Deposits and Bank Capital Structure*

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Abstract

In a model with bankruptcy costs and segmented deposit and equity markets, we endogenize the cost of equity and deposit finance for banks. Despite risk neutrality, equity capital earns a higher expected return than direct investment in risky assets. Banks hold positive capital to reduce bankruptcy costs, but there is a role for capital regulation when deposits are insured. Banks may no longer use capital when they lend to firms rather than invest directly in risky assets. This depends on whether the firms are public and compete with banks for equity capital, or private with exogenous amounts of capital.

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1 Introduction

There is a growing literature on the role of equity in bank capital structure focusing on equity as a buffer, liquidity, agency costs and various other frictions.[1] One important feature of these analyses is that they involve partial equilibrium models that do not consider the role of equity in non-financial firms and usually take the cost of equity capital as given. The standard assumption is that equity capital for banks is a more expensive form of financing than deposits.[2] However, there is no clear theoretical foundation for this assumption in this literature and many papers have questioned whether this is justified. Risky equity usually has a higher expected return than debt but, as in Modigliani and Miller (1958), this does not necessarily mean that it is more costly on a risk adjusted basis (e.g., Miller (1995), Brealey (2006), and Admati, DeMarzo, Hellwig and Pfleiderer (2010)).

To address these issues in more depth, we develop a general equilibrium model of bank and firm financing based on two main elements. First, differently from non-financial firms, banks raise funds using deposits, which are special in that the market for deposits is segmented from that of equity. Second, banks and firms incur bankruptcy costs when they fail. Our aim is to determine the optimal bank and firm capital structures and the implications of these for the pricing of equity, deposits and loans.

Although the role of deposits has varied over time, they remain an important source of funds for banks in all countries. Figure 1 shows deposits as a proportion of bank liabilities for a number of countries from 1990 to 2009. In all these countries deposits are the major form of bank finance. Deposits also play an important role in the aggregate funding structure of the economy, as shown in Figure 2 where the ratio between deposits and GDP in the period 1990-2009 is illustrated.

[1] See, e.g., Diamond and Rajan (2000), Hellmann, Murdock and Stiglitz (2000), Gale (2004), Repullo (2004), Morrison and White (2005), Allen, Carletti and Marquez (2011), Acharya, Mehran and Thakor (2012).

[2] See also Berger, Herring and Szego (1995) and the survey by Gorton and Winton (2003) for a discussion of this issue.

Despite its empirical importance, deposit finance has played a relatively small role in the theory of bank funding, where it is usually simply treated as another form of debt.[3] However, there is considerable evidence that the market for deposits is significantly segmented from other markets. While most people in developed countries have bank accounts, with the exception of the U.S. and a few other countries, the household finance literature documents that relatively few people own stocks, bonds or other types of financial assets either directly or indirectly (see, e.g., Guiso, Haliassos and Jappelli (2002) and Guiso and Sodini (2013)). The lack of participation in markets for risky financial assets, and in particular for equity, is known as the "participation puzzle." The usual explanation is that there are fixed costs of participation. In addition to deposits held by households, considerable amounts are held in this form by businesses. These amounts are held for transaction purposes and reserves. In most cases there are limited substitution possibilities with other assets, particularly equity.

The other important foundation of our analysis is the significance of bankruptcy costs. There is considerable empirical evidence that these are substantial for both banks and non-financial firms. For example, James (1991) finds that when banks are liquidated, bankruptcy costs are 30 cents on the dollar. In a sample of non-financial firms, Andrade and Kaplan (1998) and Korteweg (2010) find a range of 10-23 per cent for the ex post bankruptcy costs and 15-30 per cent for firms in or near bankruptcy, respectively. There are a number of issues that arise with the measurement of bankruptcy costs that suggest they are in fact higher than these estimates (see., e.g., Almeida and Philippon (2007), Acharya, Bharath and Srinivasan (2007) and Glover (2012)).

We start our analysis with a simple model where banks finance themselves with equity capital and (uninsured) deposits and invest in risky assets.[4] The providers of equity capital can invest directly in the risky assets, while the providers of deposits only have a storage

[3] For an exception, see Song and Thakor (2007). They show that core deposits are an attractive funding source for informationally opaque relationship loans.

[4] The case when banks invest directly in risky assets captures the idea that banks invest in a line of business with a risky income like market making, underwriting, proprietary trading or fees from advisory services such as mergers and acquisitions.

alternative opportunity with a return of one. For simplicity, both groups are risk neutral. There is a fixed supply of equity capital and deposits in the economy.

Several results hold provided that there are positive bankruptcy costs. First, the seminal results from Modigliani and Miller (1958) do not hold. Specifically, the optimal capital structure is unique and it involves banks holding a positive level of equity capital as a way to reduce bankruptcy costs. Second, equity capital has in equilibrium a higher expected return than investing directly in the risky asset, which in turn has a higher expected return than deposits. This implies that equity providers do not invest in the risky asset directly and that equity capital is "costly" relative to deposits. Third, for low expected returns of the risky asset, deposits yield the same as the storage opportunity so that deposit providers invest in both banks and storage and there is limited financial inclusion of deposits in the economy. For high expected returns of the risky asset, deposits yield an expected return greater than one and there is full financial inclusion as deposit providers only invest in banks.

We then introduce deposit insurance and analyze how regulatory distortions affect bank incentives, and what implications those have for equilibrium returns. We show that, in the absence of regulation, banks no longer have any incentive to hold capital, instead choosing to finance themselves entirely with deposits. The primary reason relates to capital's primary function, which in our setting is to reduce expected bankruptcy costs by lowering the payment that must be promised to depositors. When deposits are insured, capital has no role to play and banks will prefer not to raise any capital. This gives rise to a role for capital regulation. By requiring banks to hold capital, a regulator reduces bankruptcy costs that would otherwise be borne by the deposit insurance fund (and ultimately market participants through some form of lump sum taxation). In fact, we show that deposit insurance coupled with capital regulation can always achieve a higher level of social welfare than what is achieved in the market solution when deposits are uninsured.

We then extend the model along two important dimensions to consider the case in which firms rather than banks invest in the productive projects and need external financing. The analysis of this issue is important given banks' crucial role in channelling funds to firms through the allocation of credit. We first analyze the case of public firms, which we define as firms that have no inside equity but can attract funds both from banks and outside equity investors. Then we turn to private firms, which are firms with an initial endowment of inside equity capital but which can only raise external funds in the form of bank loans and, in particular, are unable to raise outside equity financing. While the main results of the baseline model carry over to both cases –, capital earns rents in excess of its outside option, and its equilibrium return is higher than that of deposits – there are substantial and important differences in how the funds of capital suppliers are allocated and thus in the optimal capital structure of both banks and firms.

In particular, for the case where banks make loans to public firms, the equilibrium entails that banks hold zero capital while firms hold a positive amount. In essence, all equity capital is used by firms rather than being held at the banks. When banks hold zero capital, they are conduits that transfer firm payments on loans to depositors and their bankruptcy is aligned with that of the firms. This arrangement is privately and socially optimal because banks can go bankrupt only when firms do, so it is best to use equity to minimize firm, rather than bank, bankruptcy and thus avoid unnecessary costs. This is different from the case of private firms, which have some internal capital but can raise external finance only through a bank loan. The loan itself, however, may be funded through a combination of capital and deposits, or purely through deposits. We show that banks still act as pure conduits for depositors when project returns are sufficiently low and firm bankruptcy costs are sufficiently high. Otherwise, banks hold positive amounts of capital, with an expected return that is greater than that of the deposits invested at the bank.

The paper contributes to the literature in a number of ways. First, it provides a theoretical foundation for why equity capital is costly relative to deposits, which, as explained above, is currently lacking in the literature. DeAngelo and Stulz (2013) provide an alternative rationale for why the seminal results of Modigliani and Miller (1958) may not hold for banks, so that capital structure is a relevant consideration. They show that banks may choose to be highly levered because of market frictions that lead banks to play a central role in the production of liquidity, which is highly socially valuable and thus earns a market premium. Our results abstract from any liquidity considerations and instead focus on limited market participation and bankruptcy costs, which are largely absent in the extant literature.

Second, there are relatively few empirical studies of bank capital structure. Some recent examples are Flannery and Rangan (2008), Gropp and Heider (2010) and Mehran and Thakor (2011). Flannery and Rangan (2008) document how US banks' capital ratios varied in the last decade. Gropp and Heider (2010) find that the determinants of bank capital structure are similar to those for non-financial firms. Mehran and Thakor (2011) document a positive relation between bank value and capital in the cross section. Each bank chooses an optimal capital structure and those with higher capital also have higher value. Our general equilibrium framework has many possible relationships depending on which bank investment possibility is relevant. None of these studies is designed to consider the interrelationship between asset and liability structures that is the focus of our model.

One important ingredient of our model is that depositors are segmented from capital providers in that they do not participate directly in financial markets and thus in firm financing. In this sense, the paper relates to the literature on limited participation in financial markets (see, e.g., the survey in Guiso and Sodini (2013)). In the context of banking markets, Diamond (1984) and Winton (1995) study settings where, as a result of asymmetric information, banks emerge as intermediaries between firms and uninformed depositors in order to economize on bankruptcy costs at the firm level. Our focus, however, is on the role of capital as a way to reduce bankruptcy costs either at the bank or at the firm level, and how the optimal capital structure varies depending on the organization form of banks and firms.

The paper proceeds as follows. Section 2 develops the baseline model. The equilibrium of this is considered in Section 3. Section 4 introduces deposit insurance and studies the role for capital regulation. Section 5 introduces two alternative corporate forms – public firms and private firms – and analyzes how capital is allocated and what its return is under the two alternative forms. Finally, Section 6 contains concluding remarks. All proofs are in the appendix.

2 A model of bank capital structure with direct investment

In this section we develop a simple one-period ($T = 0, 1$) model of financial intermediation where banks raise external funds through deposits and capital, and invest in a risky technology. This can either be interpreted as investment in non-publicly traded productive firms or as direct investment in a risky line of business such as market making, underwriting, proprietary trading or fees from advisory services such as mergers and acquisitions. We will refer to this model as our baseline model since we will study variations later in the paper.

The risky technology is such that for each unit invested at date 0 there is a stochastic return r at date 1 uniformly distributed on the support $[0, R]$, with $Er = \frac{R}{2} > 1$.

Since there are constant returns to scale we normalize the size of every bank to 1. Each bank finances itself with an amount of capital k_B and an amount of (uninsured) deposits $1 - k_B$. The bank has limited liability. There are two groups of risk neutral investors, capital investors and depositors. The former have an endowment of 1 each and can supply capital or deposits to banks, with the opportunity cost for investing in the bank equity or deposit market being ρ. They also have the outside option of investing directly in the risky technology so that $\rho \geq R/2$. The latter can supply deposits only. The promised per unit rate from the bank is r_D and the opportunity cost of deposits in the bank deposit market is u. Depositors have an endowment of 1 each and also have a storage option with return 1 for each unit invested so that $u \geq 1$. Banks compete for deposits and will thus always set r_D at the level required for depositors to recover their opportunity cost u. The two markets are segmented in the sense that depositors do not have access to the equity market. The idea is that they have high participation costs that make them unwilling to enter the equity market. The depositors have total wealth D. The capital providers on the other hand have zero participation costs and can access both markets. Their total wealth, and hence the total possible supply of capital, is denoted K. The ratio of the wealth of the capital providers to the wealth of the depositors is

$$\frac{K}{D} = \eta > 0. \tag{1}$$

There is free entry into banking so the banking sector is perfectly competitive. Since banks invest in a risky technology, deposits are risky. The bank repays the promised rate r_D if $r \geq \overline{r}_B$, where

$$\overline{r}_B = r_D(1 - k_B), \tag{2}$$

and it goes bankrupt otherwise. When it goes bankrupt, the proceeds from liquidation are $h_B r$ with $h_B \in [0, 1]$ and these are distributed pro rata to depositors. The bankruptcy costs are thus $(1 - h_B)r$.

3 The equilibrium with direct investment

In this section we analyze the equilibrium of the model. This requires the following:

1. Banks choose k_B and r_D to maximize expected profits.

2. Capital providers maximize expected utility.

3. Depositors maximize expected utility.

4. Banks make zero expected profits in equilibrium.

5. The equity market clears.

6. The deposit market clears.

We start by considering the individual bank's optimization problem given by:

$$\max_{k_B, r_D} E\Pi_B = \int_{\overline{r}_B}^{R} (r - r_D(1-k_B)) \frac{1}{R} dr - \rho k_B \tag{3}$$

subject to

$$EU_D = \int_0^{\overline{r}_B} \frac{h_B r}{1-k_B} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \tag{4}$$

$$E\Pi_B \geq 0 \tag{5}$$

$$0 \leq k_B \leq 1, \tag{6}$$

where $\overline{r}_B$ is as in (2). The bank chooses k_B and r_D to maximize its expected profit net of the cost of funds. The first term in (3) is what the bank obtains from the investment after paying $r_D(1-k_B)$ to the depositors. This is positive only when $r > \overline{r}_B$ and it is distributed to the shareholders. When $r < \overline{r}_B$, the bank goes bankrupt and obtains nothing. The second term ρk_B is the shareholders' opportunity cost of providing capital. Constraint (4) requires that the expected utility of depositors is at least equal to their opportunity cost u. The first term is the payoff when the bank goes bankrupt and each depositor receives a pro rata share $\frac{h_B r}{1-k_B}$ of the liquidation proceeds. The second term represents the payoff depositors receive when the bank remains solvent. Since depositors are uninsured, the promised repayment r_D must compensate depositors for the risk they face when placing their money in a bank that may go bankrupt. Constraint (5) is the requirement that the shareholders obtain their opportunity cost from providing capital to the bank. The last constraint (6) is a feasibility constraint on the amount of capital.

In equilibrium, since there is free entry into the banking market, each bank's expected profit must be zero. This means that ρ adjusts so that $E\Pi_B = 0$. Capital providers can either supply equity to the banks for a return of ρ or invest in their outside option for a return $R/2$. The sum of these two investments must be equal to K for the equity market to clear. Capital providers will invest in bank equity alone if $\rho > R/2$. They will invest both in bank equity and in the outside option if $\rho = R/2$. In other words,

$$N_B k_B \leq K, \tag{7}$$

where N_B represents the number of banks and (7) holds with an equality when $\rho > R/2$.

Similarly, depositors can either deposit their money in the banks for a promised return of r_D and an expected utility $EU = u \geq 1$, or use the storage option with a return of 1 and an expected utility $EU = 1$. The investments in deposits and in the storage option must sum to D for the deposit market to clear. The depositors will just deposit in banks and will not store if $u > 1$. They will both deposit and store if $u = 1$. It will be shown below that the form of the equilibrium depends on whether the constraint (4) binds with $u = 1$ or $u > 1$. In other words,

$$N_B(1 - k_B) \leq D,$$

where there is an equality when $u > 1$, and a strict inequality otherwise.

3.1 The Modigliani and Miller Case: No bankruptcy costs ($h_B = 1$)

We start by considering the benchmark case where there are no bankruptcy costs so that $h_B = 1$. The difference is that depositors receive the full return r when the bank goes bankrupt. This leads to the following result.

Proposition 1 *With $h_B = 1$, there are multiple equilibria. Each bank is indifferent between choosing any pair $k_B \in [0, 1]$ and $r_D = \frac{(1-\sqrt{k_B})}{1-k_B} R$ for $k_B < 1$. In any equilibrium, $\rho = u = \frac{R}{2}$, $E\Pi_B = 0$, $EU_D = \frac{R}{2}$, $N_B k_B \leq K$ and $N_B(1 - k_B) = D$.*

Depositors can only have access to the risky technology through banks. When there are no bankruptcy costs ($h_B = 1$), the efficient allocation is to channel all deposits into the risky technology given that $\frac{R}{2} > 1$. Banks simply channel funds from the deposit sector to more productive use. Competition among banks drives up the cost of deposits to the point $u = \frac{R}{2}$. Since equity providers have the option of investing directly in the risky technology and capital has no role in reducing bankruptcy costs $\rho = u = \frac{R}{2}$. With these equilibrium prices, Modigliani and Miller holds. Capital can be invested either in banks or directly in the risky technology, while all deposits are placed in the banking sector. This means that there are multiple equilibria depending on the proportion of capital invested in banks versus directly. This mix does not affect the real allocation.

3.2 Bankruptcy costs $(0 \leq h_B < 1)$

We now consider the case where there are bankruptcy costs in the banking sector. For simplicity, we start with the case where $h_B = 0$. This corresponds to zero liquidation proceeds so depositors obtain nothing in the case the bank goes bankrupt. We have the following result.

Proposition 2 *The unique equilibrium with $h_B = 0$ is as follows:*

i) For $R < \overline{R} = 4(\frac{1+\eta}{1+2\eta}) < 4, k_B = \frac{4-R}{R} \in (0,1), r_D = \frac{R}{2}, \rho = \frac{2}{4-R} > \frac{R}{2}, u = 1, E\Pi_B = 0, EU_D = 1, N_B k_B = K$ and $N_B(1-k_B) < D$.

ii) For $R \geq \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{R}{2}, \rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)}\frac{R}{2} > \frac{R}{2}, u = \frac{1+2\eta}{2(1+\eta)}\frac{R}{2} \in [1, \frac{R}{2}), E\Pi_B = 0, EU_D > 1,$ $N_B k_B = K$ and $N_B(1-k_B) = D$.

The proposition shows that once we have the friction of bankruptcy costs, Modigliani and Miller no longer holds. More equity financing leads to lower bankruptcy costs and its opportunity cost is bid up as a result so $\rho > \frac{R}{2}$. Thus, shareholders always obtain strictly more than their outside option. There is a trade-off in that equity is a relatively costly form of finance but has the advantage of reducing expected bankruptcy costs. A unique optimal bank capital structure exists and each bank uses both capital and deposits to fund itself. The bank can afford to pay $\rho > \frac{R}{2}$ for equity finance because the cost of deposit finance is $u < \frac{R}{2}$. If there was no market segmentation so that depositors could invest directly in equity, then ρ would be equal to $\frac{R}{2}$. As shown above, when there are no bankruptcy costs so that $h_B = 1$, equity has no value in reducing the bankruptcy costs so $\rho = \frac{R}{2}$. Thus, both bankruptcy costs and market segmentation are necessary for the result that equity is costly. Since in equilibrium $\rho > \frac{R}{2}$, all the capital is absorbed in the banking sector and none is invested directly in the outside option.

Unlike capital, the opportunity cost of deposits u is not always bid up above the storage option. Deposit finance is cheaper than equity but introduces bankruptcy costs. The difference between the expected returns of the outside option of equity investors and the storage option of deposit providers is low when R is low. This means that deposits are not very attractive relative to equity given the bankruptcy costs they introduce. This is why for $R < \overline{R}$ deposits are only partly placed in the banking sector where they obtain $u = 1$, and the storage option is widely used. As R is increased, more deposits are used in the banking sector. At $R = \overline{R}$ all deposits are used there so that there is full inclusion. For $R > \overline{R}$, the opportunity cost of deposits is bid up and $u > 1$.

The results on the returns to the investors hold as long as the ratio of total capital to total deposits, η, is positive and finite. For $\eta \to 0$, deposits would always be abundant so that $u \to 1$ for any value of R. By contrast, for $\eta \to \infty$, both $\rho \to \frac{R}{2}$ and $u \to \frac{R}{2}$. In other words, when there is no scarcity of capital, capital loses it main role and its equilibrium return is the same as that of deposits.

A number of comparative statics results follow easily.

Corollary 1 *The following comparative statics results hold:*

i) The optimal amount of capital, k_B, is (weakly) decreasing in the project's return R, i.e., $\frac{\partial k_B}{\partial R} \leq 0$, with the inequality strict for $R < \overline{R}$.

ii) The equilibrium return on capital, ρ, is increasing in R, i.e., $\frac{\partial \rho}{\partial R} > 0$.

iii) The equilibrium expected return on deposits, u, is (weakly) increasing in R, i.e., $\frac{\partial u}{\partial R} \geq 0$, with the inequality strict for $R > \overline{R}$.

iv) The threshold value $\overline{R}$ is decreasing in η, the ratio of total available capital to deposits, i.e. $\frac{\partial \overline{R}}{\partial \eta} < 0$.

The corollary illustrates that the optimal amount of capital and the split of the surplus generated from the banks' investments in the risky asset between the shareholders and the depositors depends on the project's return R. For $R < \overline{R}$, all the surplus is captured by the shareholders through the return ρ. As R increases up to $\overline{R}$, capital decreases and ρ rises. As the return of the risky technology increases further, it is increasingly profitable for banks to use deposits for funding. This makes capital more valuable because bankruptcy increases and ρ is bid up. For $R > \overline{R}$, all deposits are used and thus bank capital structure remains constant. As R increases beyond $\overline{R}$, the shareholders and depositors share the surplus with both u and ρ continuing to rise.

The degree of financial inclusion in terms of the proportion of deposit funds used in the banking system depends also on R. For $R = 2$ financial inclusion is zero. As R increases to $\overline{R}$, it increases to 1. Full financial inclusion is reached at lower levels of R the greater is the ratio of capital to deposits since the threshold $\overline{R}$ decreases with η.

These comparative statics results hold in all cases below so we omit explicit discussion in the following propositions.

The insights of Proposition 2 remain valid in the case of partial bankruptcy costs where $h_B \in (0, 1)$ and depositors obtain $\frac{h_B r}{1-k_B}$ when the bank goes bankrupt. We obtain the following result, which is similar to that of the previous proposition, but algebraically more complex. As the relationship between R and financial inclusion is the same as in the previous proposition, we omit the explicit discussion here again.

Proposition 3 *The unique equilibrium with $h_B \in (0, 1)$ is as follows:*

i) For $R < \overline{R}, k_B = \frac{(2-h_B R)(2(2-h_B)-R)}{2(1-h_B)^2 R} \in (0,1), r_D = \frac{2(1-h_B)R}{2(2-h_B)-h_B R}, \rho = \frac{2-h_B R}{2(2-h_B)-R} > \frac{R}{2}, u = 1, E\Pi_B = 0, EU_D = 1.$

ii) For $R \geq \overline{R}$, $k_B = \frac{\eta}{1+\eta} \in (0,1), r_D = \frac{2u(1-h_B)R}{2u(2-h_B)-h_B R}, \rho = \frac{u(2u-h_B R)}{2u(2-h_B)-R} > \frac{R}{2}, u = \frac{2(1+\eta)-(1-h_B)\left(1-h_B-\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{2(1+\eta)(2-h_B)}\frac{R}{2} \in [1, \frac{R}{2})$, $E\Pi_B = 0, EU_D > 1.$

The expression for $\overline{R}$ is given in the appendix.

The main difference from Proposition 2 is that banks' capital structure and the sharing of the surplus depend on the size of the bankruptcy proceeds as represented by h_B. For a given $R \leq \overline{R}$, the higher h_B the lower the amount of capital k_B at each bank and the higher the shareholders' return ρ. For a given $R > \overline{R}$, k_B remains constant as h_B increases, but both shareholders and depositors obtain higher returns ρ and u. The intuition is simple. As bankruptcy proceeds increase, capital becomes less necessary as a way to reduce bankruptcy costs and thus each bank uses less of it.

3.3 Efficiency of the market solution

An important question is whether the allocations of the baseline model as described in Propositions 2 and 3, which we refer to as the market solution (in contrast to the regulatory solution we will analyze in the next section), are efficient. To analyze this, we consider the case where in the baseline model the level of capital is chosen by a regulator that maximizes social welfare while deposit rates are still set by the banks in order to maximize their expected profits.

Formally, a regulator sets the level of capital k_B to maximize social welfare, which is given by

$$\max_{k_B} SW = \rho K + uD \tag{8}$$

subject to constraints (4)-(6) and

$$r_D = \arg\max_{r_D} E\Pi_B = \int_{\overline{r}_B}^{R} (r - r_D(1 - k_B)) \frac{1}{R} dr - \rho k_B, \tag{9}$$

where $\overline{r}_B$ is as in (2). All other conditions for the equilibrium remain the same and we obtain the following result.

Proposition 4 *The regulator chooses the same level of capital as banks choose in Propositions 2 and 3. The rest of the equilibrium is also the same as there.*

The proposition shows that the allocations described in Proposition 2 for $h_B = 0$ and in Proposition 3 for $h_B > 0$ are (constrained) efficient. The amount of capital that banks hold in the market solution is the same as the level chosen by a regulator who maximizes social welfare. The competitiveness of the banking sector together with the fact that the deposit rate reflects the bank's bankruptcy risk induces banks to choose the social welfare maximizing levels of capital. There is thus no need for capital regulation in our model in the absence of other possible distortions, such as those we analyze next through the introduction of deposit insurance.

4 Deposit insurance and capital regulation

So far we have considered the case where deposits are not insured so that the promised deposit rate reflects the risk taken by the bank. In that case, as we have shown above, banks have an incentive to hold a positive amount of capital as a way to reduce bankruptcy costs when investing directly in a risky technology and the resulting allocation is efficient. In this section we study the case where deposits are insured so that depositors always receive the promised deposit rate irrespective of whether their banks go bankrupt or not.

As we show below, the presence of deposit insurance results in a need for capital regulation. To see why we first introduce deposit insurance and show that there is scope for regulation as banks no longer have incentives to hold a positive level of capital. Then, we analyze the allocation with deposit insurance and capital regulation and compare it with the market allocation obtained in the baseline model.

4.1 Deposit insurance

We now study the case where deposits are fully insured so that depositors always receive the promised deposit rate irrespective of whether their banks go bankrupt or not. We interpret deposit insurance as being provided by the government: if the bank goes bankrupt, the government intervenes and pays the promised interest rate r_D to the depositors. The cost of the deposit insurance is paid from revenues raised by nondistortionary lump sum taxes.

The bank's optimization problem is still as in (3)-(6) with the difference that constraint (4) becomes

$$EU_D = \int_0^R r_D \frac{1}{R} dr \geq u. \tag{10}$$

All other conditions remain the same. We have the following result.

Proposition 5 *The unique equilibrium with deposit insurance and $0 \leq h_B < 1$ is as follows:*

i) Banks hold no capital, i.e., $k_B = 0$, and set $r_D = u = R$.

ii) Capital providers provide deposits so that $\rho = u = R$ and $N_B = D + K$.

As shown in the proposition, the introduction of deposit insurance induces banks not to hold any capital. Given that depositors are always repaid in full and banks do not internalize the cost of the deposit insurance, they have no incentives to hold capital to reduce the bankruptcy costs. The equilibrium requires depositors to obtain the whole surplus from the project, with banks making zero expected profits in equilibrium, as before. Capital providers then prefer to offer their capital in the form of deposits to the banks and obtain the same return as depositors.

The equilibrium described in the proposition implies that the bank always goes bankrupt and the government is always forced to intervene and guarantee the repayment $u = R$ to all the $D+K$ depositors. This implies a very high deadweight cost of bankruptcy. Given this, the presence of deposit insurance may give a role for capital regulation as a way of reducing the disbursement for the deposit insurance fund as in, for example, Hellmann, Murdock and Stiglitz (2000), Repullo (2004), Morrison and White (2005) and Allen, Carletti, and Marquez (2011). We study this issue next.

4.2 The role of capital regulation

The arguments above suggest a role for capital regulation in the presence of deposit insurance. As in Section 3.3, we consider the case when a regulator sets the level of capital that each bank holds at the beginning of date 0 in order to maximize social welfare while the deposit rate is still determined by the banks and is thus part of the market solution. Formally, the regulator's maximization problem is as follows:

$$\max_{k_B} SW = \rho K + uD - N_B \int_0^{\overline{r}_B} \left(r_D(1-k_B) - h_B r\right) \frac{1}{R} dr \tag{11}$$

subject to the constraints (9), (10), (5) and (6) and $\overline{r}_B$ as in (2). As in Section 3.3, social welfare is given by the sum of the returns to the capital providers and depositors, as represented by the first two terms in (11). The provision of deposit insurance, however, is internalized by the regulator in setting the capital requirement. The last term in (11) captures the insurer's disbursement when, for $r \in (0, \overline{r}_B)$, N_B banks are insolvent and each needs $r_D(1-k_B) - h_B r$ to repay r_D to its $(1-k_B)$ depositors. The expression for the social welfare in (11) can equivalently be expressed as

$$SW = N_B \left(\frac{1}{2} R - \int_0^{\overline{r}_B} (1-h_B)\, r \frac{1}{R} dr \right) + \max\{(D - (1-k_B)\, N_B)\, , 0\}, \tag{12}$$

where the first term is the total output of all the projects invested in by the N_B banks, minus the deadweight losses associated with bankruptcy; and the second term is simply the number of deposits that are invested in the storage alternative rather than being deposited at a bank. This term is zero when $u > 1$, since then all depositors place their money in a bank, while $D - (1-k_B)\, N_B$ may be positive when $u = 1$.

The rest of the equilibrium is as in the case with no deposit insurance. The returns of the capital investors and of the depositors are determined by the banks' zero profit condition and the market clearing conditions, respectively, and the regulator takes into account how the choice of k_B affects them. We obtain the following result, where we omit the discussion on financial inclusion as it is the same as in the baseline model.

Proposition 6 *In the case of deposit insurance and capital regulation, the unique equilibrium with* $0 \le h_B < 1$ *is as follows:*

i) For $R < \overline{R}^{reg}, k_B^{reg} = \sqrt{\frac{1-h_B+2R-R^2}{1-h_B}} \in (0,1), \rho^{reg} > \frac{R}{2},\ u^{reg} = r_D^{reg} = 1, E\Pi = 0,$

$EU = 1$.

ii) For $R \geq \overline{R}^{reg}$, $k_B^{reg} = \frac{\eta}{1+\eta} \in (0,1)$, $\rho^{reg} > \frac{R}{2}$, $u^{reg} = r_D^{reg} = \frac{2(1+\eta)\left(\sqrt{2+4\eta+\eta^2-h_B(1+2\eta)}-(1+\eta)\right)}{(1-h_B)(1+2\eta)}\frac{R}{2} \in [1, \frac{R}{2})$, $E\Pi = 0$, $EU > 1$.

The expressions for $\overline{R}^{reg}$ *and* ρ^{reg} *are given in the appendix.*

The proposition has the same structure as Propositions 2 and 3, which describe the market allocation. As there, Modigliani and Miller does not hold: capital structure is not irrelevant but rather there is a unique optimal capital structure for banks that maximizes social welfare. Likewise, the proposition shows that, even under the regulatory solution, market segmentation implies that in equilibrium there will be a wedge between the returns of capital and of deposits, so that $\rho^{reg} > \frac{R}{2} > u^{reg} \geq 1$.

The optimal capital requirement k^{reg} and the returns ρ^{reg} and u^{reg} to shareholders and depositors, respectively, depend on the return of the risky technology R. When R is low, the marginal benefit of adding another bank, which is achieved by having each bank hold less capital, is low relative to the increased bankruptcy risk associated with greater deposit financing. A regulator therefore trades off increasing output with reducing bankruptcy risk. As R increases, the incentive to channel funds toward productive projects increases, shifting the regulator's tradeoff toward less capital at each bank - and hence more banks - and consequently a greater amount of deposit financing. In other words, there is a push toward greater financial inclusion, with all deposits being placed at banks rather than in storage, as R increases.

Given that deposit insurance introduces a market distortion – deposits are no longer priced to reflect their risk, leading banks to want to use no capital in the absence of regulation – it becomes important to understand how social welfare is affected relative to the unregulated market solution of the baseline model, where deposits are uninsured. We therefore compare the allocation with deposit insurance and capital regulation described in the proposition above with the market allocation described in Propositions 2 and 3. We have the following result.

Proposition 7 *The regulatory solution always entails a higher level of social welfare than the market solution: $SW^{reg} > SW$. Moreover, it also entails a lower level of capital, $k_B^{reg} \leq k_B$, with the inequality strict whenever $u = 1$ in the market solution.*

An interesting implication of our analysis is that deposit insurance coupled with capital regulation is beneficial in that it improves on the market solution by yielding a higher social welfare. In the market solution, the only way to avoid bankruptcy costs is through the use of capital. As shown above, this is efficient when deposits are uninsured, and a social planner would choose the same level of capital as what the bank chooses on its own. However, deposit insurance introduces a new channel through which deadweight losses from bankruptcy can be reduced. When deposits are insured, depositors are willing to accept a lower promised repayment on their deposits since they bear no risk of default from lending to the bank. The reduction in the deposit rate directly leads to a reduction in the threshold where bankruptcy occurs, and, ceteris paribus, reduces the deadweight bankruptcy costs.

In the absence of capital regulation, banks would choose to hold no capital (Proposition 5), thus undoing much of the savings in bankruptcy costs obtained from deposit insurance. There is consequently a role for regulation: by requiring the banks to hold capital, a social welfare maximizing regulator can further reduce bankruptcy costs, complementing the benefits obtained from the reduction in deposit rates. In other words, capital regulation becomes important when deposits are insured, even if it is unnecessary when these are not insured.

Interestingly, the level of capital that maximizes social welfare when deposits are insured is never greater, and is often strictly lower, than that which maximizes bank profits in the market solution. The reason stems from the regulator's objective function, which reduces to maximizing aggregate output net of the deadweight losses of bankruptcy instead of the individual bank's expected profit. All things equal, aggregate output is increased by increasing the degree of financial inclusion, which is achieved by having more banks in operation. To accomplish this goal, the regulator has an incentive to reduce the level of capital at each bank relative to what occurs in the market solution. Therefore, a regulator whose objective is to maximize social welfare will choose a lower level of capital for each bank. Once all deposits are in use at a bank rather than invested in the storage technology in both solutions, the regulatory level of capital coincides with that of the market.

As a final point, it is worth noting that given that social welfare is higher in the regulatory solution with deposit insurance, this also means that the payments to the investors – capital suppliers and depositors – in each bank must likewise be higher than in the market solution. This occurs because each bank now has a lower loss from bankruptcy costs and thus has more surplus to allocate to the providers of funds.

5 Extensions: Lending to firms

The results so far have focused on the case where banks invest directly in the productive assets, essentially making them the owners of these projects. While useful for understanding the role of limited market participation and bankruptcy costs in determining banks' capital structures, the more common perspective on banks is that they channel funds to firms through the allocation of credit. In this section we analyze two extensions in that direction, each representing an alternative extreme in how a firm in need of financing may be organized. The first case considers public firms that have no inside equity but can attract funds both from banks and outside equity investors. The second case considers instead private firms that have an initial endowment of inside equity capital but can only raise external funds in the form of bank loans and, in particular, are unable to raise outside equity financing. As we will show, while the main results of the baseline model carry over to both cases – capital earns rents in excess of its outside option, and its equilibrium return is higher than that of deposits – there are substantial and important differences in how the funds of capital suppliers are allocated and thus in the optimal capital structure of both banks and firms.

5.1 Public firms

In this section we consider the case where a continuum of publicly traded firms in a productive sector hold the risky technology with return $r \sim U[0, R]$ as before, and can raise outside equity financing from the market. This means that capital suppliers now have a choice of investing in the bank or making equity investments directly into the public firms, so that these firms face no frictions in raising capital. As in the baseline model, deposits are uninsured.

Each firm requires 1 unit of funds and finances this with equity k_F and loans from banks of $1 - k_F$. As before, in equilibrium capital suppliers earn a return $\rho \geq \frac{R}{2}$, independently of whether they choose to invest in the firms or in the banks. The promised per unit loan rate on bank loans is r_L, which the bank receives if the firm is solvent. This is the case if $r \geq \overline{r}_F$, where

$$\overline{r}_F = r_L(1 - k_F). \tag{13}$$

If $r < \overline{r}_F$, the firm goes bankrupt and the liquidation proceeds $h_F r$, with $h_F \in [0, 1]$, are distributed pro-rata to the banks providing the $1 - k_F$ in loans.

Banks raise equity k_B and take deposits $1 - k_B$ in exchange for a promised rate r_D. When the bank receives r_L from the firms, it remains solvent and repays $r_D(1-k_B)$ to its depositors. If $r < \overline{r}_F$, firms go bankrupt and banks receive $h_F r$ for each $1 - k_F$ loaned out so that each bank receives $\frac{h_F r}{1-k_F}$ per unit loaned. If $\frac{h_F r}{1-k_F} \geq r_D(1 - k_B)$ the bank remains solvent and pays depositors in full. Differently, if $\frac{h_F r}{1-k_F} < r_D(1 - k_B)$ the bank will itself go bankrupt and each depositor obtains only $\frac{h_F h_B r}{(1-k_F)(1-k_B)}$. This implies that when the firm goes bankrupt the bank can either remain solvent for $\frac{r_D(1-k_B)(1-k_F)}{h_F} < r < \overline{r}_F$ or go bankrupt with the firm for $r < \overline{r}_F$. Formally, the bank goes bankrupt for any $r < \overline{r}_B$, where

$$\overline{r}_B = \min\left\{\frac{r_D(1 - k_B)(1 - k_F)}{h_F},\ \overline{r}_F\right\}. \tag{14}$$

Banks choose the loan rate r_L and, for simplicity, we assume that they can impose loan covenants that specify the firms' level of equity k_F.

In addition to conditions (3)-(6), the equilibrium requires that

7. Banks choose k_F and r_L in addition to k_B and r_D to maximize their expected profits.

8. Firms make zero expected profits in equilibrium.

9. The loan market clears.

As before, the equity and the deposit markets have to clear in equilibrium. Given the presence now of two sectors, market clearing requires that

$$N_F k_F + N_B k_B \leq K \tag{15}$$

and

$$N_B(1 - k_B) \leq D, \tag{16}$$

where N_F and N_B are the number of firms and banks respectively. Conditions (15) and (16) require that the total capital used in the productive and the banking sectors does not exceed the available capital K, and that the total deposits in the banking sector do not exceed the total supply D in the economy. As before, (15) and (16) hold with equality if $\rho > \frac{R}{2}$ and $u > 1$.

The loan market must clear so that

$$N_F(1 - k_F) = N_B. \tag{17}$$

This states that the total lending $N_F(1-k_F)$ needed by the firms equals the total resources available for lending at the N_B banks.

Each individual bank's maximization problem is now given by:

$$\max_{k_F, r_L, k_B, r_D} E\Pi_B = \int_{\bar{r}_B}^{\bar{r}_F} \left(\frac{h_F r}{1 - k_F} - r_D(1 - k_B) \right) \frac{1}{R} dr + \int_{\bar{r}_F}^{R} \left(r_L - r_D(1 - k_B) \right) \frac{1}{R} dr - \rho k_B \tag{18}$$

subject to

$$E\Pi_F = \int_{\overline{r}_F}^{R} (r - r_L(1-k_F)) \frac{1}{R} dr - \rho k_F \geq 0 \tag{19}$$

$$EU_D = \int_0^{\overline{r}_B} \frac{h_B h_F r}{(1-k_B)(1-k_F)} \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D \frac{1}{R} dr \geq u \geq 1 \tag{20}$$

$$0 \leq k_F \leq 1, \tag{21}$$

together with (5) and (6). The first term in (18) represents the expected payoff to the bank when firms go bankrupt but the bank remains solvent for $\overline{r}_B < r < \overline{r}_F$. In this case, the bank obtains the firms' liquidation proceeds $\frac{h_F r}{1-k_F}$ after repaying the amount $r_D(1-k_B)$ to its depositors. By contrast, when $\overline{r}_B = \overline{r}_F$, the bank goes bankrupt whenever the firm does so, and the first term in (18) becomes zero. The second term is the expected payoff to the bank from lending one unit to firms at the rate r_L after paying $r_D(1-k_B)$ to its depositors. The last term ρk_B is the opportunity cost for bank shareholders. Constraint (19) requires the expected profit of the firm to be non-negative. The first term is the expected payoff to the firm from the investment in the risky technology after paying $r_L(1-k_F)$ to the bank for $r > \overline{r}_F$. The last term ρk_F is the opportunity cost for firm shareholders. Constraint (20) is depositors' participation constraint. The first term is the payoff when the bank goes bankrupt for $r < \overline{r}_B$ and each depositor obtains a share $\frac{h_B}{(1-k_B)}$ of the $\frac{h_F r}{(1-k_F)}$ resources available at the bank. The second term is depositors' payoff for $r \geq \overline{r}_B$, when the bank remains solvent and each depositor obtains the promised repayment r_D.

We obtain the following result.

Proposition 8 *The unique equilibrium with $0 \leq h_B, h_F < 1$ in the case of public firms is as follows:*

i) Banks hold $k_B = 0$ and set $r_D = r_L$.

ii) The rest of the equilibrium is as in the case where banks hold the technology directly described in Propositions 2 and 3 with the difference that firms hold the same capital k_F as banks there.

The proposition states that in equilibrium banks hold no capital and are thus simply a conduit between depositors and firms. This minimizes overall bankruptcy costs because it aligns bank and firm bankruptcies with $\overline{r}_B = \overline{r}_F$.

The result is illustrated in Figure 3, which shows the output of a single firm as a function of the return r, and how this is split among shareholders and depositors. Consider first the case where both the bank and the firm hold positive capital and the firm goes bankrupt at a higher level of r than the bank, i.e., $\overline{r}'_F = r'_L(1-k'_F) > \overline{r}'_B = \frac{r'_D(1-k'_B)(1-k'_F)}{h_F}$. Region A represents the payoff to firm shareholders for $r \in (\overline{r}'_F, R]$, when the firm remains solvent and repays $r'_L(1-k'_F)$ to the bank. Region $B+C$ represents the payoff to the bank shareholders. For $r \in [\overline{r}'_F, R]$, the bank receives the promised repayment $r'_L(1-k'_F)$. For $r \in [\overline{r}'_B, \overline{r}'_F)$, the firm goes bankrupt and the bank receives $\frac{h_F r}{1-k'_F}$. Region $D1$ represents the deadweight loss derived from the bankruptcy of the firm. Region $E1+F$ represents the payoff to bank depositors. For $r \in [\overline{r}'_B, R]$, the bank is solvent, and each depositor receives the promised repayment r'_D. Since there are $(1-k'_B)(1-k'_F)$ depositors per firm, they obtain $r'_D(1-k'_B)(1-k'_F)$ in total. For $r \in [0, \overline{r}'_B)$ the bank goes bankrupt. Each of the $(1-k'_B)$ depositors in the bank receives a share $\frac{h_B}{1-k'_B}$ of the resources $\frac{h_F r}{1-k'_F}$ that the bank has. Thus, the $(1-k'_B)(1-k'_F)$ depositors per firm obtain $h_B h_F r$ in total.

Consider now transferring all capital from the bank to the firm and aligning the bankruptcy points of the bank and the firm. This entails setting $k^*_B = 0$ and $k^*_F = k'_B(1-k'_F) + k'_F$. The firm then has a transfer of $k'_B(1-k'_F)$, which is the amount of capital that the bank has per firm, in addition to its original amount k'_F. Since the bank has zero capital, it is possible to set $r^*_D = r^*_L = r'_D$ so that the bank becomes a conduit with zero profit. This aligns the firm and bank bankruptcy points and changes them to $\overline{r}^*_F = r^*_L(1-k^*_F) = \overline{r}^*_B = r'_D(1-k'_B)(1-k'_F) < \overline{r}'_B = \frac{r'_D(1-k'_B)(1-k'_F)}{h_F}$. It is immediate to see that this allows the deadweight losses in Region $D1+D2$ and $E2$ to be eliminated and improves the allocation.

This argument shows that in any equilibrium it must be the case that $k_B = 0$ and $r_L = r_D$. The optimal choice of k_F and r_L is then the same as the bank's choice of k_B and r_D when the bank invests directly in the risky asset except that the liquidation proceeds

$h_B r$ are replaced by $h_B h_F r$. The equilibrium is then as described in Proposition 8.

5.2 Private firms

In this section we consider a slightly different setup from the one in the previous section in that we study the case where firms are "private," meaning that, while they may possess some capital already, they are unable to raise additional outside equity from capital suppliers. Specifically, we assume that each private firm is endowed with capital $0 \leq k_F < 1$, but can only raise the remaining $1 - k_F$ as a bank loan rather than being able to obtain direct equity investments from capital suppliers. In the context of the discussion from the previous section, this can be interpreted as entrepreneurs/firms that face frictions in raising outside equity. Finally, to be consistent with our analysis above of public firms, we focus on the case where capital and deposits are in short supply relative to the number of private firms that would like to borrow, meaning that the number of entrepreneurs is large relative to the number of banks, which is at most $K + D$, but may be less for the case where $u = 1$.

The bank's maximization problem is still given by (18)-(20) with the difference that k_F is now fixed and that capital providers to banks and firms may obtain different returns denoted, respectively, as ρ_B and ρ_F. The latter is set equal to $\frac{R}{2}$ because of the assumption on the abundance of productive firms relative to capital and deposits. Finally, to simplify the problem, we focus on the case where $h_B = 0$ so that there is no recovery if the bank is unable to meets its obligations to depositors. This eliminates the first term in depositors' expected utility in (20).

We can now obtain the following result, which is illustrated in Figure 4.

Proposition 9 *The unique equilibrium in the case of private firms, for Regions A through D, is as follows:*

A. $k_B = 0$, $r_L = r_D, u = u_C \in (1, \frac{R}{2}), E\Pi_B = 0, EU > 1$ *and* $N_B = D$;

B. $k_B > 0, r_L > r_D, \rho_B(u) \geq \rho_B(u_C) > \frac{R}{2}$, $u \geq u_C \in (1, \frac{R}{2}), E\Pi_B = 0, EU > 1, N_B k_B = K, N_B(1 - k_B) = D$;

C. $k_B \geq \frac{\eta}{1+\eta}, r_L > r_D, \rho_B(u) > \frac{R}{2}, u \in [1, \frac{R}{2}), E\Pi_B = 0, EU \geq 1, N_B k_B = K, N_B(1 - k_B) \leq D$;

D. There is no intermediation.

The boundaries $\underline{R}_C, \underline{R}_{k_B}, \underline{h}_F$ *defining Regions A through D are shown in Figure 4 and, together with the various expressions for* $\rho_B(u), \rho_B(u_C), u,$ *and* u_C *are defined in the appendix.*

Proposition 9 demonstrates that while our main results concerning the costs of bank capital relative to deposits carry over to a setting where firms are private in the sense of being unable to raise outside equity. However, it also shows that the introduction of private firms raises new issues for banks' capital structures that were not present when studying public firms in Section 5.1. In that case, Proposition 8 establishes that in equilibrium banks always act as conduit banks, with all capital flowing directly to the firms in order to minimize the deadweight costs of bankruptcy. When firms are private, however, capital cannot freely flow to firms needing financing and must instead be channeled through the banking sector in the form of loans.

As illustrated in Region D in Figure 4, when projects' returns are very low, no intermediation is possible. In the region labeled $A1$, intermediation becomes possible, but only for a bank that holds no capital and acts purely as a conduit between depositors and firms. As h_F increases so that bankruptcy costs are reduced, a bank holding a positive level of capital becomes feasible when Region $A2$ is reached. However, this capital structure is not yet optimal because the bank cannot provide depositors with the same utility u_C as the conduit bank. When h_F reaches $\underline{h}_F$ in Region B, the bank with positive capital becomes optimal as it can offer at least u_C to depositors and, at the same time, $\rho_B(u) \geq \rho_B(u_C) > \frac{R}{2}$ to capital providers. Region B can be thought of as "contestable" since banks can only attract deposits by paying at least what a conduit bank would pay. This limits banks' ability to remunerate capital suppliers, so that ρ_B may be constrained at a lower level than what would be optimal if there were no contestability. Only when conduit banks are not feasible, such as in Region C positive capital banks behave in an unconstrained manner, holding the optimal amount of capital and ignoring the possible entry of a conduit bank.

6 Concluding remarks

We have developed a general equilibrium model of banks and firms to endogenize the equity cost of capital in the economy. The two key assumptions of our model are that deposit and equity markets are segmented and there are bankruptcy costs for banks and firms. We have shown that in equilibrium equity capital has a higher expected return than investing directly in the risky asset. Deposits are a cheaper form of finance as their return is below the return on the risky asset. This implies that equity capital is costly relative to deposits. When banks directly finance risky investments, they hold a positive amount of equity capital as a way to reduce bankruptcy costs.

Much of the recent literature on bank capital structure has been concerned with issues of regulation (e.g., Hellmann et al. (2000), Van den Heuvel (2008), Admati et al. (2010), Acharya et al. (2012)). In our baseline model there are no benefits from regulating bank capital. The market solution is efficient since there are no pecuniary or other kinds of externalities. Requiring banks to hold higher levels of equity capital would reduce the number of banks and possibly the amount of deposits used in the banking sector. This is different once deposits are insured since then banks no longer have any incentive to hold capital and the market solution is not efficient. Capital regulation restores efficiency and, in fact, improves upon the market outcome.

As a final step, we extend the model to consider the case in which firms, rather than banks, invest in the productive assets and need external financing. We first consider the case of public firms that have access to financial markets and can raise both outside capital and bank loans; and then that of private firms that have a given amount of inside capital but can raise external funds only through bank loans. The main results of the baseline model remain valid in that equity capital is still a costly form of finance but the optimal capital structure differs significantly depending on the corporate structure of firms.

In our analysis we have assumed that the supplies of capital and deposits are given. An important issue is what would determine these in a full general equilibrium analysis. As discussed in the introduction, the justification for market segmentation is that the participation costs for equity markets are much higher than for deposits. One way to model this explicitly is to assume an increasing marginal cost of participating in equity markets. This would determine the proportion of the population that supplies equity and the proportion that would supply deposits. Another important factor in determining the supplies of capital and deposits is the different services that the two savings instruments provide. Deposits provide transaction services that equity does not. For example, bank customers do not have to continually check that they have enough funds in their accounts to make payments. Providing a full understanding of the determinants of the supplies of capital and deposits is an important topic for future research.

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A Proofs

Proof of Proposition 1: Since there are no bankruptcy costs, there are no efficiency gains from having capital in the banks. This means it is always possible to set up a bank with $r_D = R$ and $k_B = 0$ such that

$$EU_D = \int_0^R r\frac{1}{R}dr = \frac{R}{2}.$$

Thus, in equilibrium depositors must always receive $EU = \frac{R}{2}$. Since capital providers can always invest directly in the risky technology, they receive at least $\frac{R}{2}$ as well. Since total output with no bankruptcy costs is $\frac{R}{2}$ for each unit invested, the capital providers will earn exactly $\frac{R}{2}$. So one equilibrium involves all depositors using banks with no capital and all capital providers investing in their alternative opportunity. However, there exist many other equilibria. In these, banks choose a pair k_B and r_D such that $E\Pi_B = 0$ and $EU_D = \frac{R}{2}$. Substituting $\rho = \frac{R}{2}$ in (3) and solving $E\Pi_B = 0$ with respect to r_D gives r_D as in the proposition. Given $\rho = u = \frac{R}{2}$, we have $N_B k_B \leq K$ and $N_B(1 - k_B) = D$. $\square$

Proof of Proposition 2: Solving (4) with equality for k_B after setting $h_B = 0$, we find

$$k_B = 1 - \frac{(r_D - u)}{r_D^2}R. \tag{22}$$

Substituting this into (3), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{u(2\rho - u)}{\rho}. \tag{23}$$

Substituting this into (22) gives

$$k_B = 1 - \frac{\rho R(\rho - u)}{u(2\rho - u)^2}. \tag{24}$$

Using (23) and (24) in (3), we obtain

$$E\Pi_B = \frac{\rho^2 R}{2u(2\rho - u)} - \rho. \tag{25}$$

Equating this to zero since $E\Pi_B = 0$ in equilibrium, and solving for ρ gives

$$\rho = \frac{2u^2}{4u - R}. \tag{26}$$

Substituting (26) into (23) and (24) leads to $r_D = \frac{R}{2}$, and

$$k_B = \frac{4u}{R} - 1. \tag{27}$$

If $\frac{k_B}{1-k_B} > \eta$ depositors use their alternative opportunity and $u = 1$. In this case, banks will be formed until all the capital is used up. To find when this is the case, we solve

$$\frac{k_B}{1-k_B} = \eta, \tag{28}$$

with respect to R, where k_B is given by (27) after setting $u = 1$. We then obtain that for $u = 1$ is an equlibrium for

$$R < \overline{R} = \frac{4(1+\eta)}{1+2\eta}.$$

Putting $u = 1$ in (26) and (27) gives $\rho = \frac{2}{4-R}$ and $k_B = \frac{4}{R} - 1$. It can easily be checked that $\rho > \frac{R}{2}$ and $k_B \in (0,1)$. Given $\rho > \frac{R}{2}$ and $u = 1$, we have $N_B k_B = K$ and $N_B(1-k_B) < D$. This gives the first part of the proposition.

For $R \geq \overline{R}$, deposits are in short supply and in this case $u \geq 1$, with the inequality strict for $R > \overline{R}$. The equilibrium level of u is then found from solving (28) with respect to u, where k_B is given by (27). We obtain

$$u = \frac{1+2\eta}{2+2\eta}\frac{R}{2}.$$

Using this in (26) and (27) gives $\rho = \frac{1+4\eta(1+\eta)}{4\eta(1+\eta)}\frac{R}{2}$ and $k_B = \frac{\eta}{1+\eta}$. It can easiliy be checked that $\rho > \frac{R}{2}$, $k_B \in (0,1)$ and $u \in (1, \frac{R}{2})$ for any $R > \overline{R}$. Given $\rho > \frac{R}{2}$ and $u > 1$, we the have $N_B k_B = K$ and $N_B(1-k_B) = D$ for $R \geq \overline{R}$. This gives the second part of the proposition. □

Proof of Proposition 3: Solving (4) with equality for k_B, we find

$$k_B = 1 - \frac{2(r_D - u)}{(2-h_B)r_D^2}R. \tag{29}$$

Substituting this into (3), differentiating with respect to r_D, and solving for r_D gives

$$r_D = \frac{2u\left((2-h_B)\rho - u\right)}{(2-h_B)\rho - uh_B}. \tag{30}$$

Using (30) in (29) and then both expressions into (3), and solving the resulting expression for ρ after setting it to zero since $E\Pi_B = 0$ in equilibrium gives

$$\rho = \frac{u\,(2u - h_B R)}{2u(2 - h_B) - R}. \tag{31}$$

Substituting (31) into (30) leads to

$$r_D = \frac{2u(1 - h_B)R}{2u(2 - h_B) - h_B R}, \tag{32}$$

and substituting this into (29) gives

$$k_B = \frac{(2u - h_B R)(2u(2 - h_B) - R)}{2u(1 - h_B)^2 R}. \tag{33}$$

As in the case with $h_B = 0$, depositors use their alternative opportunity and thus $u = 1$ when $\frac{k_B}{1-k_B} > \eta$. To find when this is the case, we solve (28) with respect to R, where k_B is given by (33) after setting $u = 1$. We then obtain that for $u = 1$ is an

$$R < \overline{R} = \frac{2(1+\eta) - (1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)h_B}. \tag{34}$$

Then, substituting $u = 1$ into (33), (32) and (31) gives $k_B = \frac{(2-h_B R)(2(2-h_B)-R)}{2(1-h_B)^2 R}$, $r_D = \frac{2(1-h_B)R}{2(2-h_B)-h_B R}$ and $\rho = \frac{2-h_B R}{2(2-h_B)-R}$. To show that r_D, ρ and k_B are positive, we start by showing that $2 - h_B R > 0$ and $2(2 - h_B) - R > 0$ for any $R < \overline{R}$. Substituting (34) into $2 - h_B R$, we obtain

$$\begin{aligned} 2 - h_B\overline{R} &= \frac{2(1+\eta)h_B - h_B\left(2(1+\eta) - (1-h_B)^2 - (1-h_B)\sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)h_B} \\ &= \frac{(1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)} > 0. \end{aligned} \tag{35}$$

Then, substituting (34) into $2(2-h_B)-\overline{R}$, we obtain

$$2(2-h_B)-\overline{R}=\frac{(4-2h_B)(1+\eta)h_B-\left(2(1+\eta)-(1-h_B)\left(1-h_B+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)\right)}{(1+\eta)h_B}$$
$$=(1-h_B)\frac{\sqrt{4\eta(1+\eta)+(1-h_B)^2}-(1+2\eta)(1-h_B)}{(1+\eta)h_B}.$$

The sign of the numerator is the same as the sign of

$$4\eta(1+\eta)+(1-h_B)^2-(1+2\eta)^2(1-h_B)^2.$$

This simplifies to

$$4\eta\left(1+\eta\right)\left(1-(1-h_B)^2\right)>0,$$

so that

$$2(2-h_B)-\overline{R}>0. \tag{36}$$

This implies that r_D is positive and less than R as $h_B<1$ and

$$R-r_D=\frac{(2-h_BR)R}{(2(2-h_B)-h_BR))}>0 \text{ for } R\leq\overline{R}.$$

Finally, it can be seen that $\rho>\frac{R}{2}$, as

$$\rho-\frac{R}{2}=\frac{(R-2)^2}{2(2(2-h_B)-R)}>0 \text{ for } R\leq\overline{R}.$$

It follows from (35) and (36) that $k_B>0$. Also, $k_B<1$ since, using the expression for k_B in the proposition, we have

$$2(1-h_B)^2R-(2-h_BR)(2(2-h_B)-R)=(R-2)(2(2-h_B)-h_BR)>0 \text{ for } R<\overline{R}.$$

This completes the first part of the proposition.

For $R\geq\overline{R}$, deposits are in short supply and in this case $u\geq 1$, with the inequalit strict for $R\geq\overline{R}$. The equilibrium level of u solves (28), where k_B is given by (33). This gives

$$u=\frac{2(1+\eta)-(1-h_B)\left(1-h_B+\sqrt{4\eta(1+\eta)+(1-h_B)^2}\right)}{2(1+\eta)(2-h_B)}\frac{R}{2}. \tag{37}$$

As usual, it holds $u < \frac{R}{2}$ since, given (36). $(2(2-h_B) - h_B R) > 0$ and thus $\frac{h_B \overline{R}}{2(2-h_B)} < 1$. Substituting then u as in (37) into (33) gives $k_B = \frac{\eta}{1+\eta}$. Similarly, closed form solutions for r_D and ρ can be found from substituting (37) into the expressions (32) and (31). To check that $r_D < R$, we calculate

$$R - r_D = \frac{(2u - h_B R)R}{(2u(2-h_B) - R)}.$$

Substituting for u from (37), the numerator becomes

$$\begin{aligned} 2u - h_B R &= \left(\frac{2(1+\eta) - (1-h_B)\left(1 - h_B - \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)(2-h_B)} - h_B \right) R \\ &= \left(\frac{1 + \eta h^2 + (1-h_B)\left(2\eta + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)(2-h_B)} \right) R > 0, \end{aligned}$$

while the denominator is

$$\begin{aligned} 2u(2-h_B) - R &= \left(\frac{2(1+\eta) - (1-h_B)\left(1 - h_B - \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)} - 1 \right) R \\ &= \left(\frac{1 + \eta - (1-h_B)\left(1 - h_B - \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)} \right) R > 0. \end{aligned}$$

This implies that $r_D < R$ for $R > \overline{R}$. Moreover, it is easy to see that $\rho > \frac{R}{2}$, since

$$\rho - \frac{R}{2} = \frac{(R-2u)^2}{2\left(2u(2-h_B) - R\right)} > 0.$$

This completes the second part of the proposition. $\square$

Proof of Proposition 4: We consider the more general case when $h_B > 0$. The case when $h_B = 0$ can be derived similarly. The model is solved backward. Solving (4) with equality for r_D, we find

$$r_D = \frac{R - \sqrt{R\left[R - 2u(2-h_B)(1-k_B)\right]}}{(2-h_B)(1-k_B)}. \tag{38}$$

Substituting this into (3) and solving for ρ after equating the bank's expected profit to zero gives

$$\rho = \frac{(2-2h_B+h_B^2)R - 2u(2-h_B)(1-k_B) + 2(1-h_B)\sqrt{R\left[R-2u(2-h_B)(1-k_B)\right]}}{2k_B(2-h_B)^2}. \tag{39}$$

Substituting this then into (8) and differentiating it with respect to k_B gives

$$k_B = \frac{(2u - h_B R)(2u(2-h_B) - R)}{2u(1-h_B)^2 R}, \tag{40}$$

which is the same as in (33).

It is then easy to see that the regulatory solution coincides with the market solution in Proposition 3. As there, if depositors use their alternative opportunity then $u = 1$, which occurs for

$$R < \overline{R} = \frac{2(1+\eta) - (1-h_B)\left(1 - h_B + \sqrt{4\eta(1+\eta) + (1-h_B)^2}\right)}{(1+\eta)h_B}.$$

Substituting then $u = 1$ into (40), (38) and (39) gives k_B, r_D and ρ as in part i) of Proposition 3.

For $R \geq \overline{R}$, $u \geq 1$, with the inequality strict for $R > \overline{R}$. Substituting (40) into (28) and solving it with respect to u gives u as in (37). The rest of part ii) of Proposition 3 follows from substituting the expression for u into those for k_B, r_D and ρ given above. □

Proof of Proposition 5: Solving (10) with equality for r_D gives $r_D = u$. Substituting this into (3) and differentiating it with respect to k_B gives

$$\frac{\partial E\Pi_B}{\partial k_B} = \frac{-(1-k_B)u^2 - (\rho - u)R}{R},$$

which is negative for any $\rho \geq u$. This implies $k_B = 0$. Substituting this into (3) gives

$$E\Pi_B = \frac{R}{2} - u + \frac{u^2}{2R}.$$

Equating this to zero since $E\Pi_B = 0$ in equilibrium and solving for u gives $u = R$. This gives the first part of the proposition.

Given $u = R$, the capital providers prefer to provide deposits to the bank and obtain $\rho = u = R$ instead of investing in the technology and obtain $\frac{R}{2}$. Thus, the number of banks is given by $N_B = D + K$. This gives the second part of the proposition. □

Proof of Proposition 6: Solving (10) with equality for r_D gives $r_D = u$. Substituting this into (3) and solving this equal to zero with respect to ρ gives

$$\rho^{reg} = \frac{R\left[R - 2u(1-k_B)\right] + u^2\left[1 - k_B(2-k_B)\right]}{2k_B R}. \tag{41}$$

Substituting this into (11) and differentiating it with respect to k_B gives

$$k_B^{reg} = \sqrt{\frac{u^2(1-h_B) + 2uR - R^2}{u^2(1-h_B)}}. \tag{42}$$

To have a real non-negative solution for k_B^{reg}, it must hold that $u^2(1-h_B)+2uR-R^2 \geq 0$, which implies $R \leq u(1+\sqrt{2-h_B})$.

As usual, $u = 1$ as long as $\frac{k_B^{reg}}{1+k_B^{reg}} > \eta$ is satisfied. Substituting (42) with $u = 1$ into (28) and solving it for R, we obtain that $u = 1$ holds in equilibrium for

$$R < \overline{R}^{reg} = \frac{1 + \eta + \sqrt{2 + 4\eta + \eta^2 - h_B(1+2\eta)}}{1+\eta}. \tag{43}$$

As required above, $\overline{R}^{reg} < 1 + \sqrt{2-h_B}$. To see this, we substitute the expression for $\overline{R}^{reg}$ and, after rearranging the expression, we obtain

$$\sqrt{2 + 4\eta + \eta^2 - h_B(1+2\eta)} < (1+\eta)\sqrt{2-h_B}.$$

Squaring both terms and rearranging them gives

$$\eta^2 < (2-h_B)\eta^2.$$

This implies that $\overline{R}^{reg} < 1 + \sqrt{2-h_B}$.

Substituting then $u = 1$ into (42) gives $k_B^{reg} = \sqrt{\frac{1-h_B+2R-R^2}{1-h_B}}$. This satisfies the feasibility constraint in (6) with strict inequality since $\frac{1-h_B+2R-R^2}{1-h_B} > 0$ for $R < \overline{R}^{reg}$ $< 1 + \sqrt{2-h_B}$ and $\frac{1-h_B+2R-R^2}{1-h_B} < 1$ for $R > 2$. Substituting the expression for k_B^{reg} into (41) gives

$$\rho^{reg} = \frac{2 - h_B(2 - 2R + R^2) + 2(R-1)\sqrt{(1-h_B)(1-h_B+2R-R^2)}}{2R\sqrt{(1-h_B)(1-h_B+2R-R^2)}}. \tag{44}$$

To show that $\rho^{reg} > \frac{R}{2}$, we first note that ρ^{reg} is increasing in h_B since

$$\frac{\partial \rho}{\partial R} = \frac{(2-h_B)(R-2)^2 R}{4\sqrt{(1-h_B)^3(1-h_B+2R-R^2)^3}} > 0.$$

It is then enough to show that $\rho^{reg} > \frac{R}{2}$ for $h_B = 0$. Substituting $h_B = 0$ into (44) and rearranging the expression we obtain $\rho^{reg} = 1 + \frac{1-\sqrt{1+2R-R^2}}{R\sqrt{1+2R-R^2}}$. To show that this is greater than $\frac{R}{2}$, we differentiate it with respect to R and obtain

$$\frac{\partial \rho}{\partial R} = \frac{-1-3R+2R^2+(1+2R-R^2)\sqrt{1+2R-R^2}}{R^2(1+2R-R^2)\sqrt{1+2R-R^2}} > 0$$

since $-1-3R+2R^2 > 0$ for any $R > 2$. This, together with the fact that $\rho^{reg} = 1$ for $R = 2$, implies $\rho^{reg} > \frac{R}{2}$ for $h_B = 0$ and thus for any $0 < h_B < 1$. This completes the first part of the proposition.

For $R \geq \overline{R}$, $u \geq 1$ with the inequality strict for $R > \overline{R}$. The expression for u is found by substituting k_B^{reg} as in (42) into (28) and solving it with respect to u. We obtain

$$u^{reg} = \frac{2(1+\eta)\left(\sqrt{2+4\eta+\eta^2-h_B(1+2\eta)}-(1+\eta)\right)}{(1-h_B)(1+2\eta)}\frac{R}{2}.$$

As usual, it holds that $u < \frac{R}{2}$ for $R > \overline{R}$ since $\frac{2(1+\eta)\left(\sqrt{2+4\eta+\eta^2-h_B(1+2\eta)}-(1+\eta)\right)}{(1-h_B)(1+2\eta)} < 1$ for any $h_B < 1$. To see this, it is enough to note that this coefficient is increasing in h_B and tends to 1 for $h_B \rightarrow 1$. The equilibrium return to capital, ρ^{reg}, can now be obtained by substituting the above expression for u^{reg} into (41). To show that $\rho^{reg} > \frac{R}{2}$, note that for $u > 1$ we must have $k_B = \frac{\eta}{1+\eta}$ since there is full inclusion of capital and deposits. Substituting into (41) yields

$$\begin{aligned}
\rho^{reg} &= \frac{R\left(R-2u\left(1-\frac{\eta}{1+\eta}\right)\right)+u^2\left(1-\frac{\eta}{1+\eta}\left(2-\frac{\eta}{1+\eta}\right)\right)}{2\frac{\eta}{1+\eta}R} \\
&= \frac{R}{2}+\frac{(R-2u)}{2\eta}+\frac{u^2\left(1-\frac{\eta}{1+\eta}\left(2-\frac{\eta}{1+\eta}\right)\right)}{2\frac{\eta}{1+\eta}R}.
\end{aligned}$$

The sum of the first two terms is clearly greater than $\frac{R}{2}$ since $u < \frac{R}{2}$. The last term is strictly positive for any η. Therefore, $\rho^{reg} > \frac{R}{2}$, as desired. This completes the second part of the proposition. □

Proof of Proposition 7: We first show that social welfare is always higher under deposit insurance with capital regulation than under the market solution where deposits are uninsured. For an arbitrary fixed R, suppose that the regulator chooses the same level of capital at each bank as in the market solution, which we denote by k_B^M, that is $k_B^{reg} = k_B^M$. This will imply that the number of banks will also be the same, i.e., $N_B = \frac{K}{k_B^{reg}} = \frac{K}{k_B^M}$, which means that total output, gross of bankruptcy costs, will be the same as well, and equal to $N_B \frac{R}{2} + D - \left(1 - k_B^M\right) N_B$. Banks now maximize

$$\max_{r_D} E\Pi_B = \int_{\overline{r}_B}^{R} \left(r - r_D(1 - k_B^M)\right) \frac{1}{R} dr - \rho k_B^M, \tag{45}$$

where $\overline{r}_B = \left(1 - k_B^M\right) r_D$, and subject to the same constraints as before except that the depositors' participation constraint is given by

$$EU_D = \int_0^R r_D \frac{1}{R} dr \geq u. \tag{46}$$

Compare this to the problem the bank maximizes in the market solution:

$$\max_{k_B, r_D} E\Pi_B = \int_{\overline{r}_B}^{R} \left(r - r_D(1 - k_B)\right) \frac{1}{R} dr - \rho k_B. \tag{47}$$

If $\{k_B^M, r_D^M\}$ are solutions to (47), then choosing $\{r_D^M\}$ for the problem given in (45) must give the bank the same value. But in that case, depositors are better off since $\int_0^R r_D^M \frac{1}{R} dr > \int_0^{\overline{r}_B} \frac{h_B r}{1-k_B} \frac{1}{R} dr + \int_{\overline{r}_B}^R r_D^M \frac{1}{R} dr = u^M$, where u^M is the equilibrium return depositors make in the market solution. Therefore, the bank can increase its value by lowering r_D below r_D^M and transferring some of the surplus to itself. With no change in the total amount of investment, the reduction in r_D reduces deadweight costs of bankruptcy, thus raising SW. Raising r_D beyond r_D^M cannot be optimal as it would increase the bankruptcy threshold and lead to lower value for the bank. Therefore, by choosing $k_B^{reg} = k_B^M$, the regulator can increase social welfare when deposits are insured relative to the market solution when deposits are uninsured.

Finally, the optimal regulatory solution may be different from the market solution k_B^M, but cannot do worse than the SW obtained when choosing k_B^M. Therefore, deposit insurance coupled with capital regulation improves upon the market solution.

To show that the optimal level of capital under regulation is always (weakly) lower than in the market solution, $k_B^{reg} \leq k_B^M$, consider again the maximization problem under regulation, which is to maximize (12) with respect to k_B, subject to (46) and

$$r_D = \arg\max_{r_D} E\Pi_B = \int_{\overline{r}_B}^{R} \left(r - r_D(1 - k_B)\right) \frac{1}{R} dr - \rho k_B.$$

Now consider the problem in the market solution, which is to maximize (3) subject to depositors' participation constraint in (4). Start by rewriting the participation constraint for the depositors by multiplying both sides by $(1 - k_B)$:

$$(1 - k_B)EU_D = \int_0^{\overline{r}_B} h_B r \frac{1}{R} dr + \int_{\overline{r}_B}^{R} r_D(1 - k_B)\frac{1}{R} dr \geq u(1 - k_B).$$

Setting this with equality, we can solve as follows:

$$\int_{\overline{r}_B}^{R} r_D(1 - k_B)\frac{1}{R} dr = u(1 - k_B) - \int_0^{\overline{r}_B} h_B r \frac{1}{R} dr.$$

We can now substitute this into (3) to get a maximization problem that depends only on k_B:

$$\max_{k_B} E\Pi_B = \int_{\overline{r}_B}^{R} r\frac{1}{R} dr - u(1-k_B) + \int_0^{\overline{r}_B} h_B r \frac{1}{R} dr - \rho k_B = \int_{\overline{r}_B}^{R} r\frac{1}{R} dr + \int_0^{\overline{r}_B} h_B r \frac{1}{R} dr - \rho k_B - u(1-k_B).$$

We can add and subtract $\int_0^{\overline{r}_B} r\frac{1}{R}dr$ to obtain

$$\max_{k_B} E\Pi_B = \frac{1}{2}R - \int_0^{\overline{r}_B} (1 - h_B)\, r\frac{1}{R} dr - \rho k_B - u(1 - k_B).$$

Note now that we can write SW as (we ignore the expectation term, E, for ease of notation)

$$\begin{aligned} SW &= N_B\left(\Pi_B + \rho k_B + u(1 - k_B)\right) + \left(D - (1 - k_B)\, N_B\right) \\ &= N_B \Pi_B + \left(D - (1 - k_B)\, N_B\right) + N_B\left(\rho k_B + u(1 - k_B)\right). \end{aligned}$$

For an interior solution in the market problem the standard first order condition $\frac{\partial E\Pi_B}{\partial k_B} = 0$ must be satisfied. Call this solution k_B^M. Now consider the first order condition for the SW problem, assuming again an interior solution:

$$\begin{aligned} \frac{\partial SW}{\partial k_B} &= N_B\frac{\partial \Pi_B}{\partial k_B} + \frac{\partial N_B}{\partial k_B}\Pi_B + N_B - (1 - k_B)\frac{\partial N_B}{\partial k_B} + \frac{\partial N_B}{\partial k_B}\left(\rho k_B + u(1 - k_B)\right) + N_B(\rho - u) \\ &= N_B\left(\frac{\partial \Pi_B}{\partial k_B} + 1 + (\rho - u)\right) + \frac{\partial N_B}{\partial k_B}\left(\Pi_B + \rho k_B + (u - 1)(1 - k_B)\right). \end{aligned}$$

We know from the Envelope Theorem that, at k_B^M, $\frac{\partial \Pi_B}{\partial k_B} = 0$. Recall as well that $N_B = \frac{K}{k_B}$, and that therefore $\frac{\partial N_B}{\partial k_B} = \frac{-K}{k_B^2}$. Substituting, we get:

$$\begin{aligned}\frac{\partial SW}{\partial k_B} &= \frac{K}{k_B}\left(1+(\rho-u)\right) - \frac{K}{k_B^2}\left(\Pi_B + \rho k_B + (u-1)(1-k_B)\right) \\ &= -\frac{K}{k_B}\left(\frac{u-1}{k_B} + \frac{\Pi_B}{k_B}\right) < 0.\end{aligned}$$

This means that, at the market solution for the level of capital (assuming an interior solution), a regulator would prefer to reduce the amount of capital each bank holds. In other words, the incentive to hold capital is lower when maximizing social welfare, and the regulatory solution entails $k_B^{reg} < k_B^M$. The strict inequality holds as long as $u = 1$ in the market solution since, as it can easily be shown, $\overline{R} > \overline{R}^{reg}$, that is the critical value of R above which u becomes greater than 1 (and thus $k_B = \frac{\eta}{1+\eta}$) is lower in the regulatory solution than in the market solution. $\square$

Proof of Proposition 8: As argued, any equilibrium must involve $\overline{r}_F \geq \overline{r}_B$. We show that $\overline{r}_F > \overline{r}_B$ cannot hold in equilibrium and that equilibrium entails $k_B = 0$ and $r_L = r_D$.

Suppose there exists a candidate equilibrium, defined as X, with

$$k_B' > 0, k_F' > 0, r_L' > r_D', \rho' \geq \frac{R}{2}, u' \geq 1,\ \overline{r}_F' = r_L'(1-k_F') > \overline{r}_B' = \frac{r_D'(1-k_B')(1-k_F')}{h_F}. \tag{48}$$

This cannot be an equilibrium because, by transferring the capital of the bank to the firm and aligning the bankruptcy thresholds of the bank and the firm, it is possible to reduce overall bankruptcy costs. To see this, consider the following deviation, which we denote Z, where

$$k_B^* = 0, k_F^* = k_B'(1-k_F') + k_F', r_D^* = r_L^* = r_D', \overline{r}_F^* = \overline{r}_B^* = r_L^*(1-k_F^*) < \overline{r}_B'. \tag{49}$$

It can be seen from Figure 3 that this deviation eliminates the firm bankruptcy costs represented by Region $D1 + D2$, and the bank bankruptcy costs represented by $E2$. The shareholders are better off by the amount $D1 + D2$ and the depositors are better off by the amount $E2$. This implies that the deviation Z represents a Pareto improvement.

When $k_B' = 0$, it must be the case that $r_L' = r_D'$ for bank expected profits to be zero. In this case, $\overline{r}_F' = \overline{r}_B'$ and this is the equilibrium since no profitable deviation is possible. The choice of the optimal value of k_F and r_L are then identical to the choice of k_B and r_D in the case when the bank invests directly in the risky asset except the liquidation proceeds are $h_F h_B r$ instead of $h_B r$. $\Box$

Proof of Proposition 9: We start by noting that in order to satisfy the zero profit condition of the firm, the loan rate must be set so that

$$r_L = \frac{R}{1+\sqrt{k_F}}. \tag{50}$$

We now distinguish between two cases depending on whether $\overline{r}_B = \overline{r}_F = r_L(1-k_F)$ or $\overline{r}_B = \frac{r_D(1-k_B)(1-k_F)}{h_F} < \overline{r}_F$, and we first analyze when either case is feasible.

Suppose first that $\overline{r}_B = \overline{r}_F$ holds in equilibrium. Then, the bank's maximization problem simplifies to:

$$\max_{k_B, r_D} E\Pi_B = \int_{\overline{r}_F}^{R} \left(r_L - r_D(1-k_B)\right) \frac{1}{R} dr - \rho k_B \tag{51}$$

subject to

$$EU_D = \int_{\overline{r}_F}^{R} r_D \frac{1}{R} dr \geq u. \tag{52}$$

Solving (53) with equality with respect to r_D after substituting r_L as in (50) gives

$$r_D = \frac{Ru}{R-(1-k_F)r_l} = \frac{u}{\sqrt{k_F}}. \tag{53}$$

We now substitute (50) and (53) into the bank's profit as in (51) and differentiate it with respect to k_B. We obtain

$$\frac{\partial E\Pi_B}{\partial k_B} = -\rho + u \leq 0$$

for $\rho \geq u$. This implies $k_B = 0$, which is consistent with $\overline{r}_B = \overline{r}_F$, and also that $r_L = r_D$ so that the bank makes zero expected profit. This solution is feasible when the bank can offer at least $u = 1$ to its depositors. To see when this is the case, we substitute $u = 1$ into $r_L = r_D$, where r_L and r_D are given in (50) and (53), and solve the equality with respect to R. This gives the minimum level of R, denoted $\underline{R}_C$, that allows a bank with no capital to be feasible:

$$\underline{R}_C = \frac{1+\sqrt{k_F}}{\sqrt{k_F}}.$$

Thus, the solution with $k_B = 0$ is feasible for $R \geq \underline{R}_C$ while it is not feasible for $R < \underline{R}_C$. Depositors obtain $u = 1$ for $R = \underline{R}_C$ and $u = u_C > 1$ for $R > \underline{R}_C$. The value for u_C is found by equating r_L in (50) to r_D in (53) and solving the equality with respect to u.

This gives

$$u_C = \frac{\sqrt{k_F}}{1+\sqrt{k_F}} R < R/2. \tag{54}$$

Now suppose that $\overline{r}_B = \frac{r_D(1-k_B)(1-k_F)}{h_F}$ so that $k_B > 0$ must hold. We first find k_B and r_D as the solutions to the bank problem in (18)-(20) for given ρ_B and u, and we then analyze when such a solution is feasible. Solving (20) with equality with respect to k_B after setting $h_B = 0$ gives

$$k_B = 1 - \frac{h_F(r_D - u)}{(1-k_F)r_D^2} R.$$

Substituting this expression for k_B and r_L as in (50) into (18) and differentiating it with respect to r_D gives

$$r_D = \frac{u(2\rho - u)}{\rho}. \tag{55}$$

Substituting (55) into the expression for k_B above gives

$$k_B = 1 - \frac{h_F \rho R(\rho - u)}{(1-k_F)u(2\rho - u)^2}. \tag{56}$$

This solution is feasible when capital providers and depositors obtain at least $\rho = \frac{R}{2}$ and $u = 1$, respectively, and the bank makes non-negative profits. We therefore substitute $\rho_B = \frac{R}{2}$ and $u = 1$, (55) and (56) into (18), set it equal to zero and solve for R. This gives the minimum value of R, denoted $\underline{R}_{k_B}$, that is needed for a bank with $k_B > 0$ to be feasible:

$$\underline{R}_{k_B} = \frac{2}{h_F}\sqrt{\left(1 - 2(1-h_F)\sqrt{k_F} + (1-h_F)k_F\right)} \left(\begin{array}{c} \sqrt{1 - 2(1-h_F)\sqrt{k_F} + (1-h_F)k_F} \\ +(1-\sqrt{k_F})\sqrt{(1-h_F)} \end{array} \right).$$

Thus, the solution with $k_B > 0$ is feasible for $R \geq \underline{R}_{k_B}$, while it is not feasible for $R < \underline{R}_{k_B}$. Capital providers and depositors obtain, respectively, $\rho = \frac{R}{2}$ and $u = 1$ for $R = \underline{R}_{k_B}$ and $\rho > \frac{R}{2}$ and $u \geq 1$ for $R > \underline{R}_{k_B}$. The boundaries $\underline{R}_C$ and $\underline{R}_{k_B}$ meet for h_F equal to

$$\underline{h}_F = \frac{4\sqrt{k_F}}{1+4\sqrt{k_F}}. \tag{57}$$

It follows that $\underline{R}_C < \underline{R}_{k_B}$ for $h_F < \underline{h}_F$ and $\underline{R}_C > \underline{R}_{k_B}$ for $h_F > \underline{h}_F$. This implies that there is no intermediation in Region D of Figure 4 as defined by $R < \min[\underline{R}_C, \underline{R}_{k_B}]$; that only the solution with $k_B = 0$ and $u = u_C$ given in (54) is feasible in Region $A1$ of Figure 4 as defined by $\underline{R}_C < R < \underline{R}_{k_B}$; and that only the solution with $k_B > 0$ is feasible in

Region C as defined by $\underline{R}_{k_B} < R < \underline{R}_C$. In the latter, depositors obtain $u > 1$ or $u = 1$ depending on whether (28) binds at $u = 1$ or at $u > 1$, while capital providers always obtain a return $\rho_B > \frac{R}{2}$. This also implies $k_B \geq \frac{\eta}{1+\eta}$ when $u = 1$ and $k_B = \frac{\eta}{1+\eta}$ when $u > 1$, while $r_L > r_D$ must hold for the bank to make non-negative profits with $k_B > 0$.

It remains now to establish which solution, $k_B = 0$ or $k_B > 0$, is optimal in the sense that it provides a higher return when both are feasible for $R > \underline{R}_{k_B} > \underline{R}_C$. Recall first that for any $R > \underline{R}_C$, $u_C > 1$. Any bank with $k_B > 0$ can therefore compete with the bank with $k_B = 0$ only if it can offer depostiors at least $u = u_C$, while at the same time offering at least $\rho = \frac{R}{2}$ to the capital providers. In other words, the bank with positive capital is constrained by the potential entry of the bank with zero capital. To analyze this contestability argument formally, we substitute the expression for u_C in (54) and $\rho_B = \frac{R}{2}$ into (18), set it equal to zero, and solve for h_F. This gives the minimum value of h_F that allows a bank with positive capital to offer $u = u_C$ while still attracting capital providers with $\rho_B = \frac{R}{2}$ and making non-negative profits:

$$h_F = \underline{h}_F = \frac{4\sqrt{k_F}}{1 + 4\sqrt{k_F}}.$$

This critical value of h_F coincides with the value $\underline{h}_F$ in (57) at which the boundaries $\underline{R}_C$ and $\underline{R}_{k_B}$ are equal. Thus, in Region $A.2$ defined by $R > \underline{R}_{k_B}$ and $h_F < \underline{h}_F$ and illustrated in Figure 4, a bank with $k_B = 0$ offering $u = u_C$ is optimal as it can offer a higher return to its depositors. By contrast, a bank with $k_B > 0$ is optimal in Region B of Figure 4 as defined by $R > \underline{R}_C$ and $h_F > \underline{h}_F$. In this region, the bank will offer $u \geq u_C$ to the depositors and $\rho_B(u) \geq \rho_B(u_C)$, depending or whether it is constrained by the threat of entry of a zero capital bank, where $\rho_B(u)$ is the expected return to capital suppliers when depositors earn a return of u. This completes the proof of the proposition. $\square$

Figure 1: Deposits over bank liabilities

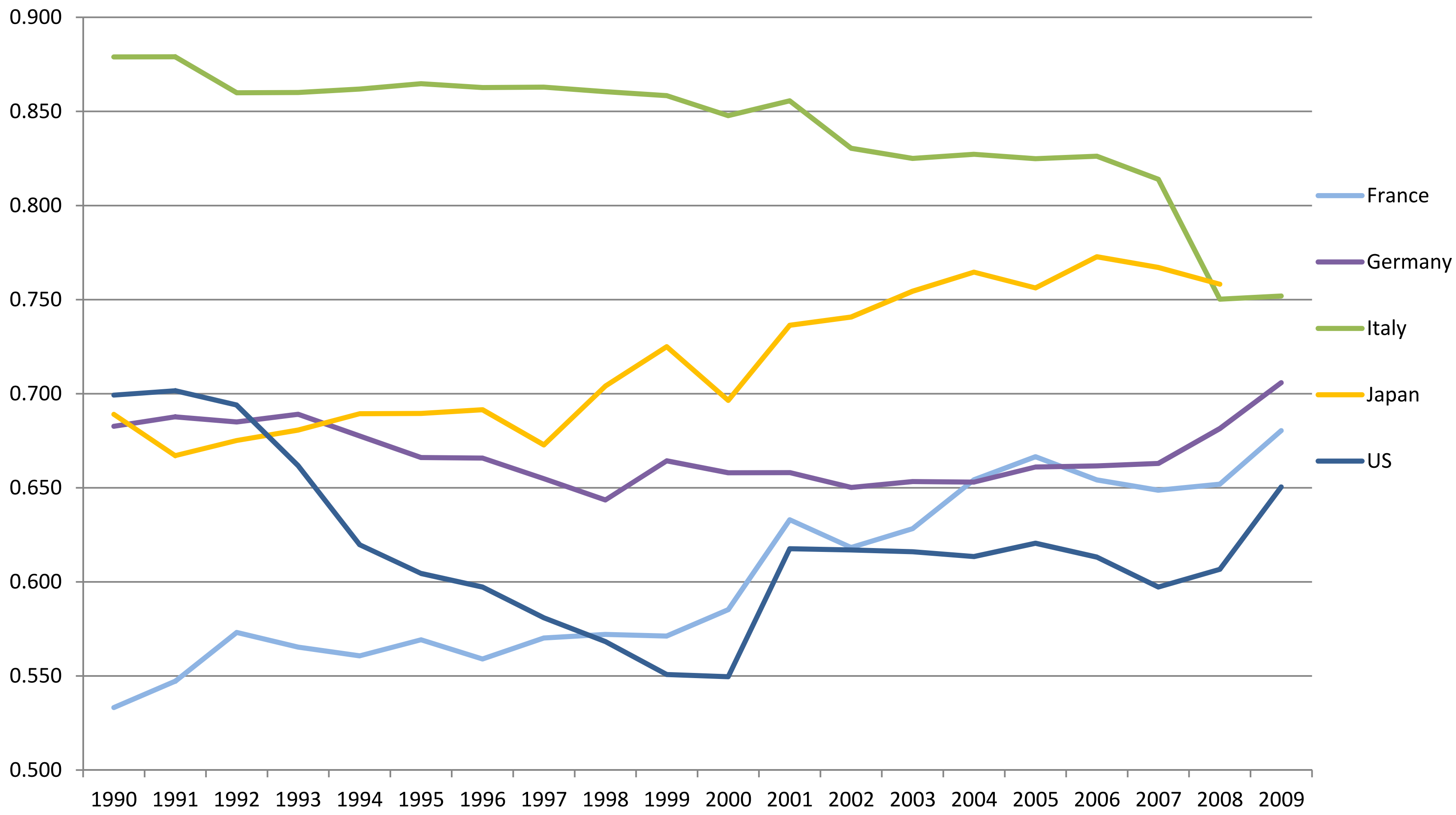
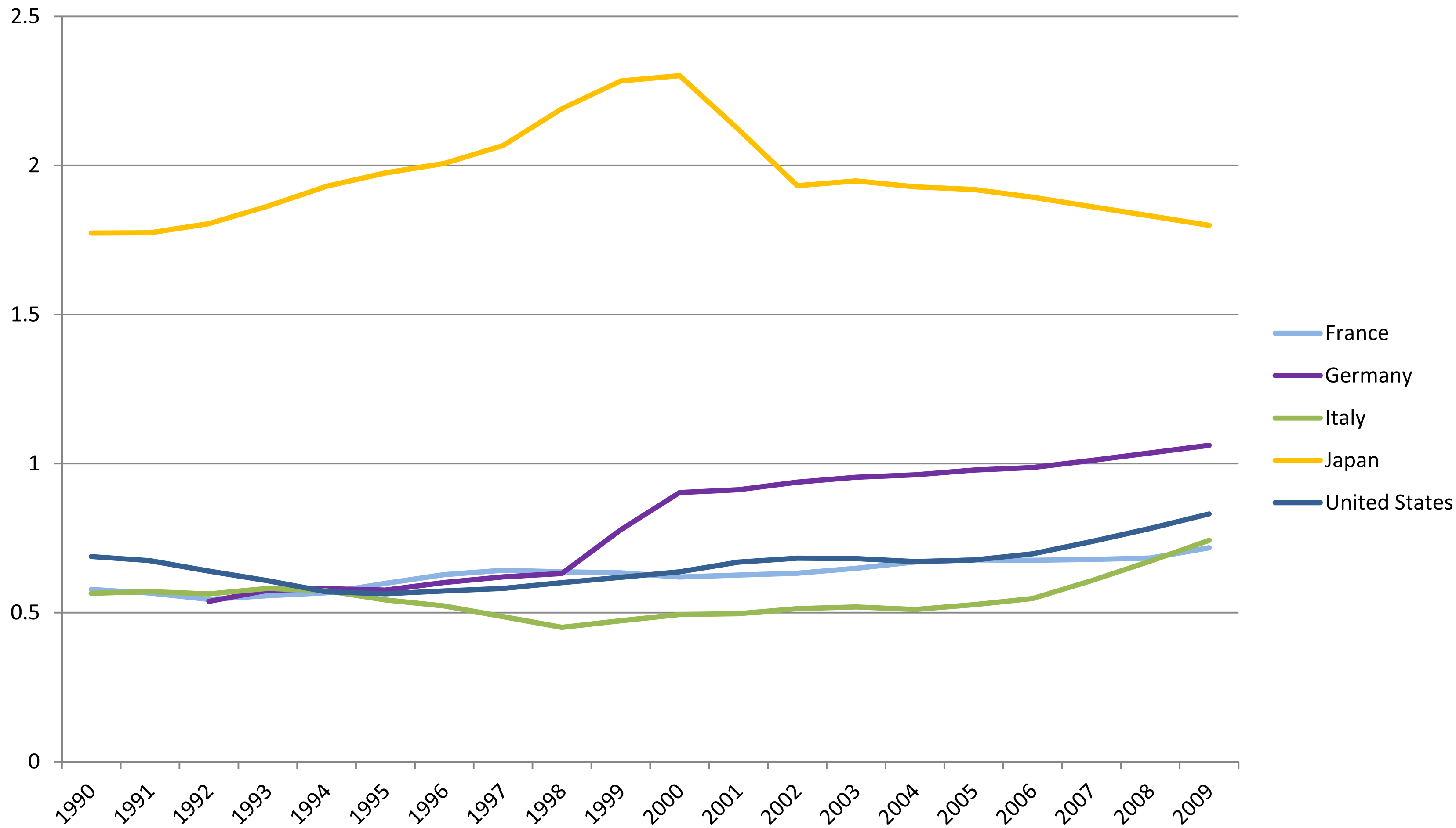


Figure 2: Bank deposits over GDP



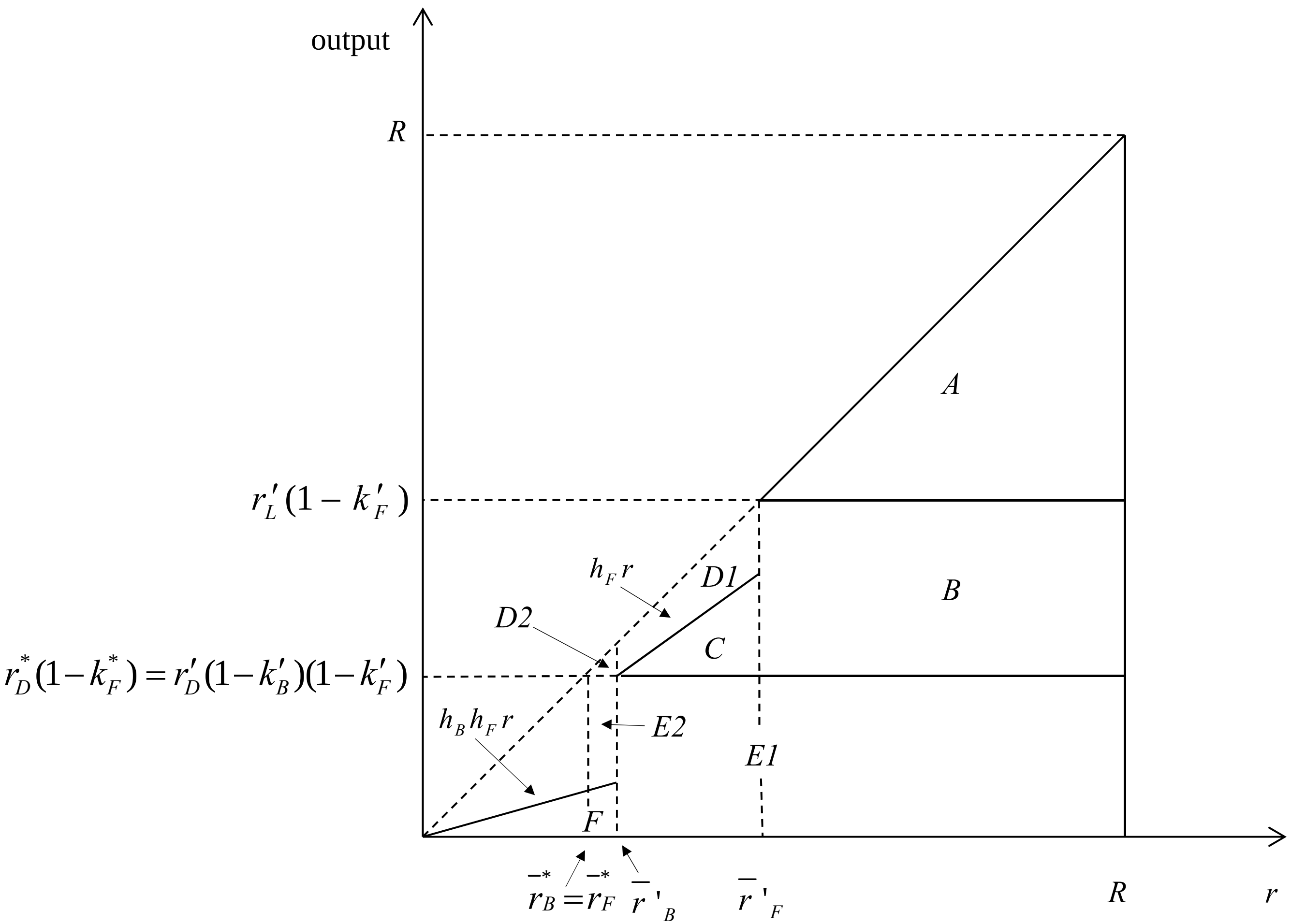


Figure 3: Output of a single firm and returns to shareholders and depositors as a function of the return r in the case of a single productive sector.

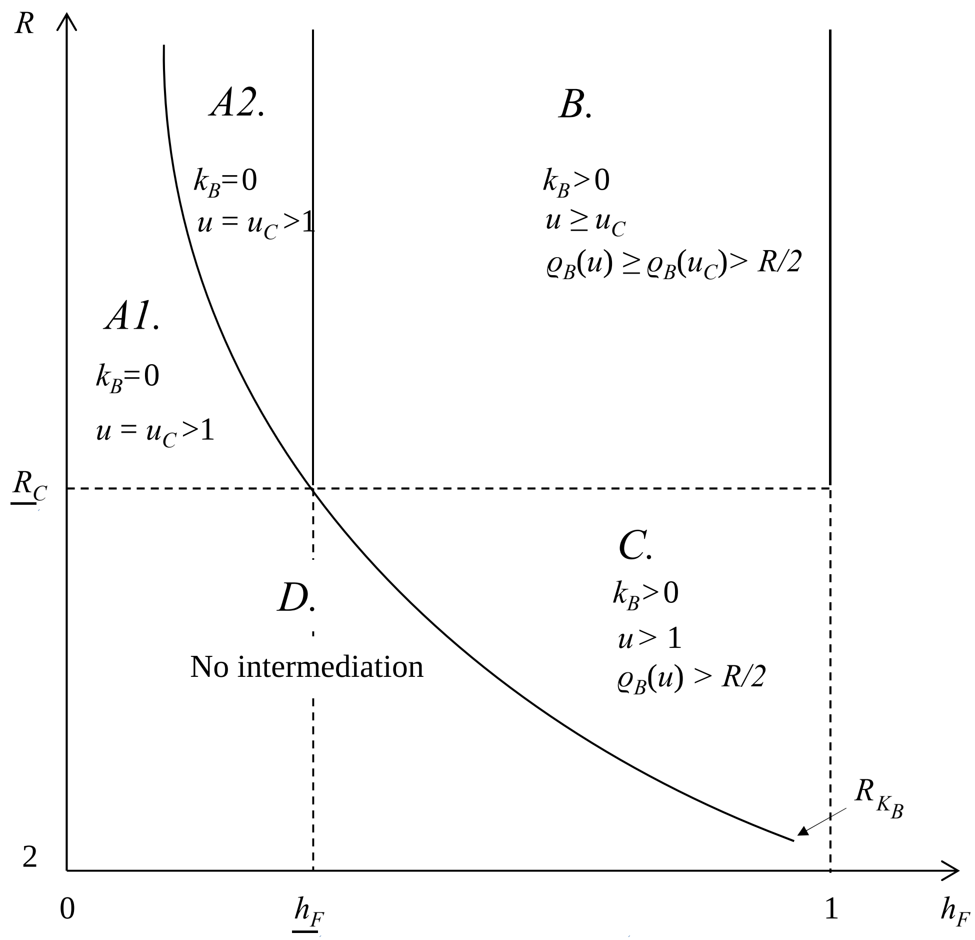


Figure 4: The case of private firms as a function of the recovery rate h_F and the asset return R.