Risk management optimization for sovereign debt restructuring

Andrea Consiglio *
Stavros A. Zenios †

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The Wharton School, University of Pennsylvania, PA

Abstract

Debt restructuring is gaining acceptance as a policy tool for resolving sovereign debt crises. In this paper we propose a scenario analysis for debt sustainability and integrate it with scenario optimization for the risk management of re-profiling sovereign debt. The scenario dynamics of debt are used to define a risk metric –conditional Debt-at-Risk– for the tail of debt-to-GDP ratios, and a multi-period stochastic programming model optimizes the expected cost of financing a debt structure, subject to limits on the risk. The model handles important technical aspects of debt restructuring: it collects all debt issues in a common framework, and can include embedded options and contingent claims, multiple currencies and step-up or linked contractual features. Alternative debt profiles are then analyzed for their cost vs risk tradeoffs. With a suitable re-calculation of the efficient frontier, debt sustainability of a given debt profile can then be ascertained. The model is applied to two stylized examples drawn from an IMF publication and from the Cyprus debt crisis.

Keywords: sovereign debt; debt restructuring; scenario analysis; portfolio optimization; stochastic programming.

*University of Palermo, Palermo, IT. andrea.consiglio@unipa.it
†University of Cyprus, Nicosia, CY and The Wharton Financial Institutions Center, University of Pennsylvania, USA. zenios.stavros@ucy.ac.cy
1 The sovereign debt restructuring challenge

On May 20, 2013, the International Monetary Fund issued a public notice that its Executive Board had discussed developments in sovereign debt restructuring and the implications for the Fund’s legal and policy framework, IMF (2013). There had not been any discussions at the IMF about sovereign debt restructuring since 2005, and it had been more than ten years since Anne Krueger’s signature proposal on a sovereign debt restructuring mechanism was rejected under strong opposition from creditor countries, Krueger (2002). If approved, the new Fund policy will introduce the possibility of maturity extensions for countries whose debt is not considered sustainable. Debt rescheduling and, perhaps, restructuring will become an official policy tool.

The renewed interest was prompted by the realization, in the aftermath of the Greek crisis, that “debt restructurings have often been too little and too late [...] thus failing to re-establish debt sustainability and market access in a durable way”. Sovereign debt defaults are prevalent as shown in two databases compiled recently, Beers and Nadeau (2014); Trebesch (2011). Figure 1 summarizes statistics on debt default worldwide from 1975 to 2013. Greece holds the record with the largest sovereign debt restructuring in history (in 2012) and other recent restructurings include Belize (in 2007 and 2013), Jamaica (in 2010 and 2013), and St. Kitts and Nevis (2012). The litigation against Argentina in New York courts by holdouts from the restructuring of 2005 is expected to have significant ramifications for future sovereign debt restructurings; the latest ruling against Argentina came in June and pushed the country back into (technical) default.

The eurozone crisis highlighted that sovereign defaults are not the privilege of emerging markets, developing economies or ill-run autocracies. It happens to the best economies as Sturzenegger and Zettelmeyer (2006) show studying ten years of crises; the record is held by a eurozone country after all. The literature documents historical track records of advanced economies where debt restructuring, financial repression, and high inflation were an integral part of the resolution of debt crisis. The depositor bail-in and capital controls imposed on Cyprus in March 2013, see Zenios (2014), and the Greek debt restructuring, are recent episodes of financial repression in advanced economies.

Given the complexity of inter-related risk factors in the debt of sovereigns, there is a need for advances in the risk management of public debt. In general, it has been assumed that markets will impose discipline on sovereigns and keep their debt in sustainable territory. However, for many reasons, this does not happen and when a sovereign debtor enters a crisis zone the creditors bolt for the exit. No one is left to roll over debt and the official sector is left to handle the crisis. In the case of Greece, the Troika of IMF-EC-ECB demanded involvement of private creditors before the official sector would bail out the country. The Greek PSI saw 106 billion euro transferred from private creditors to Greece and an international assistance package of 164 billion under strict austerity measures that drove the Greek economy into recession for five consecutive years with unemployment rising to 27%.

While Greece has been an extreme example of debt crisis -and of mismanagement of doing too little and too late, see, e.g., Zettelmeyer et al. (2013)- it is not a unique case of advanced economies facing debt crises. Figure 2 illustrates the increase of sovereign debt of selected eurozone countries after the international crisis of 2008. The current average debt of eurozone countries exceeds the debt-to-GDP ratio of 60% prescribed in the stability and growth pact of the European Union. At 96% it exceeds the 90% threshold of Reinhart and Rogoff (2010), and while this rigid threshold was recently discredited by Herndon et al. (2014), the conclusion that excessive debt is a drag on growth is supported by other studies, Cecchetti et al. (2011).

Debt is fragile and when a sovereign loses market access —it has a “Minsky moment” to quote the popular term coined in McCulley (2013)— the results are not easily reversed. Many factors come into play: higher borrowing rates for the distressed sovereign, snowball effect of debt servicing on the country’s GDP and its ability to re-pay, the bank-sovereign “diabolical loop” of Brunnermeier et al. (2011) discussed in Mody and Sandri (2011) for eurozone, capital flight
De Grauwe (2012). All these factors create negative feedback loops and reversing the vicious circle requires drastic action. This is where debt restructuring comes into play. Recognizing that several eurozone countries are in the crisis zone, credible proposals have emerged for debt restructuring, see, e.g., Gianviti et al. (2010); Paris and Wyplosz (2014). It is in this setting that the IMF discussions take place.

In this paper we argue for a risk management approach to the re-profiling of public debt. We use the broader term re-profiling to include both rescheduling and restructuring of debt, whereby creditors may suffer losses in present or nominal value. Risk management is not restricted to the static analysis of debt sustainability. Instead, it requires policymakers to postulate plausible scenarios and develop risk metrics that ensure high probability of debt sustainability. We adopt for sovereign debt crisis management the same quantitative approach used for risk management of portfolios or the asset/liability management of financial institutions, see, e.g. Mulvey and Ziemba (1998); Zenios and Ziemba (2007).

Of course, risk management has also failed us in many instances and we do not imply that what we suggest is a foolproof method. Nevertheless, it is a much richer framework. Our approach is in line with recent trends in public debt management that are moving beyond simulation modelling to integrate simulations with optimization, and tradeoff expected cost with Cost-at-Risk, Balibek and Köksalan (2010); Bolder and Rubin (2011); Consiglio and Staino (2012).

We start with a discussion of debt restructuring issues in section 2 and then discuss scenario analysis for debt re-profiling in Section 3. We develop a scenario optimization model for minimizing Conditional Debt-at-Risk and explain how various issues arising in debt restructuring can be addressed either by the model or by suitable modifications and post-optimality analysis. The use of the model is illustrated on two stylized examples obtained from an IMF publication and for restructuring the Cyprus debt in section 4, and conclusions are summarized in section 5.

2 Sovereign debt re-profiling: why and how

When a sovereign faces a debt crisis it may be advisable to re-profile its debt in a manner that preserves value for both debtor and creditors. There is certainly no benefit to the creditors if
the sovereign defaults on its obligations. Similarly, a defaulting sovereign faces severe penalties: sanctions including loss of market access, litigation costs, reputation punishment; see, e.g. Bulow and Rogoff (1989), but also Cruces and Trebesch (2011) for critiques. Hence, re-profiling the debt can be beneficial to both sides.

By re-profiling we mean a change of the terms of the debt obligation that allow the debtor to continue servicing it. For instance, maturities may be extended, interest rates renegotiated or nominal values written down. When creditors do not suffer losses of nominal value then we talk about debt rescheduling, while debt restructuring implies haircut in nominal value. Rescheduling may involve losses in present value to creditors or it may be loss-free and simply allow the debtor to ride past a concentration of payments around specific years. Figure 3 illustrates an IMF example of debt restructuring that involves a nominal debt reduction of 33% and a lengthening of maturities, Das et al. (2012).

To deal with debt restructuring we need to consider both legal and policy issues, as well as technical issues; see Das et al. (2012); Wright (2012) for reviews. There is no universally applicable legal framework for dealing with sovereign defaults – akin to Chapter 11 for corporations and Chapter 9 for municipalities in the USA – and coordination between creditors and debtor is a major challenge in sovereign debt restructuring. This is not to say that debt restructuring is done on a totally ad hoc basis. Some principles have emerged from experience to ensure stable capital flows, groups have been formed to facilitate creditor-debtor coordination such as the Paris and London Clubs and, recently, we have seen renewed thinking on the process in the work of Gianviti et al. (2010). Legal provisions such as Collective Action Clauses have been introduced to coordinate multiple creditors in dealing ex post with debt crises, and state-contingent debt provisions attempt to ex ante chart appropriate courses of action depending on the sovereign’s capacity to service its debt.

Assuming that coordination is achieved and legal obstacle can be overcome, the problem then becomes one of specifying the restructured debt series and ensuring that its financing is...
Das et al. (2012) provide the key parameter specifications of the problem:

1. Face and market value of bonds or loans
2. Interest rate and coupon (fixed vs flexible, step-up or linked features)
3. Amortization schedule (bullet vs amortization, existence of a sinking fund)
4. Currency of denomination of the instruments (local vs foreign currency)
5. Enhancements, including embedded options or collateral
6. Legal clauses (CACs, non-default clauses, exit consents).

2.1 The need for a model

The issues relating to the implementation of debt restructuring are discussed in depth by Wright (2012) who makes several suggestions on the need for reform. Perhaps his most important suggestion, and the one most relevant to our work, is the development of “criteria for an “optimal” debt restructuring process. “Optimal” is put within quotes by the author, recognizing that in such a complex setting there is no unique optimality criterion. His work addresses institutional, legal and instrumental issues of restructuring, for which one can not expect a mathematical optimum. However, the suggestions point to the need for a model to coordinate the process. Such a model would capture the tasks of the “international debt referee”, can address “the greatest problem which is restricted to restructuring a single debt”, and account for the central role of “state contingent debt”. It would provide the decision support tool for the sovereign debt restructuring framework discussed at the IMF in Krueger (2002), and for the eurozone in Gianviti et al. (2010).

The scenario optimization model we develop next handles the technical aspects of debt restructuring. First, it collects all debt issues in a common framework and under a set of scenarios that capture consistently the relevant risk factors. Second, it develops optimal financing strategies for alternative debt profiles and trades off their cost and associated risks using an optimality criterion that captures debt sustainability. Third, embedded options and contingent claims can
readily be modelled conditioned on the scenarios. Fourth, the currency of denomination can be modelled via exchange rate scenarios. Fifth, the model is multi-period so we can model sinking funds and step-up or linked contractual features. We do not include all these features in the current model. The basic model developed in this paper deals with the first two items, but extensions are discussed.

3 Scenario analysis for optimal debt restructuring

3.1 The choice of a model

The model is a multi-period stochastic program. Decisions are made here-and-now based on all available information, and anticipating future uncertain information. As new information is obtained and uncertainty resolved we have the recourse decisions that are conditioned on the realized scenario and on prior decisions. Stochastic programming models have a rich history in financial management over the last twenty years; see Zenios (2007); Zenios and Ziemba (2007) and the extensive bibliography therein. Recently such models received attention for public debt management in the works of Balibek and Köksalan (2010); Consiglio and Staino (2012). Our model extends Consiglio and Staino (2012) to debt re-profiling. A multi-period model captures some key features of the debt re-profiling problem: it allows us to integrate multiple debt issues with different maturities that may extend well into the future, and can account for clustering of maturities around specific dates with the associated roll-over risk. Clustering of debt maturities around specific dates, usually following government elections, is prevalent in sovereign debt management, as the political process borrows during a government’s term and pushes payment to the next government. Note, for instance, in Figure 8 of Cyprus debt, the large payments due in 2014, just after the presidential elections of 2013.

3.2 The scenario setting

We consider now the setting where the key economic and financial variables evolve according to some stochastic processes. The processes may be correlated. In scenario modeling we adopt a discrete set of time-stages when decisions are taken, denoted by $T = \{0, 1, 2, \ldots, T\}$. $t = 0$
denotes *here-and-now* where all information is known, $T$ is the *risk horizon* and $t$ is the time index, see, e.g. (Zenios, 2007, chap. 5). At each time instance $t$, data evolve on a scenario tree, such as the one illustrated in Figure 4, whereby problem data can take values from a set indexed by the set of nodes $\mathcal{N}_t$. Each node $n \in \mathcal{N}_t$ represents possible future states of the economy at time $t$. Not all nodes at $t$ can be reached from every node at $t-1$ and we define *paths* from the root node 0 to some final node in the set $\mathcal{N}_T$ to denote the unique way of reaching a particular node. Each of these paths is a *scenario*. Our example has 12 scenarios, two possible states at $t = 0$, three possible states at $t = 1$ and six at $t$. We denote by $n \in \mathcal{N}_t$ nodes in the sets $\mathcal{N}_t$ for each $t = 0, 1, 2, \ldots T$. We denote by $\mathcal{P}(n)$ the set of nodes on the unique path from the root node to $n \in \mathcal{N}_t$, and by $p(n)$ the unique predecessor node for $n$, with $p(0)$ being empty. At any given node $n$ all information contained in its predecessor $p(n)$ and the path $\mathcal{P}(n)$ is known.

With this notation we now define state-dependent fiscal variables for each node $n \in \mathcal{N}_t$:

**GDP**, denoted by $G^n$ in nominal value with growth rate $g^n$.

**Debt**, denoted by $D^n$ in nominal value and $d^n$ as a ratio to GDP.

**Interest on debt**, denoted by $r^n$. For countries under an assistance program the interest may be fixed at $r^0$ for all time periods until the end of the program or even beyond.

**Government net budget**, denoted by $NB^n$, and the following relation applies:

$$NB^n = GT^n - GE^n.$$ (1)

$GT$ denotes government revenues, typically taxes, and $GE$ denotes government expenditures excluding debt servicing costs. For states of the economy with $NB^n > 0$ the government is running a structural surplus that can be used to pay down debt or accumulate reserves. Conversely, $NB^n < 0$ denotes structural deficit that increases debt.

**Stock flow adjustment of debt**, denoted by $SF^n$ in nominal value and $sf^n$ as a ratio to GDP.

We assume that all debt is in domestic currency. Technical modifications of everything that follows are possible to account for foreign debt and exchange rate risks; see, e.g., Sturzenegger and Zettelmeyer (2006) eqns. (A.10)-(A.18) for the inter-temporal dynamics on a single deterministic scenario, or Topaloglou et al. (2002) eqns. (13)-(14) for multi-currency scenario modeling including exchange rate hedging. We develop our model in a single currency for ease of notation and avoid one more risk factor in the stylized examples consider later.

### 3.3 Scenario arithmetic for fiscal dynamics

The general debt stock recursive equation is as follows, see, e.g., Ley (2010):

$$D_t = (1 + r_t)D_{t-1} - NB_t + SF_t.$$ (2)

Ley uses seignorage as $SF_t$. We use stock flow adjustment to represent adjustments to the debt profile via restructuring or rescheduling. If a sovereign can collect seignorage, then $SF$ needs to be split in a seignorage term and a debt restructuring term, and seignorage modeled separately. In the scenario setting the debt dynamics are conditioned on the nodes:

$$D^n = (1 + r^n)D^{p(n)} - NB^n + SF^n.$$ (3)

This equation is defined for every node on the tree. It can be solved recursively for each path leading from the root to each terminal node $n \in \mathcal{N}_T$. While there is only one solution to
equation (2) for the deterministic case, we have as many solutions to equation (3) as there are paths to terminal states in the scenario setting.

We express the debt dynamics as a ratio to GDP to account for “snowball effect”, i.e., the improvement/deterioration of the debt situation of a country by growth/contraction of the economy. GDP growth is a significant risk factor in debt crises and the debt-to-GDP ratio stock dynamics are expressed conditioned on the state of the economy denoted by $n$, derived directly from the scenario-independent debt dynamics.

$$
\frac{D^n}{G^n} = (1 + r^n) \frac{D^{p(n)}}{G^{p(n)}} G^{p(n)} - \frac{NB^{p(n)}}{G^n} + SF^n.
$$

(4)

GDP growth is given by

$$
g^n = \frac{G^n - G^{p(n)}}{G^n},
$$

(5)

and we express the debt dynamics in proportional growth instead of nominal value by:

$$
d^n = \frac{1 + r^n}{1 + g^n} d^{p(n)} - nb^n + sf^n.
$$

(6)

We can use this equation to derive conditions for debt sustainability to answer questions such as “How can a government maintain a constant debt-to-GDP ratio?”, or, “How to reduce debt-to-GDP ratio to a level that is sustainable?”. For instance, debt is stable if

$$
d^n = d^{p(n)}
$$

for the paths to all terminal nodes, and the primary surplus to ensure this is given by:

$$
\hat{nb}^n = \frac{r^n - g^n}{1 + g^n} d^{p(n)} + sf^n.
$$

(7)

When primary balance satisfies $nb^n > \hat{nb}^n$ debt will be reduced in direct proportion to the primary balance. Assuming no debt restructuring, i.e., $sf^n = 0$ and balanced budget, i.e., $nb^n = 0$, the debt is stable if growth equals effective interest rate on debt $r^n = g^n$. Growth is suppressed during crises, requiring strictly positive primary balance to maintain constant debt. Fiscal consolidation, in turn, exerts downward pressure on GDP exacerbating the crisis. Hence, there is a limit to how much can be achieved with surplus-generating austerity once a country is in crisis. That is when debt restructuring enters as a policy option. The question is then posed on which combination of primary surplus and debt restructuring will reduce debt by a proportion $\beta$ to bring it to sustainable level.

To reduce debt by $d^n = \beta d^{p(n)}$ we have the following debt stock equation:

$$
nb^n = \frac{1 + r^n}{1 + g^n} d^{p(n)} + sf^n.
$$

(8)

This equation provides the relationship between the two key policy variables: primary surplus achieved with austerity measures and debt restructuring. However, the use of scenarios highlights the difficulty in making deterministic statements for the policy variable. The equation provides different values for each scenario and we need to adopt some risk metric, such as using the worst case value, or the expected value or some acceptable quintile.

### 3.4 Scenario optimization

#### 3.4.1 Model equations

We consider now the funding of government debt with a sequence of decisions $x = \{x_t\}_{t=0,1,\ldots,T}$. These decisions could include borrowing from the international market, loans of different maturities and contractual obligations from international organizations as part of an assistance program, or bi-lateral agreements with friendly governments. $x_0$ is the here-and-now decision
and \(x_t\) are the recourse decisions that are conditioned on the state of the economy. \(x_t\) is a set of possible decision vectors \(\{x^n_t\}_{n \in N_t}\). Without ambiguity we drop the time index from \(x^n_t\) since each node \(n\) takes values from a time-indexed set \(N_t\). If we assume there are \(J\) available options for funding debt, then the decision vector is given by

\[
x^n = (x^{n1}, x^{n2} \ldots x^{nJ}).
\]

We now express the debt dynamics on the scenario tree in terms of funding decisions. For each \(t = 0, 1, \ldots, T - 1\), and each \(n \in N_t\) we have:

\[
O^n = \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} CF_j(n, m),
\]

where \(CF_j(n, m)\) denotes cash flows at node \(n\) for any debt issued at some node \(m \in \mathcal{P}(n)\).\(^1\) \(O^0\) is the nominal debt due here-and-now. This equation accounts for the total debt to be covered at each node \(n\) due to decisions made at previous time periods.

The debt stock equation takes into account obligations created by previous funding decisions and existing debt, and requires that this stock is financed from new funding decisions:

\[
\sum_{j=1}^{J} x^{nj} = D^n + O^n. \tag{10}
\]

At the end of the horizon \(T\) and for each \(n \in N_T\) we have the cost of our decisions:

\[
C^n = D^n + O^n + \sum_{m \in \mathcal{P}(n)} \sum_{j=1}^{J} x^{mj} P_j(n, m). \tag{11}
\]

\(P_j(n, m)\) is the state-dependent value of debt and it can be given in nominal value, if we follow the accounting standards for sovereign debt reporting, or by market value if we are interested in fair valuation for the sovereign’s creditors. The use of market values requires contingent re-pricing of debt at the different states, see Mulvey and Zenios (1994). The problem of choice between book and market valuation is prevalent in the sovereign debt management literature, so much so that there are two approaches of computing sovereign debt haircuts; Das et al. (2012). Using market value is more appropriate for debt buyback situations, while book value is more appropriate for contractual restructuring.

### 3.4.2 Objective function

We develop the objective function on the debt-to-GDP ratio \(c^n = C^n / G^n\) which is the key indicator for debt sustainability; (Sturzenegger and Zettelmeyer, 2006, p. 308-313). Debt-to-GDP is a random variable whose discrete distribution depends on debt financing decisions, on the schedule of existing debt, on any debt restructuring, and the economic and financial random variables. The decision maker wants to shape the distribution of this random variable to match some views. For instance, they may want to tradeoff expected value against volatility. Or, more appropriately, consider a target Value-at-Risk that should not be exceeded or a target on the coherent risk measure of conditional Value-at-Risk (CVaR), of Artzner et al. (1999). Alternatively, the public debt management office may have a multitude of objectives

\(^1\)The calculations of \(CF\) are tedious. They take into account coupon and principal payments, perhaps adjustable rates, contingency provisions and so on. However, all these are exogenous to the debt funding decision. Once a scenario tree is built and a path \(\mathcal{P}(n)\) specified \(CF\) is obtained using standard cash-flow calculators; see (Consiglio and Staino, 2012, eqn. 6) for cash-flow calculations of government bonds and Topaloglou et al. (2002) for bonds in multiple currencies.
and wish to trade-off expected cost, duration of debt, volatility of cost or VaR and so on. The public debt management office of the Turkish Ministry of Finance adopts a multi-objective optimization framework, Balibek and Köksalan (2010), while Consiglio and Staino (2012) optimize conditional Cost-at-Risk for the Italian treasury.

Debt sustainability analysis establishes a threshold debt-to-GDP ratio below which the sovereign can service its debt without resorting to additional borrowing. In general 120% is considered the threshold for advanced economies, and this was used for the Greek debt restructuring. For Cyprus the threshold was estimated at 100%, and to avoid exceeding this value with a bail-out of banks by the state, the depositors were bailed-in instead. These targets are based on a unique projection of debt-to-GDP which, in our scenario setting, would be the mean value. To assess the risk of deviation from a sustainable debt-to-GDP threshold we need a risk measure. We define the stress debt for each terminal state of the economy as the non-negative difference of the state-dependent debt-to-GDP ratio from its expected value. Stress debt is a signal of problems that debt-to-GDP ratio deviates from a sustainable mean value, and we now formulate its conditional Value-at-Risk, which we call conditional Debt-at-Risk. Let

\[ sd^n = c^n - E[c], \]  

where \( E[C] \) is the expected value of the terminal final debt-to-GDP ratio, i.e.,

\[ E[c] = \sum_{n \in \mathcal{N}_T} \pi^n c^n, \]  

where \( \pi^n \) are the probabilities of the terminal states. For uniform distributions of child nodes at each non-terminal node, the probabilities are given by \( \frac{1}{|\mathcal{N}_T|} \) for symmetric trees. For non-symmetric trees, such as the one of Figure 4, terminal probabilities are easily computed as the joint probability of all nodes on the path \( P(n) \). The conditional Debt-at-Risk is the expected excess debt over its Value-at-Risk of debt at a given confidence level \( \alpha \) and, following Rockafellar and Uryasev (2000), this is obtained as CDaR from the following set of equations:

\[ \text{CDaR} = \zeta + \frac{1}{1 - \alpha} \sum_{n \in \mathcal{N}_T} \pi^n y^n_+, \]  

\[ y^n_+ \geq sd^n - \zeta, \]  

\[ y^n_+ \geq 0, \]  

where \( y^n_+ \) is a dummy variable denoting the non-negative values of debt in excess of \( \zeta \), and \( \zeta \) is the Value-at-Risk of the debt.

3.4.3 Model

We have expressed debt dynamics and conditional Debt-at-Risk in terms of exogenous GDP dynamics and debt restructuring decisions, and endogenous debt financing. The following optimization model minimizes the expected cost of debt with limits on the risk metric CDaR:
Minimize $\mathbb{E}[c]$
\[\text{s.t.} \quad O^n = \sum_{m \in P(n)} \sum_{j=1}^{J} x^{mj} CF^j(n, m) \quad \text{for all } n \in \mathcal{N}_t, t \in \mathcal{T} \setminus 0,\]
\[D^n + O^n = \sum_{j=i}^{J} x^{nj}, \quad \text{for all } n \in \mathcal{N},\]
\[C^n = D^n + O^n + \sum_{m \in P(n)} \sum_{j=1}^{J} x^{mj} P^j(n, m), \quad \text{for all } n \in \mathcal{N}_T,\]
\[sd^n = c^n - \mathbb{E}[c], \quad \text{for all } n \in \mathcal{N}_T,\]
\[y_+^{n} \geq sd^n - \zeta, \quad \text{for all } n \in \mathcal{N}_T,\]
\[\zeta + \frac{1}{1 - \alpha} \sum_{n \in \mathcal{N}_T} \pi^n y_+^{n} \leq \rho,\]
\[x^n, O^n, c^n, y_+^{n} \geq 0, \quad \text{for all } n \in \mathcal{N}.\]

Varying the bounds on CDaR we trace an efficient frontier and from that we can evaluate different policies and identify those that are sustainable at the $\alpha$ confidence level.

4 Stylized examples from Cyprus and the IMF

We use two examples drawn from an IMF publication and the debt crisis in Cyprus. The applications are stylized and further validation is needed before the results can be used to guide specific policies. However, key features of debt restructuring are captured and allow us to illustrate how risk profiles change with alternative debt restructuring policies and to show how sustainable policies can be identified. Interest rate scenarios were generated using the simulator of Bernaschi et al. (2007). The main input data are (i) the current term structure, (ii) the ECB official rate, and (iii) the current inflation rate. Using this information and fitted parameters for the short rate model, we generate a set of scenarios of lengths 12 months. The next stage scenarios are simulated by starting from the relevant data of the previous month and the procedure is repeated for each scenario and for each stage. Figure 5 illustrates the scenarios and the input term structure we use. The confidence level is set to 95%.

4.1 The IMF example

We work with a three-period discretization of the example from Das et al. (2012) shown in Figure 6. In this example we are not given any information on the size of the economy, so instead of optimizing our risk metric for debt-to-GDP ratios, we work directly with debt (i.e., GDP is set equal to 1). We run the model to optimize the financing of three different debt structures and compare the efficient frontiers (i) for the original debt, (ii) the rescheduled debt, and (iii) the restructured debt with nominal debt reduction. Figure 7 illustrates the improvements in the tradeoffs. The improvements, in this example, are very large, reflecting the fact that the original debt was heavily front loaded and therefore rescheduling was very effective. Further savings of about 40% of expected cost are realized by the 33% haircut. However, without information on size of GDP we can not tell which points on the frontiers correspond to sustainable debt.
Figure 5: Sample scenarios and the input term structure for the implementation.

Figure 6: Discretized three-period IMF example: original debt, rescheduled debt and restructured debt with 33% nominal value haircut.
4.2 The Cyprus example

The Cyprus debt crisis, while small in absolute numbers –GDP was EUR18 billion at the onset of the crisis corresponding to 0.2% of the eurozone economy– is one of the most complex in the eurozone. Zenios (2013) gives a review of the crisis as it developed until March 2013 when a rescue package was agreed with international lenders and explains why Cyprus faces a “perfect crisis”. With the Greek PSI of 2011 the two major banks of Cyprus suffered significant losses and required about EUR4 bil. to satisfy capital requirements while a subsequent due diligence by PIMCO –taking into account nonperforming loans under deteriorating economic conditions– estimated capital shortfalls of 5.98 bil. and 8.87 bil. under a base and adverse scenario, respectively. These estimates were criticized in Zenios (2014) as being excessive by a factor of two, and an investigation by NY Times also questioned the estimates, but nevertheless these were the numbers used to define bank recapitalization needs 2. They represent 35-55% of GDP to be added to 2012 public debt of 85.6% GDP. The resulting capital needs could not be resolved with an IMF-ECB-EC bail-out and Cyprus became the first eurozone country where bank depositors were bailed-in, making international headlines 3. The debt profile in Figure 8 is based on Cyprus debt position at the end of 2013 and we use GDP projections and fiscal variables from the debt sustainability analysis of Commission (2013) shown in Figure 9.

We run the optimization model for three test cases of (i) the original debt, (ii) rescheduled debt, and (iii) restructured debt. The results are summarized in Figure 10. For the original debt we also report the range of values for each point on the frontier, based on a sensitivity analysis of the results by re-running the simulations sixteen times.

What do these results tell us about the sustainability of Cyprus debt? For the country’s GDP of 17 billion in 2013 we could judge that all points on the efficient frontier for the original

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Figure 8: Cyprus debt profile (top) and three-period discretization with rescheduling and restructuring with 20% haircut (bottom).
Figure 9: Projections for Cyprus GDP growth and primary surplus. (Data from Commission (2013).)

Figure 10: Risk profiles for financing the Cyprus debt with and without restructuring.
Figure 11: Risk adjusted cost of financing the Cyprus debt. The horizontal line is the sustainability threshold corresponding to 100% debt-to-GDP ratio.

debt structure in Figure 10 lie below the GDP level. As debt-to-GDP ratio of 100% is considered sustainable by the IMF, we conclude from this figure that Cyprus debt is sustainable. However, this comparison ignores the tail of the distribution. Recall that the vertical axis is the expected cost but in a crisis we are interested in the tail and not in the mean. While the tail is a rare event, a country going through a debt crisis is experiencing this rare event, and the tail is the new normal. We therefore redraw the efficient frontiers by adjusting the expected cost for its associated risk by adding $1 \times CDaR$. That is, we consider uncertainty around the average of the tail, showing the results in Figure 11. For the original debt we also show the range of each value. This representation of results allows us to make two observations:

1. Note the convexity of these curves, which indicates that reducing expected cost and increasing risk on the efficient frontiers has the effect of originally reducing the cost of the extreme scenarios, but then these costs increase. Governments that myopically reduce the short term expected cost are likely to face problems when entering the crisis zone.

2. The extreme frontier for the original debt lies above the threshold and is unsustainable. Rescheduling the debt, even without a haircut, pushes the frontier below the threshold and makes the debt sustainable.

5 Conclusions

Scenario analysis and stochastic programming allow us to model the key technical aspects of debt restructuring questions. Current policy debates indicate that there is need for such models. We have shown how scenario-based debt sustainability analysis can be integrated within an optimization model to trace the tradeoffs between the cost of debt financing and the associated risks. The model allows us to compare alternative debt structures and identify those that are sustainable with a high probability. Hence, the models can be used for the risk management of debt restructuring decisions.

The model integrates in a common framework, and under a set of consistent scenarios, multiple debt issues over different time horizons, allows the embedding of options and contingent
claims, can deal with multiple currencies, sinking funds and step-up or linked contractual features. While not all of these features were modeled and tested, they require only straight-forward technical modifications of the base model.

There are two extensions of the model that deserve further work for the model to be deployed in large-scale real-world applications in crisis countries. First, to simulate the feedback loop between austerity and GDP scenarios. At the simplest level this can be done for each state of the postulated scenarios through fiscal multipliers for the economy of interest. Second, to optimize the debt restructuring decisions, as oppose to doing risk management optimization of alternative exogenously given debt structures. This would require a parametrization of the debt structure and embedding our optimization model into a hierarchical optimization of the debt structure parameters.

The results with the two stylized examples are very informative, especially for the case of Cyprus where they cast doubt on current beliefs on the sustainability of the country’s debt. Given the insights obtained for the examples, the two extensions discussed above are worth pursuing.
References


