

In our theoretical analysis, we show that the necessary condition for industry capacity to be maximized is that all insurers hold a proportionate share of the industry underwriting portfolio. The sufficient condition for capacity maximization, given a level of total resources in the industry, is for all insurers to hold a net of reinsurance underwriting portfolio which is perfectly correlated with aggregate industry losses. Based on these theoretical results, we derive an option-like model of insurer responses to catastrophes, leading to an insurer response-function where the total payout, conditional on total industry losses, is a function of the industry and company expected losses, industry and company standard deviations of losses, company net worth, and the correlation between industry and company losses. The industry response function is obtained by summing the company response functions, giving the capacity of the industry to respond to losses of various magnitudes.

Based on our theoretical model, we utilize 1997 insurer financial statement data to estimate the capacity of the industry to respond to catastrophic losses. Two samples of insurers are utilized – a national sample, to measure the capacity of the industry as a whole to respond to a national event, and a Florida sample, to measure the capacity of the industry to respond to a Florida hurricane. In the estimation, we recognize that many insurers are organized in the form of insurance groups, where several insurers operate under common ownership. Insurance groups hold a valuable option, i.e., the option to allow financially vulnerable group members to fail. Accordingly, we estimate capacity under two polar assumptions about the behavior of groups: (a) that groups always bail out failing subsidiaries, so that each group operates as a single firm, and (b) that groups never bail out failing subsidiaries, so that members of groups operate independently. These two assumptions lead to upper and lower bounds, respectively, on the capacity of the industry to respond to catastrophic loss.

The empirical analysis estimates the capacity of the industry to bear losses ranging from the expected value

of loss up to a loss equal to total company resources. We develop a measure of industry *efficiency* equal to the difference between the loss that would be paid if the industry acts as a single firm and the actual estimated payment based on our option model. The results indicate that national industry efficiency ranges from about 78 to 85 percent, based on catastrophe losses ranging from zero to $300 billion, and from 70 to 77 percent, based on catastrophe losses ranging from $200 to $300 billion. The industry has more than adequate capacity to pay for catastrophes of moderate size. E.g., based on both the national and Florida samples, the industry could pay at least 98.6 percent of a $20 billion catastrophe. For a catastrophe of $100 billion, the industry could pay at least
92.8 percent. However, even if most losses would be paid for an event of this magnitude, a significant number of insolvencies would occur, disrupting the normal functioning of the insurance market, not only for property insurance but also for other coverages.

We also compare the capacity of the industry to respond to catastrophic losses based on 1997 capitalization levels with its capacity based on 1991 capitalization levels. The comparison is motivated by the sharp increase in capitalization following Hurricane Andrew and the Northridge earthquake. In 1991, the industry had $0.88 in equity capital per dollar of incurred losses, whereas in 1997 this ratio had increased to $1.56. To compare 1991 and 1997, we proportionately reduce the capital of the insurers in our sample to achieve an industry-wide capital-to-loss ratio of $0.88 in 1997. The results indicate a dramatic increase in capacity between 1991 and 1997. For a catastrophe of $100 billion, our lower bound estimate of industry capacity in 1991 is only 79.6 percent, based on the national sample, compared to 92.8 percent in 1997. For the Florida sample, we estimate that insurers could have paid at least 72.2 percent of a $100 billion catastrophe in 1991 and 89.7 percent in 1997. Thus, the industry is clearly much better capitalized now than it was prior to Andrew.

The relatively high capacity of the industry at the present time has significant implications for insurance and capital markets as well as for public policy regarding the financing of catastrophic risk. It does <u>not</u> appear that the gaps in catastrophic risk financing are presently sufficient to justify government intervention in private insurance markets in the form of Federally sponsored catastrophe reinsurance. However, even though the industry could adequately fund the “Big One,” if defined as a catastrophe in the range of $100 billion, doing so would disrupt the functioning of insurance markets and cause price increases for all types of property-liability insurance. Thus, it appears that there is still a gap in capacity that provides a role for privately and publicly traded catastrophic loss derivative contracts. Accordingly, both the state and Federal governments should focus on removing regulatory barriers to the further development of private capital market solutions to the financing of large catastrophes. Capital market solutions thus are needed both to shore up industry capacity and to reduce the societal costs of dealing with catastrophic risk.

References

Borch, K. 1962, Equilibrium in a Reinsurance Market, *Econometrica* 30: 424-444.

Cummins, J. David and Patricia M. Danzon, 1997, "Price Shocks and Capital Flows in Liability Insurance," *Journal of Financial Intermediation* 6: 3-38.

Cummins, J. David and Francois Outreville, 1987, "An International Analysis of Underwriting Cycles." *Journal of Risk and Insurance* 54:

Cummins, J. David, Christopher M. Lewis, and Richard D. Phillips, 1999, "Pricing Excess of Loss Reinsurance Contracts Against Catastrophic Loss," in Kenneth Froot, ed., *The Financing of Catastrophe Risk* (Chicago: The University of Chicago Press.

Cummins, J. David and David Sommer, 1996, "Capital and Risk in Property-Liability Insurance Markets." *Journal of Banking and Finance* 20 (1996): 1069-1092.

Doherty, Neil A. and Seha Tinic, 1981, "A Note on Reinsurance under Conditions of Capital Market Equilibrium" , *Journal of Finance*, 36, 949-953.

Easterbrook, Frank H. and Daniel R. Fischel. 1985. "Limited Liability and the Corporation." *University of Chicago Law Review* 52: 89- 117

Hogg, Robert V. and Allen T. Craig, 1995, *Introduction to Mathematical Statistics*, 5th ed. (Upper Saddle River, NJ: Prentice-Hall).

Froot Kenneth A. and Paul J. G. O'Connell, 1996 "The Pricing of U.S. Catastrophe Reinsurance," in Kenneth Froot, ed., *The Financing of Catastrophe Risk* (Chicago: The University of Chicago Press .

Froot, Kenneth, David Scharfstein, and Jeremy Stein, 1993, "Risk Management: Co-ordinating Investment and Financing Problems," *Journal of Finance*, 48, 1629-1658.

Guy Carpenter, 1998, "1998 Review: United States Property Catastrophe Programs," *Guy Carpenter's Monitor* (New York).

Jaffee, Dwight M. and Thomas Russell, 1997, "Catastrophe Insurance, Capital Markets, and Uninsurable Risks," *Journal of Risk and Insurance 64: 205-30.*

Lewis, Christopher M. and Kevin C. Murdock, 1996, "The Role of Government Contracts in Discretionary Reinsurance Markets for Natural Disasters," *Journal of Risk and Insurance* 63: 567-597.

Merton, Robert and Andre Perrold, 1993, "The Theory of Risk Capital in Financial Firms," *Journal of Applied Corporate Finance*.

Myers, Stewart C. 1977, "Determinants of Corporate Borrowing", *Journal of Financial Economics*, 5, 147-175

SwissRe, 1997, "Too Little Reinsurance of Natural Disasters in Many Markets," *Sigma* 7: 3-22.

Winkler, Robert L., Gary M.Goodman, and Robert R.Britney, 1972, The Determination of Partial Moments", *Management Science*, 19, 290-296.

Table 1
Summary Statistics: Losses and Equity Capital

Case	1997 Losses	% of Total Industry Losses	1997 Equity	% of Total Industry Equity	Number of Firms
National Net Loss:Groups and Unaffiliated Cos	201,905,979,763		370,993,421,647	96.6%	1,248
National Net Loss: All Companies	201,905,979,763		370,993,421,647	96.6%	2,256
Florida Net Loss: Groups and Unaffiliated Cos	156,404,436,985		306,861,844,005	79.9%	431
Florida Net Loss: All Companies	156,404,436,985		306,861,844,005	79.9%	898

Note: Equity has not been adjusted for intra-group consolidations. An adjustment for consolidations was made in estimating capacity.
Florida losses are the total (countrywide) losses of insurers operating in Florida.

Table 2
Detrended and Raw Parameter Estimates: Property-Liability Insurance Industry

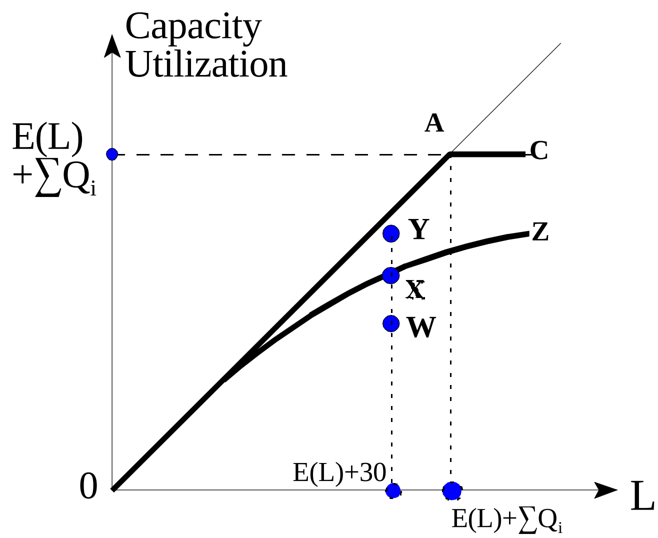
				Averages				
Case	DTSigma	DTRho	DTCorr	Mu	Sigma	Rho	Correl	N
National Net Loss: Groups and Unaffiliated Cos	0.4112	0.3386	0.1751	15.57	0.5188	0.4762	0.4366	1,248
National Net Loss: All Companies	0.4203	0.3694	0.1916	15.67	0.5303	0.4724	0.4320	2,256
Florida Net Loss: Groups and Unaffiliated Cos	0.4110	0.3429	0.1308	16.97	0.4760	0.5407	0.4921	431
Florida Net Loss: All Companies	0.4442	0.3888	0.1821	16.71	0.5177	0.5097	0.4683	898

Table 3
Regression Models of Raw Standard Deviations: Net Losses Incurred

Variable	Florida Companies	National Companies	Florida Groups	National Groups
Intercept	1.825	1.328	1.297	0.970
	8.047	9.588	5.253	5.367
Equity Capital/Assets	-0.817	-0.800	-0.761	-0.540
	-5.502	-8.358	-3.320	-3.423
Ln(Net Losses Incurred)	-0.296	-0.357	-0.338	-0.355
	-10.126	-20.645	-6.219	-12.040
Ln(Equity Capital)	0.221	0.309	0.293	0.317
	7.124	17.223	5.333	10.467
Return on Assets	0.930	1.852	1.339	3.966
	2.358	6.270	1.155	7.509
Net Income/Net Premiums Written	0.194	-0.183	-0.564	-0.545
	1.711	-3.730	-2.499	-5.405
Reinsurance Accounts Receivable/Assets	1.133	0.722	1.698	0.770
	3.417	3.258	3.335	2.008
Stocks/Assets	-0.036	-0.194	-0.012	-0.269
	-0.313	-2.629	-0.075	-2.222
Adjusted R^2	37.9%	40.1%	33.1%	44.6%
No. of obs.	511	953	172	286

Note: The dependent variable is the standard deviation of losses over the period 1983-1997.

Figure 1: Capacity Utilization



Note: The line OAC represents maximum capacity utilization. The line 0Z = E(L) +∑Q-∑E(T_i|L) represents estimated capacity utilization.

Figure 2: Response Functions: National Net Loss

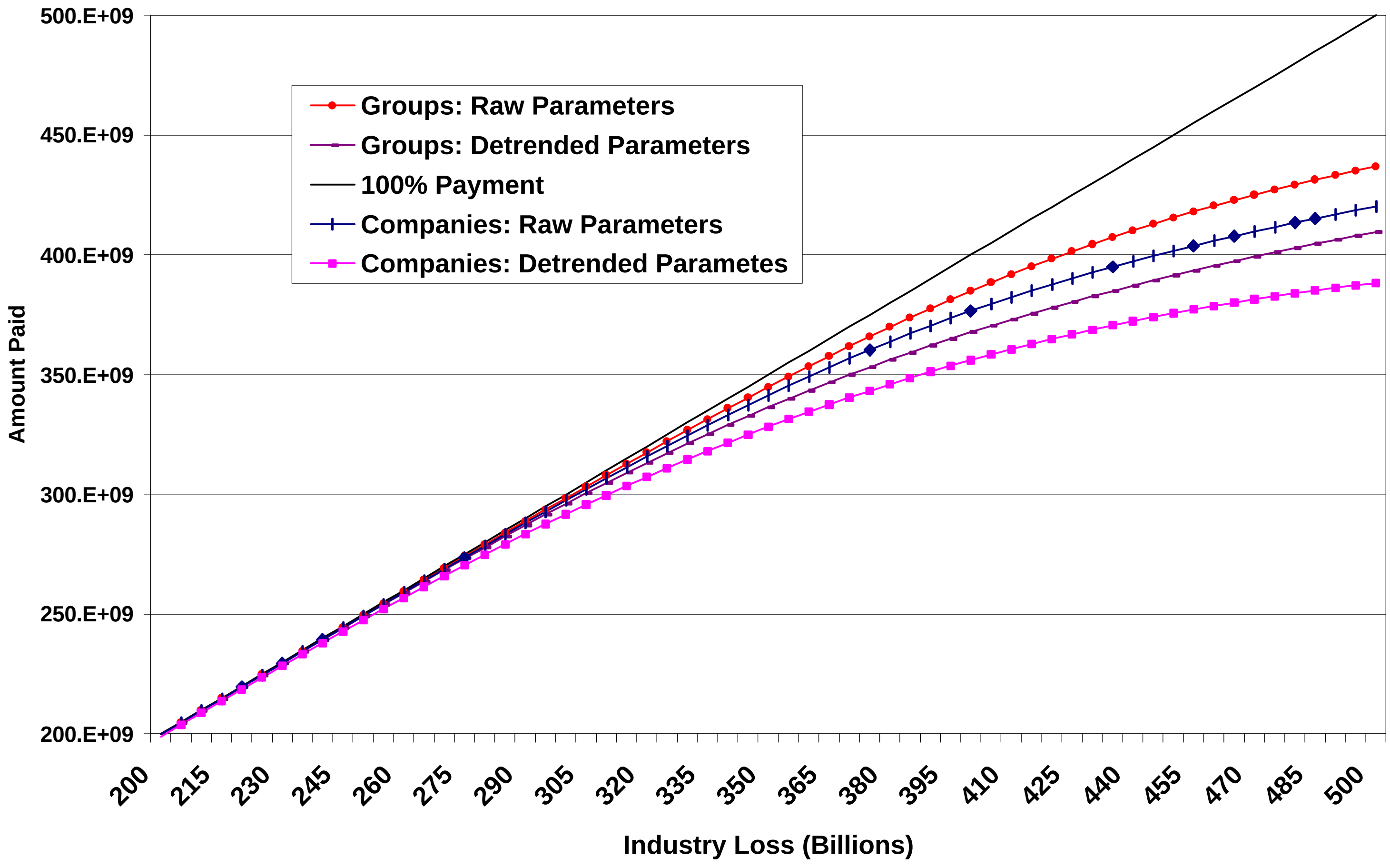


Figure 3: Efficiency - National Net Loss

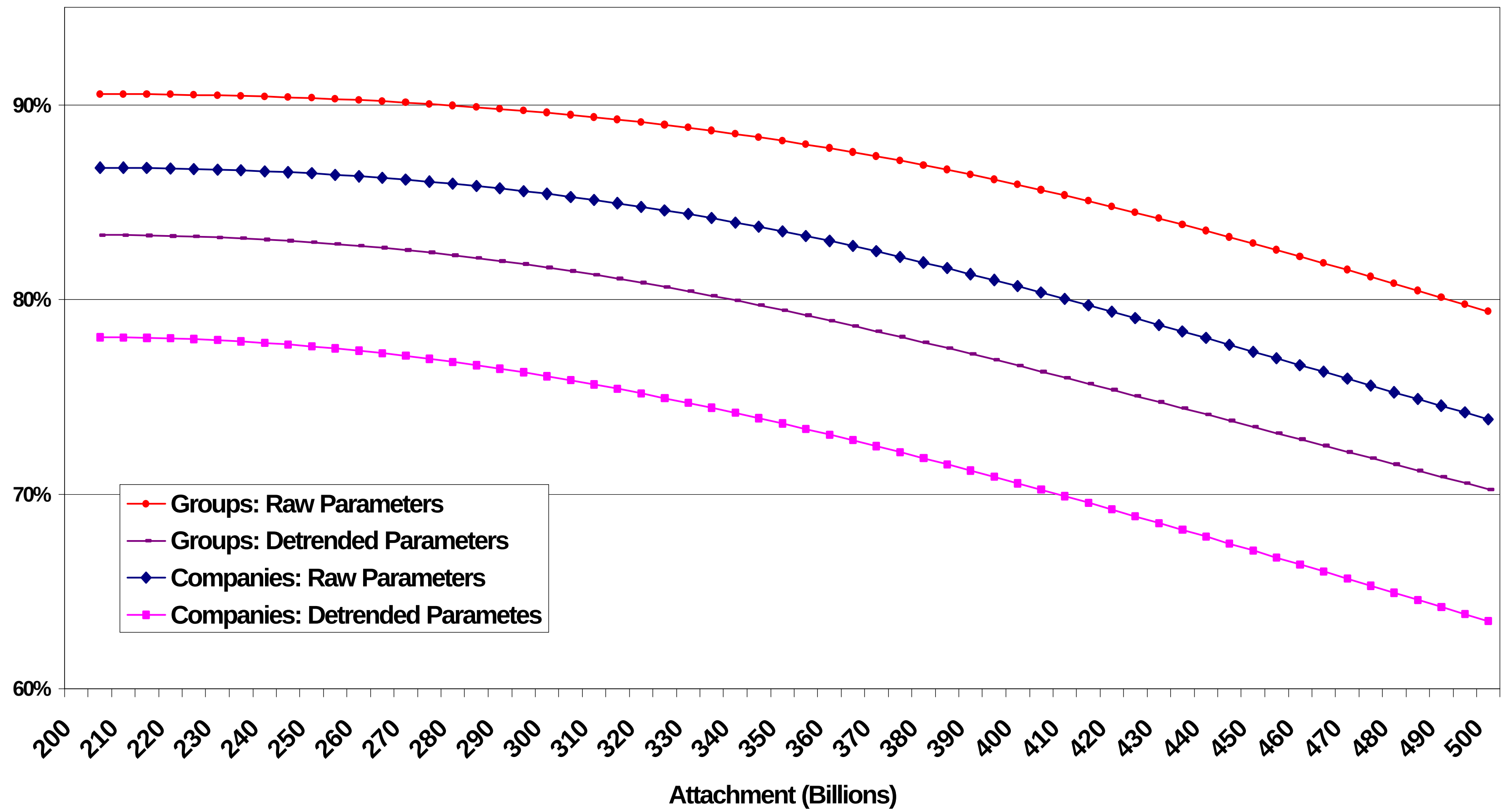
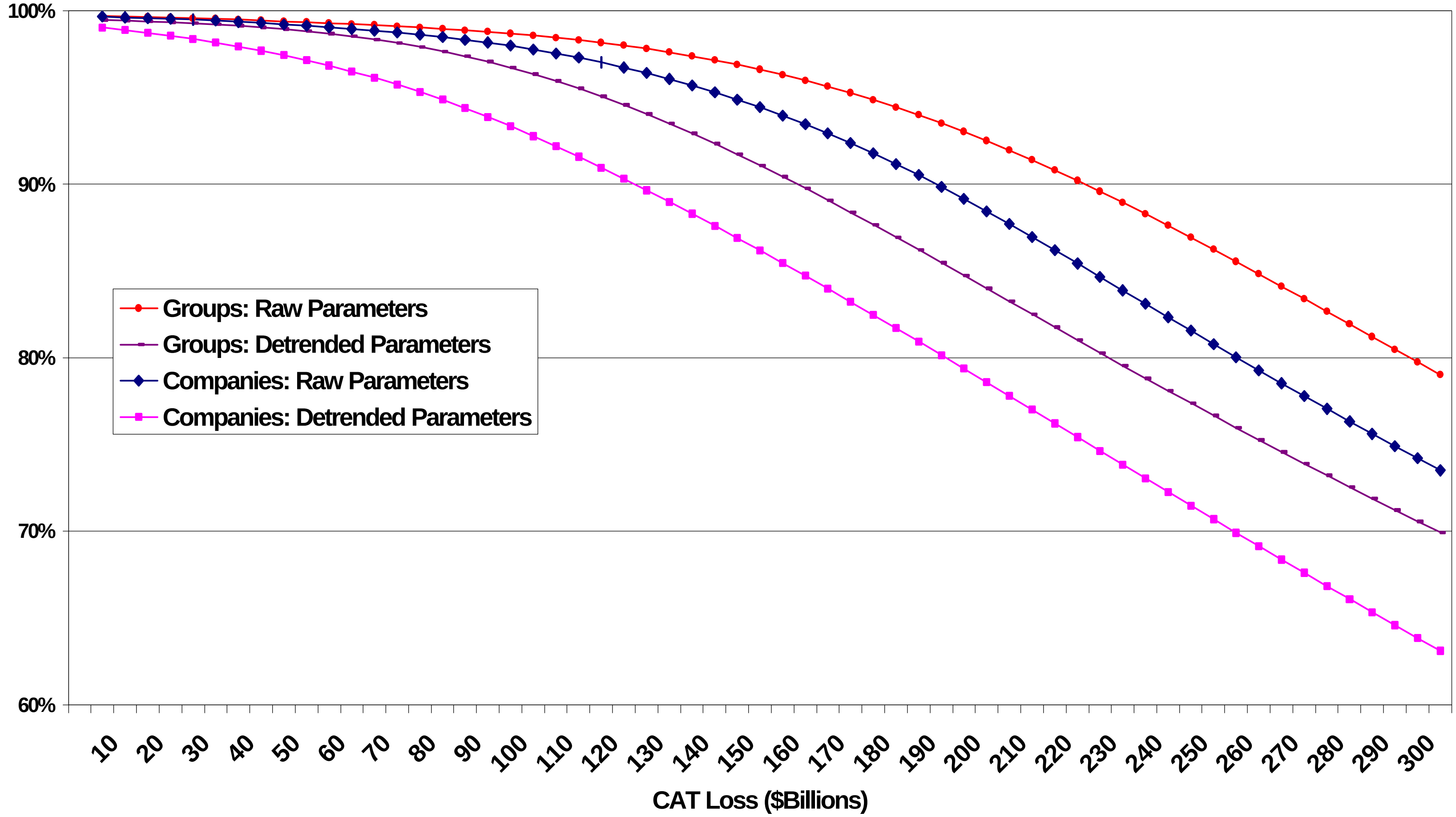


Figure 4: Percent Paid by CAT Loss Size - National Net Loss



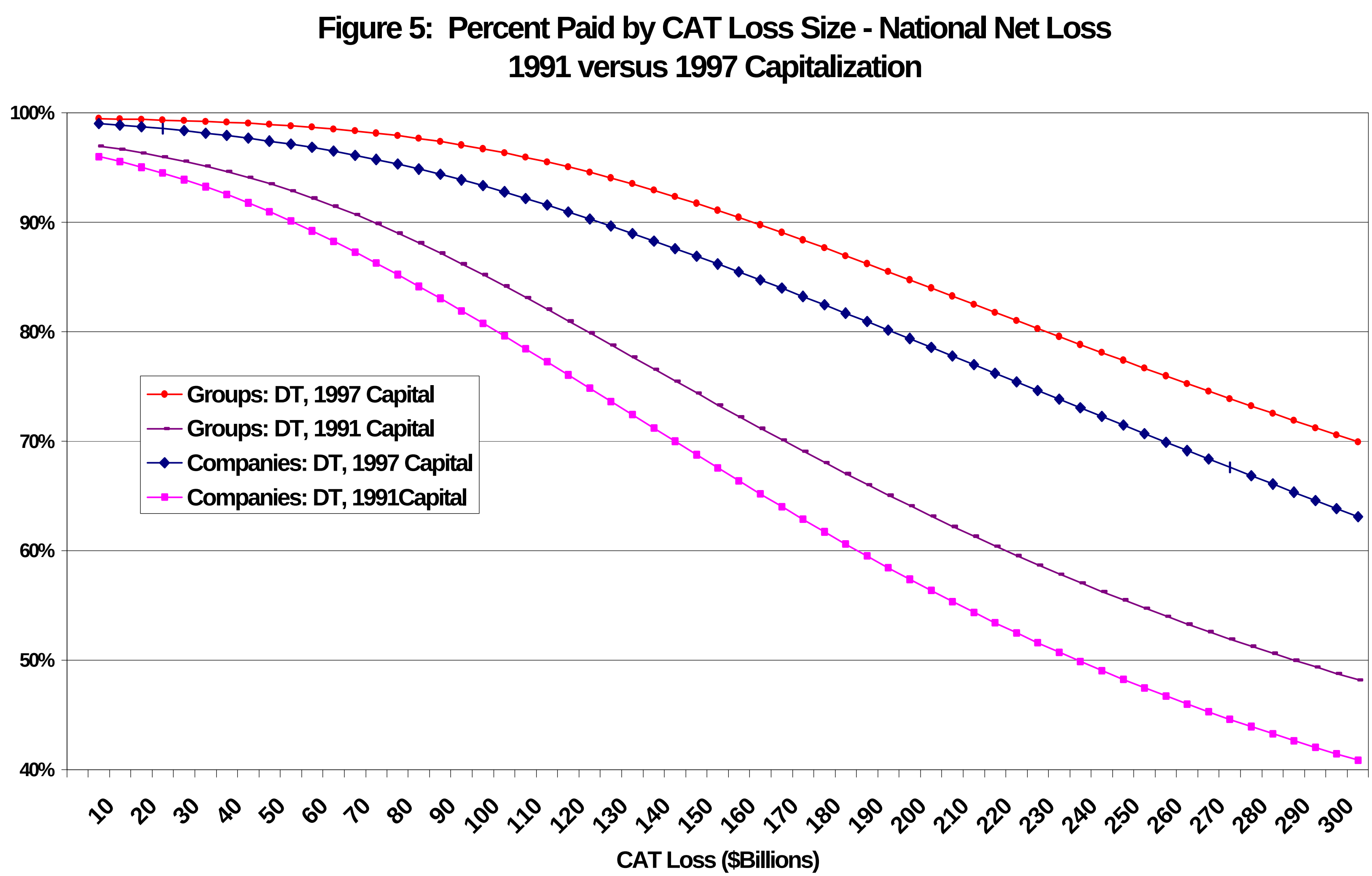


Figure 5: Percent Paid by CAT Loss Size - National Net Loss
1991 versus 1997 Capitalization
100%
90%
80%
70%
60%
50%
40%
Groups: DT, 1997 Capital
Groups: DT, 1991 Capital
Companies: DT, 1997 Capital
Companies: DT, 1991Capital
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
270
280
290
300
CAT Loss ($Billions)

Figure 6: Amount Paid - Florida Net Loss

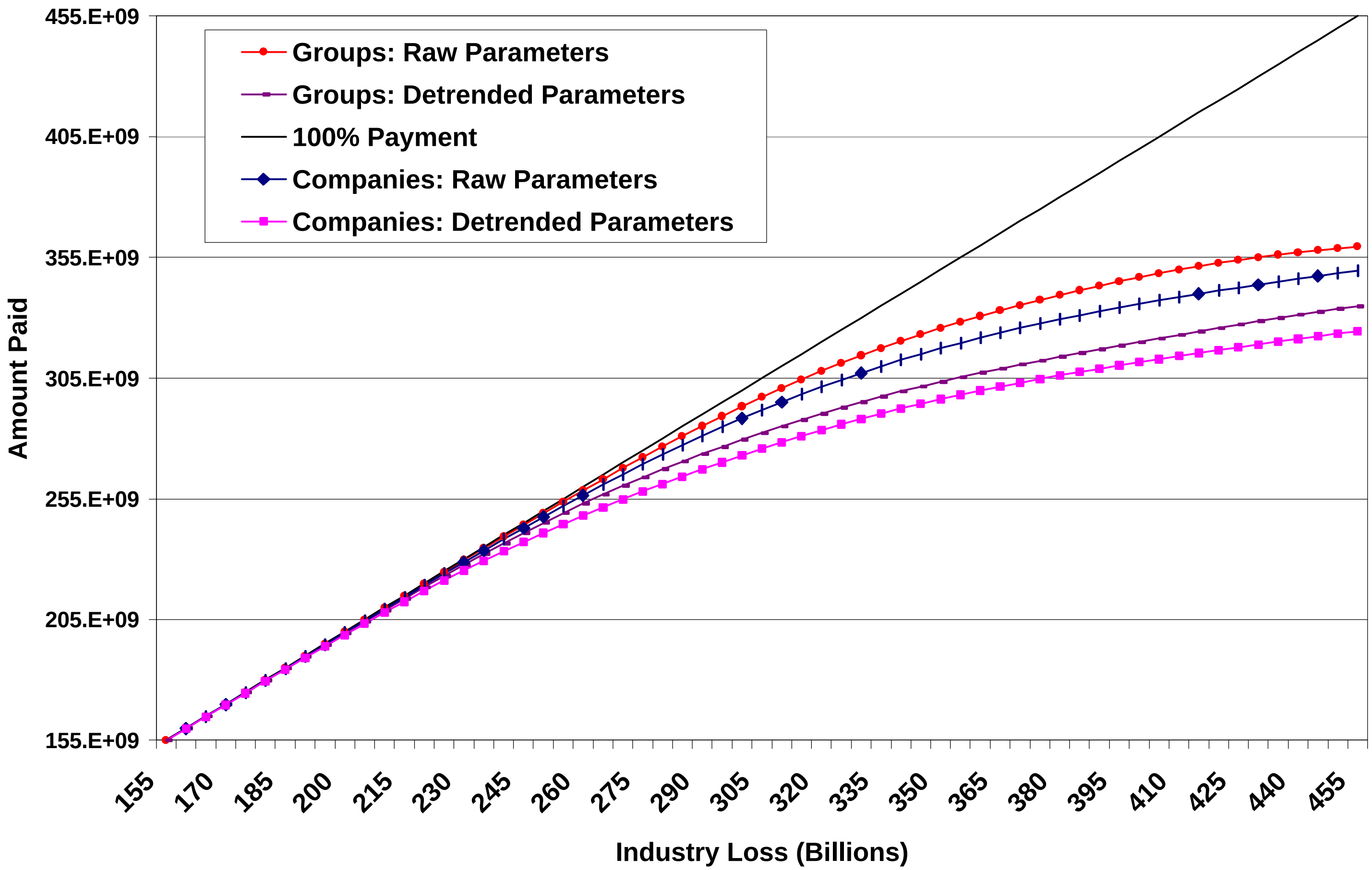
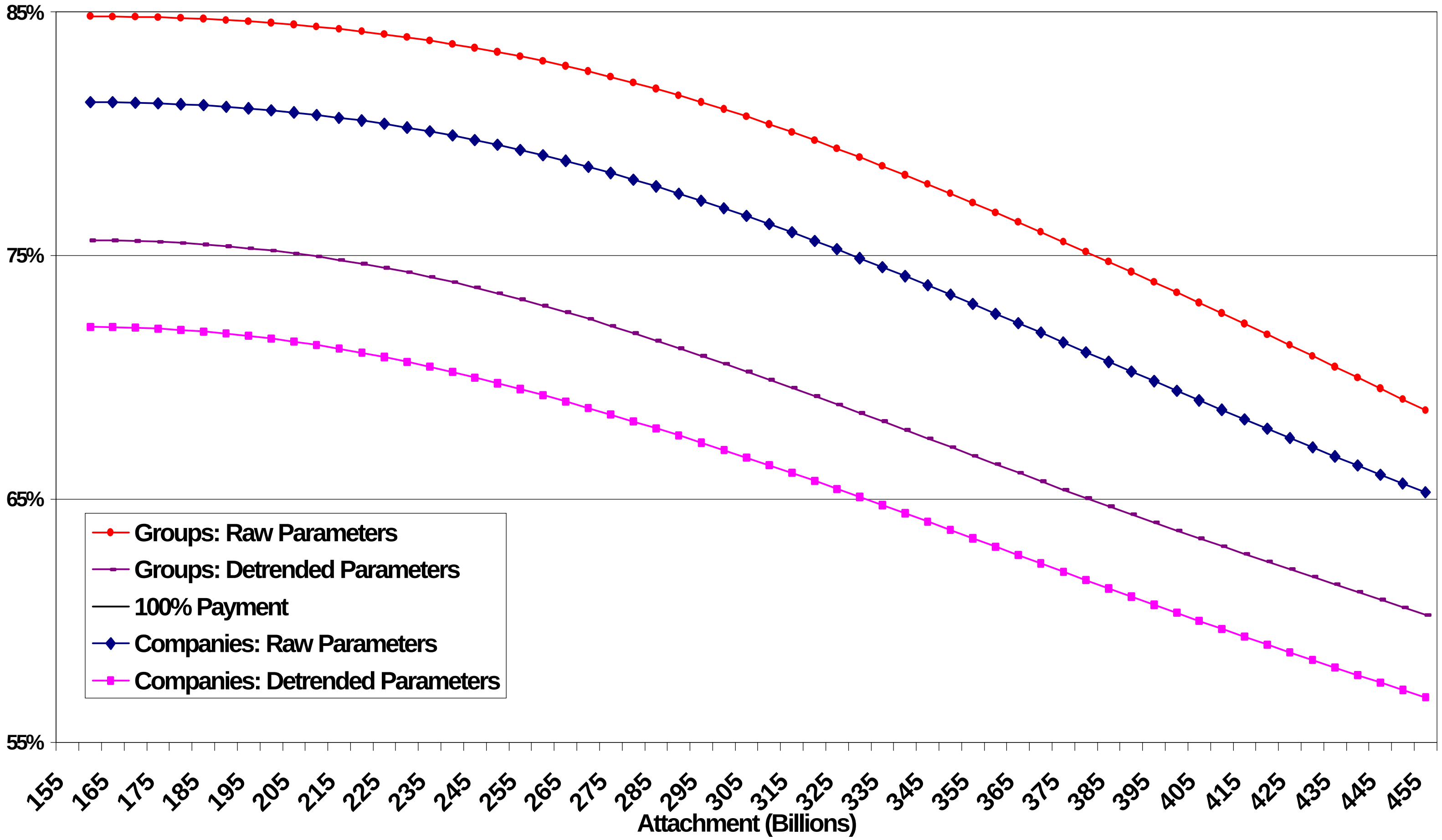


Figure 7: Efficiency - Florida Net Loss



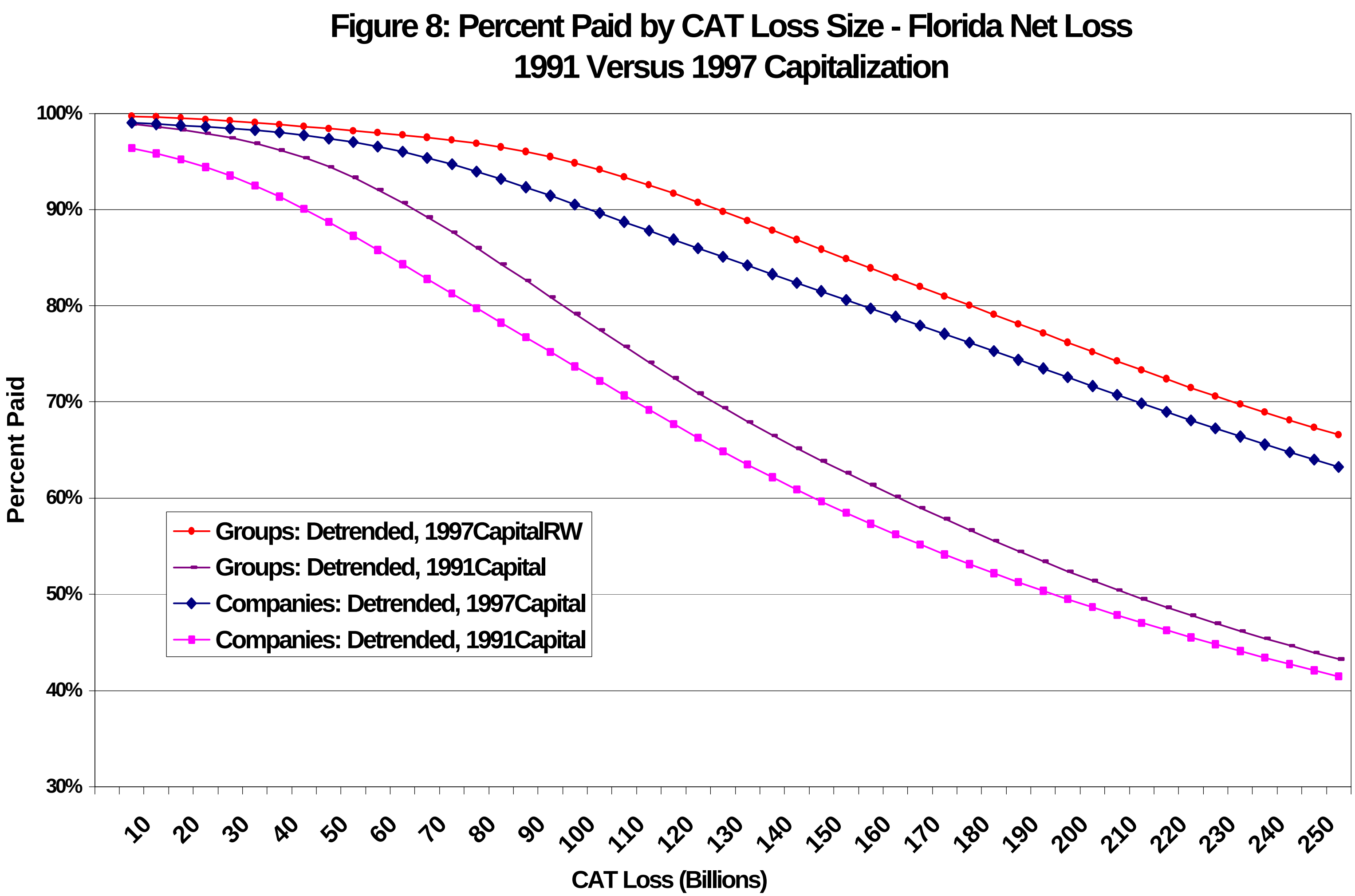


Figure 8: Percent Paid by CAT Loss Size - Florida Net Loss
1991 Versus 1997 Capitalization
Percent Paid
100%
90%
80%
70%
60%
50%
40%
30%
Groups: Detrended, 1997CapitalRW
Groups: Detrended, 1991Capital
Companies: Detrended, 1997Capital
Companies: Detrended, 1991Capital
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
CAT Loss (Billions)

APPENDIX

The Model

To estimate the capacity of the industry, we would like to estimate the payments made by individual insurers conditional on the total industry loss. We make use of some general results relating to conditional probability distributions to provide an overview of the problem and then propose specific models based on the normal and lognormal distributions.

In general, let L_i = the loss paid by company i, where $E(L_i) = \mu_i$, $Var(L_i) = \sigma_i^2$, and i = 1, 2, . . ., N, for a sample of N firms. Define the total losses of the industry as the sum of the losses of the individual companies, i.e.,

$$L = \sum_{i=1}^{N} L_i \qquad \textbf{(8)}$$

where L = the total losses of the industry. The usual formulas for the mean and variance of a sum of random variables apply, so that

$$E(L) = \sum_{i=1}^{N} \mu_i$$
$$Var(L) = \sum_{i=1}^{N} \sigma_i^2 + 2 \sum_{i<j}\sum \sigma_{ij} \qquad \textbf{(9)}$$

where $\sigma_{ij} = Cov(L_i,L_j)$. To conserve parameters and focus on the industry loss, we rewrite the variance of the industry loss as follows:

$$Var(L) = \sum_{i=1}^{N} Cov(L_i, L) = \sum_{i=1}^{N} Cov(L_i, \sum_{i=1}^{N} L_i) \qquad \textbf{(10)}$$

The well-known formula for the conditional expected value (see Hogg and Craig, p. 94) can be used to obtain:

$$E(L_i | L) = \mu_i + \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) = \mu_i + \beta_i (L - \mu_L)$$
$$where \quad \beta_i = \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) = \frac{Cov(L_i, L)}{\sigma_L^2} \qquad \textbf{(11)}$$

Summing over the N firms in the industry, we find that $\Sigma_i E(L_i|L) = L$, because $Var(L) = \Sigma_i Cov(L_i,L)$ and $\mu_L = \Sigma_i \mu_i$. Thus, we can use these formulas to allocate any given industry loss among the firms in the sample.

The above results are not distribution dependent, i.e., we did not have to assume that losses follow any particular probability distribution in order to obtain the results. To calculate the capacity of the industry, however, it is helpful to have a distributional assumption. We develop the model under two assumptions: (1) the distribution of the L_i is multivariate normal and (2) the distribution of the L_I is multivariate lognormal. We develop the model in general and then specify the formulas for the normal and lognormal cases.

Insurers are assumed to begin the period with premiums, P, and beginning equity (surplus), Q_0. The insurer is assumed to pay claims up to the point where these resources are exhausted and to declare bankruptcy and default if the claims exceed its resources. The expected equity of the insurer at the end of the period, conditional on an industry loss of L, is given by:

$$E(T_i \mid Q_{i0}, L) = \int_0^{P_i + Q_{i0}} (P_i + S_{i0} - L_i)\, f(L_i \mid L)\, dL_i \qquad \textbf{(12)}$$

where $f(L_i|L)$ = the distribution of the losses of a given insurer (L_i), conditional on the losses of the industry, L.

In the case where the L_i are jointly normally distributed, the conditional distribution in (5) is given by:

$$f(L_i \mid L) = \frac{1}{\sqrt{2\pi}\,\sigma_i \sqrt{1-\rho_i^2}}\, e^{\frac{1}{1-\rho_i^2}\left[\frac{L_i-\mu_i}{\sigma_i} - \rho_i \frac{L-\mu_L}{\sigma_L}\right]^2} \qquad \textbf{(13)}$$

So $E(L_i|L) = \mu_i + (\rho_i\sigma_i/\sigma_L(L - \mu_L)$. Inserting equation (6) into equation (5) and simplifying, we obtain the expression for the expected ending surplus under the assumption of multivariate normality:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0} - m)\, N\left[\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right] + \sigma_{L_i|L} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{P + Q_{i0} - \mu_{L_i|L}}{\sigma_{L_i|L}}\right)^2} \qquad \textbf{(14)}$$

$$\textit{where} \quad \mu_{L_i|L} = \mu_i + \frac{\rho_i \sigma_i}{\sigma_L}(L - \mu_L) \quad , \textit{and} \quad \sigma^2_{L_i|L} = \sigma_i(1 - \rho_i^2)$$

.N[·] = the standard normal distribution function.

Using an analogous approach for the case where L_i and L are jointly lognormal, we obtain:

$$E(T_i \mid Q_{i0}, L) = (P + Q_{i0})\, N(C_i) - e^{D_i} N(C_i - \xi_i \sqrt{1 - \gamma_i^2}) \qquad \textbf{(15)}$$

where

$$C_i = \frac{\ln(P + Q_{i0}) - \nu_i - \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L)}{\xi_i \sqrt{1 - \gamma_i^2}} \quad \textbf{(16)}$$

$$D_i = \nu_i + \frac{\xi_i \gamma_i}{\xi_L}(\ln L - \nu_L) + \frac{\xi_i^2 (1 - \gamma_i^2)}{2} \quad \textbf{(17)}$$

and υ_i, υ_L = the location parameters of the joint lognormal distribution of L_i and L,

ξ_i, ξ_L = the dispersion parameters of the joint lognormal distribution of L_i and L, and

γ_i = the correlation coefficient between $\ln(L_i)$ and ln(L).

In the lognormal case, we have the complication that L_i and L cannot be jointly lognormal if the L_i, i = 1, . . . , N, are jointly lognormal, because sums of lognormals are not lognormal. Hence, the formula (equation (8)) is only approximate in this case.

The normal and lognormal models yield estimates of the expected end-of-period surplus of the insurers in the sample. However, our ultimate objective is to estimate the amount of claims paid. The amount of claims paid can easily be estimated using the following relationship:

$$E(L_i \mid Q_{i0}, L) = P_i + Q_{i0} - E(T_i \mid Q_{i0}, L) \quad \textbf{(18)}$$

That is, the expected payment equals the resources at the beginning of the period minus the expected amount of surplus at the end of the period.