Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach*

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Abstract

This paper uses risk-shifting models to analyze policy responses to asset price booms and bubbles. We argue that the presence of risk shifting can generate many of the features of such booms. Our analysis offers several insights. First, in our setting policymakers do not need to determine whether an asset is a bubble to intervene, since risk-shifting leads to the same inefficiencies whether asset prices equal fundamentals or exceed them. Second, while risk shifting offers a reason for intervention, it implies the leading interventions policymakers are debating have ambiguous welfare implications. We show tighter monetary policy may exacerbate some inefficiencies due to risk shifting even as it mitigates others, and that leverage restrictions can lead to higher asset prices and more rather than less excessive leverage. A key takeaway is that effective policies are those that disproportionately affect speculation.

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*The views here do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

Introduction

Policymakers have long debated how to respond to asset price booms and potential bubbles, or when asset prices surge to levels that seem to exceed the value of the dividends that assets are expected to yield. One view, summarized in Bernanke and Gertler (1999) and Gilchrist and Leahy (2002), argues that policymakers should wait and see what happens to asset prices, and then step in only if asset prices collapse and drag down economic activity. An alternative view, summarized in Borio and Lowe (2002), argues that policymakers should not stand idly by during these episodes. They argue that asset booms, especially those that coincide with credit booms, are likely to end in financial crises and recessions, a claim corroborated in subsequent work such as Jorda, Schularick, and Taylor (2015) and Mian, Sufi, and Verner (2017). By intervening to dampen asset prices, they reason, policymakers might be able to mitigate the eventual crash.

The severity of the Global Financial Crisis in 2007 and the difficulty central banks faced in its wake to provide stimulus after lowering short-term nominal interest rates to zero shifted opinion in policy circles toward favoring a more proactive response to asset booms. But this led to a new debate over how to intervene as opposed to whether to do so. The two policies that attracted the most attention are monetary policy and macroprudential regulation. Svensson (2017) argues against monetary tightening during asset booms because its costs exceed its benefits. Stein (2013) argues that even if regulatory policy could work in principle, in practice it is likely to be circumvented through clever financial engineering.

This paper explores how policy should respond to asset booms through the lens of risk-shifting models, i.e., models in which the agents who finance asset purchases cannot gauge the risk from any individual borrower they fund, and as a result take on more risk than they would like. We focus on risk-shifting because it seems relevant for these episodes. First, asset booms feature extensive lending against assets. Second, booms tend to be associated with assets that are difficult for lenders to evaluate, i.e. assets tied to new and imperfectly understood technologies or assets like housing that are valued idiosyncratically. With new technologies, it will be hard to distinguish genuinely productive applications of new technologies (dot-com, blockchain, tranched securities) from speculative investments that may not pan out. With housing, lenders may not be able to distinguish illiquid borrowers who value home ownership from speculators betting that house prices will rise and willing to walk away if they fall. In both cases, lenders will have a hard time discerning how likely the agent they lend to is to default if asset prices fall.[1] To be clear, our focus on risk-shifting is not meant to deny other mechanisms that can give rise to bubbles and asset booms. Risk-shifting is complementary with these mechanisms, a point we elaborate on in the Conclusion.

A large literature on risk-shifting has developed since it was first introduced by Jensen and Meckling (1976). Much of this literature focuses on how leverage affects investment and risk-taking. A smaller and more recent literature has examined how investment decisions might affect asset prices. Allen and Gorton

[1]While we describe situations where limted information is an exogenous feature of an asset, Asriyan, Laeven, and Martin (2018) argue asset booms can reduce the incentive to screen borrowers, so information about assets deteriorate endogenously.

(1993) first showed that risk-shifting allows asset prices to exceed the fundamental value of the dividends assets can generate. Subsequent work by Allen and Gale (2000), Barlevy (2014), Dow and Han (2015), Dubecq, Mojon, and Ragot (2015), and Bengui and Phan (2018) has built on this original result.[2]

This paper contributes to the above literature in two ways. First, we explore risk-shifting in a general equilibrium setting that allows us to study monetary policy and macroprudential regulation, something previous work on risk-shifting has yet to do. Second, unlike previous work, we allow default to be costly. These costs can be viewed as a reduced-form way to capture a fall in output when an asset boom ends. Previous work has identified various reasons why a collapse in asset prices can lead to lower output. For example, financial intermediaries who lent against assets during the boom may be reluctant to finance new investments given the overhang of debt they face. Alternatively, when prices are rigid, indebted households may be forced to delever in a way that erodes aggregate demand and output.[3] Default costs similarly imply that a collapse in asset prices leaves agents with fewer resources, even if for the more mundane reason that lenders must expend resources to recover payment. Allowing for output to fall once a boom ends is important, since it can justify intervention during booms when forgone output is sufficiently large. Empirically, Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that asset price crashes are associated with a decline in GDP per capita of between 9 and 16%, while Atkinson, Luttrell, and Rosenblum (2013) estimate the cumulative loss in output in the US in the recent crisis was even larger.

At the heart of our model is an information asymmetry in which borrowers know the risks of their investments better than lenders. This encourages some agents to borrow and gamble on risky assets, knowing that it will be lenders who bear the losses if their gamble fails. As speculators buy up assets, they drive the price of these assets up and the expected return on these assets down. This process leads to two distinct inefficiencies. The first is misallocation. Since lenders cannot distinguish among the different types of investments, borrowers ultimately decide how funds are allocated. They will direct too many resources to risky investments that offer high private returns to speculation but low overall returns. This is consistent with evidence of misallocation during credit booms in Borio, Kharroubi, Upper, and Zampolli (2015) and Charles, Hurst, and Notowidigdo (2018). Second, even if resources are properly allocated, agents borrow too much because they fail to internalize the costs that others incur when they default. In our model these are the costs lenders incur in recovering assets, but more generally speculators would likely not internalize how their actions lead to lower output due to debt overhang or lower aggregate demand.

Our analysis yields new insights on when and how policymakers should address these inefficiencies. First,

[2]Shleifer and Vishny (1997) also study asset pricing with credit and limited information, but without involving risk-shifting. Asset prices in their model deviate from fundamentals because of noise traders and limits to arbitrage. They show how lender uncertainty can shape what creditors lend against assets and so how much asset prices deviate from fundamentals.

[3]See Phillipon (2010) on debt overhang and financial crises and Korinek and Simsek (2016) and Farhi and Werning (2016) on aggregate demand externalities and deleveraging. Rognlie, Shleifer, and Simsek (2018) suggest another channel involving investment overhang, whereby a glut of assets during the boom dampens the production of assets after the crash. The latter is tricky to capture in our setup, since we assume assets are either endowed or created at date 0 but not thereafter.

we find that even though risk-shifting can give rise to bubbles, it can remain a source of inefficiency even without giving rise to bubbles. When default costs are large, assets will not exhibit bubbles. Intuitively, large default costs discourage lending, helping keep asset prices in check and equal to fundamentals. But these costs don't discourage lending against risky assets altogether, and the lending that remains is distortionary. This provides a rigorous basis for the assertion in Borio and Lowe (2002) that policymakers should intervene during booms even if they cannot be sure asset prices exceed fundamentals.

The other key insight is that since risk-shifting models require productive uses of credit to cross-subsidize the lending that finances speculation, policy interventions that affect both activities can have ambiguous and even surprising effects. In particular, we find that tighter monetary policy helps alleviate excessive leverage associated with risk-shifting, but that it also exacerbates resource misallocation by reducing more productive investments that are already underfunded. This reveals a counterproductive aspect of tighter monetary policy. Nevertheless, tighter monetary policy can play a useful role if the first effect dominates, and there may be ways to mitigate some of the counterproductive aspects of tighter monetary policy if instead of tightening immediately, policymakers promise to tighten only if the boom continues into the future (and, by implication, to ease if the price of the asset collapses). We similarly find that macroprudential regulation can be counterproductive, increasing speculation rather than curbing it. This is because restricting leverage might especially curb productive investments, leaving more resources for speculation. Macroprudential regulation can thus make things worse, although in other circumstances it can play a useful role. Hence, while risk-shifting offers a reason to intervene during an asset boom, it also implies intervening will be difficult since policymakers will be hampered by the same informational frictions that give rise to risk-shifting in the first place. The takeway is that effective policies must disproportionately impact speculation, which is what policy seeks to discourage. Indeed, one of the reasons a promise to tighten if a boom persists is more useful in our framework than tightening immediately is that it targets speculators who stand to gain only if the boom persists.

The paper is organized as follows. Section 1 introduces the basic setup, focusing on a simple case where assets are riskless. We then build on this framework in Section 2 and consider the case of risky assets. There, we argue that the equilibrium of our model can capture some of the key features of asset booms. Section 3 describes the ways in which the equilibrium of our model is inefficient and allow a possible role for policy intervention. Section 4 considers monetary policy, and Section 5 considers macroprudential regulation, specifically restrictions on leverage. Section 6 concludes.

1 Credit, Production, and Assets

We now set up a framework that includes credit, production, and assets. In this section we will consider the simple case where the asset involves no risk. In the next section we build on this setup and consider the more interesting case where assets are risky. It is in the latter case where credit and asset booms emerge.

Consider an overlapping generations economy where agents live for two periods and only value consumption when old. That is, agents born at date t value consumption c_t and c_{t+1} at dates t and $t+1$ at

$$u\left(c_t, c_{t+1}\right) = c_{t+1} \tag{1}$$

There is a fixed supply of identical assets normalized to one. For now, we assume these already exist at date 0, although later on we will consider the case where they must be produced. Each asset yields a constant real dividend $d > 0$ per period. In the next section we will allow for a stochastic dividend.

There is a cohort of old agents at date 0 who start out owning all the assets. A new cohort is born at each $t = 0, 1, 2, ...$ Each cohort consists of two types of agents. The first, whom we call savers, are endowed with an aggregate e units of the good when young. They cannot produce or store goods, and must either buy assets or trade intertemporally to convert their endowment into consumption when old. The second type, whom we call entrepreneurs, can convert a good at date t into $1 + y$ goods at date $t + 1$ where $y > 0$, but only up to a finite capacity of one unit of input. Each entrepreneur is endowed with $w < 1$ goods while young. Since this is below their productive capacity, there is scope for savers and entrepreneurs to trade.

In principle, w and y can vary across entrepreneurs. For most of the analysis, we assume $w = 0$ for all entrepreneurs, so they must borrow all of their inputs. As will become clear in Section 5 when we allow for $w > 0$, allowing entrepreneurs to have wealth greatly complicates the analysis even though the results are essentially unchanged. We do allow y to vary across entrepreneurs. Let $n\left(y\right)$ denote the density of entrepreneurs with productivity y. We assume $n\left(y\right) > 0$ for all $y \in [0, \infty)$ and

$$e < \int_0^\infty n\left(y\right) dy < \infty \tag{2}$$

Condition (2) implies entrepreneurs could deploy more inputs than savers are endowed with.

We assume trade between savers and entrepreneurs is subject to several frictions. At this point, when dividends are deterministic and there is no uncertainty nor default, these frictions will be largely irrelevant. But once when we assume stochastic dividends in the next section, these frictions will matter.

1. **Transaction Costs**: Agents incur a fixed utility cost ϕ to trade with savers, where we let $\phi \to 0$.
2. **Information Frictions**: Savers cannot monitor if those they finance produce or buy assets. They also cannot observe any of the agent's wealth beyond the particular project the lender finances.
3. **Contracting Frictions:** Trade is restricted to non-contingent debt contracts, i.e., for each unit of funding agents receive at date t they must pay a fixed amount $1 + R_t$ at date $t + 1$.
4. **Default Costs:** If borrowers fail to pay their obligation, lenders can collect any proceeds from the project borrowers invested in, but the seizure wastes Φ resources per unit invested in the project.

Assuming transactions costs ϕ ensures only agents who expect to earn positive profits will seek to borrow. We then take the limit as $\phi \to 0$ to keep this cost small. This cost eliminates equilibria in which agents borrow even when they do not stand to gain anything, including scenarios in which they borrow knowing they will surely default. Our information frictions capture the idea that lenders cannot easily evaluate risk for individual loans. For example, mortgage lenders might not distinguish illiquid agents who value homeownership from speculators who would default on their loans if prices fell. The return $1+y$ would then be analogous to the surplus to the former from homeownership.[4] Assuming wealth is unobservable implies borrowers face limited liability, since lenders can only go after the resources they know about. Effectively, we are allowing agents to borrow using non-recourse loans, or, alternatively, via shell entities that hide their other resources so their liability is limited to the project they borrow for. The contracting frictions we assume capture the prevalence of non-contingent debt, which presumably is due to the prohibitive costs of measuring and verifying contingencies. If lenders could offer contingent debt, then when we consider risky assets in the next section, lenders could demand higher repayment when the return on the asset is high to ensure borrowing to buy assets is never profitable. By contrast, with non-contingent debt, agents who borrow to buy assets might profit when the return on the asset is high, and would be willing to borrow even if $\phi > 0$. Finally, our assumption that default costs are proportional to the scale of the project and not the amount agents borrow captures the idea that auditing a borrower requires inspecting their projects.

Each period, then, young savers allocate their endowment e to buying assets and making loans, young entrepreneurs choose whether to borrow to produce, and all young agents choose whether to borrow to buy assets. An equilibrium for this economy consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$ that ensure both asset and credit markets clear when agents act optimally.

To facilitate our exposition, suppose these paths are deterministic. We confirm in Appendix A that this is true in equilibrium. To solve for an equilibrium, we need supply and demand for assets and credit. Agents in their last period of life neither supply nor demand credit. They do own all assets, though, and will sell them if the asset price $p_t > 0$. Young savers are the only agents who can lend. They compare the return to lending $1 + R_t$ with the return to the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ and invest in whatever offers the highest return. Young entrepreneurs choose whether to borrow to produce. Finally, all young agents choose whether to borrow to buy assets. Agents will borrow for any activity they expect to profit from. Entrepreneurs with productivity $y \geq R_t + \phi$ will find it profitable to borrow to produce, while those with productivity $y < R_t$ will not. In the limit as $\phi \to 0$, entrepreneurs will produce iff their productivity $y \geq R_t$.

Savers use their endowment to either buy assets or make loans. Their borrowers will then either produce or buy assets. Hence, the endowment is ultimately used to either finance production or buy assets, implying

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{3}$$

[4] Of course, the surplus agents who value homeownership obtain are not a constant but depend on the price of housing. For an example of a proper risk-shifting model of housing, see Barlevy and Fisher (2018).

Since we assume $n(y) > 0$ for all y, there is a unique interest rate $R_t = \rho(p_t)$ that satisfies (3) for any asset price p_t. Moreover, $\rho(p_t)$ is increasing in p_t. Intuitively, a higher p_t reduces the amount of goods available for productive investment, so the interest rate on loans R_t must rise to lower demand from entrepreneurs.

Next, we argue the interest rate on loans $1 + R_t$ must equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$. For suppose $R_t < r_t$. Then agents could earn positive profits by borrowing to buy assets. Even if the fixed cost ϕ were positive, as long as agents borrow enough, their profits will exceed this cost. Hence, demand for borrowing would be infinite, yet the supply of credit is at most e, so this cannot be an equilibrium. Next, suppose $R_t > r_t$. Savers would then earn more from lending than from buying the asset, they would refuse to buy assets. Moreover, no agent would borrow to buy the asset knowing she would default. Since the old sell the asset whenever its price is positive, this would require $p_t \leq 0$. But if the price were nonpositive, demand for the asset would be infinite. For both the credit and asset market to clear, then, we must have

$$1 + R_t = \frac{d + p_{t+1}}{p_t} = 1 + r_t \tag{4}$$

Note that (4) holds for any ϕ. For $\phi > 0$, no agent would incur the cost ϕ to borrow to buy assets and earn no profits. Evaluating the equilibrium as $\phi \to 0$ implies agents will not borrow to buy assets in equilibrium, even though they would be willing to do so when $\phi = 0$. Substituting (3) into (4) implies

$$\begin{aligned} p_{t+1} &= (1 + \rho(p_t))\, p_t - d \\ &\equiv \psi(p_t) \end{aligned} \tag{5}$$

where $\psi'(p_t) > 1$, $\psi(0) = -d < 0$, and $\lim_{p\to\infty} \psi(p) > e$. The graph of $\psi(p)$ is illustrated in Figure 1 together with the 45$^{\text{o}}$ line. The two lines intersect at the unique value p^d at which $p^d = \psi(p^d)$. For any initial condition, the law of motion $p_{t+1} = \psi(p_t)$ defines a unique path of asset prices. For any initial condition other than $p_0 = p^d$, the path will reach in finite time a value that is either negative or exceeds e, neither of which can be an equilibrium. Hence, the unique deterministic equilibrium is $p_t = p^d$ and $R_t = \rho(p^d)$ for all t. We make note that the steady state price p^d is increasing in the dividend d. Formally, setting $p_t = p_{t+1} = p^d$ in the zero-profit condition (4), implies

$$d = \rho(p^d)\, p^d$$

The right hand side is increasing in p^d, so a higher d must imply a higher p^d. Graphically, a larger d will shift the curve $p_{t+1} = \psi(p_t)$ in Figure 1 down, and so the steady state p^d will rise.

In Appendix A, we confirm that there are no stochastic equilibria, so $p_t = p^d$ for all t is the unique equilibrium for this economy. We can summarize this result as follows.

Proposition 1 *When $d_t = d$ for all t, in the limit as $\phi \to 0$, the unique equilibrium features a constant price $p_t = p^d$ and constant interest rate $R_t = \rho(p^d)$ for all t. Only entrepreneurs with productivity $y > R_t$ produce, only savers hold assets, and no agents borrow to buy assets.*

In equilibrium, the return on assets and loans are equal. Denote the common return to both activities by $R^d = \rho\left(p^d\right)$. Consider the present value of dividends discounted at this return. This is given by

$$f_t \equiv \sum_{j=1}^{\infty}\left(\frac{1}{1+R^d}\right)^j d = d/R^d = p^d$$

The value of dividends discounted according to the return agents can earn thus coincides with the price of the asset. When $d_t = d$ for all t, the asset will not be associated with a bubble.

Remark 1: We can easily allow for multiple riskless assets. Suppose there were J assets indexed $j = 1, ..., J$, each with fixed supply of 1 but potentially different fixed dividends d_j. Let p_{jt} denote the price of the j-th asset at date t. Define $d \equiv \sum_{j=1}^{J} d_j$ as the total dividends from all J assets and $p_t \equiv \sum_{j=1}^{J} p_{jt}$ as the value of all J assets. Resources that don't finance production will be used to buy assets, so (3) continues to hold. In addition, the return on each asset $1 + r_{jt} \equiv \frac{d_j + p_{j,t+1}}{p_{jt}}$ must equal the interest rate on loans $1 + R_t$. Combining these equalities implies (4). Hence, the equilibrium conditions for p_t and R_t are unchanged. We can reinterpret p_t from our model as the value of all assets, each offering return R_t. ■

Remark 2: With some modifications, we can also allow for a growing set of assets. This will be relevant in the next section, where we will argue that the periodic arrival of new types of assets can trigger asset booms. Suppose each period's old are endowed with a stock of new assets normalized to 1. Stocks pay dividends one period after arrival. For aggregate dividends to remain constant, dividends on any single asset must decay over time. Let d_{st} denote the dividend at date t on assets that arrived at date s, and set

$$d_{st} = \begin{cases} (1-\theta)^{t-1} d & \text{if } s = 0 \\ (1-\theta)^{t-(s+1)} \theta d & \text{if } s = 1, 2, 3, ... \end{cases} \qquad \text{for } t \geq s+1$$

By design, total dividends $\sum_{s=0}^{t-1} d_{st}$ in each period t sum to d. Let p_{st} denote the date-t price of the asset that arrived at date s, and set $p_t = \sum_{s=0}^{t} p_{st}$ as the total value of all assets around at date t. The market clearing condition (3) is unchanged. The return on each asset $1 + r_{st} \equiv \frac{d_{s,t+1} + p_{s,t+1}}{p_{st}}$ will equal the interest rate on loans $1 + R_t$. Aggregating over all assets available at date t yields the following alternative to (4):

$$1 + R_t = \frac{d + (p_{t+1} - p_{t+1,t+1})}{p_t}$$

The equilibrium value of all assets p_t will be constant and equal to $\frac{d}{R^d+\theta}$, where R^d denotes the equilibrium interest rate on loans. The price of any individual asset equals $p_{st} = \frac{d_{s,t+1}}{d} p_t = \frac{d_{s,t+1}}{R^d+\theta}$. ■

We conclude our discussion with a brief comment on welfare. In equilibrium, the amount savers spend on assets equates the return on the asset to the productivity of the marginal entrepreneur. Is this efficient? At first, it might seem that any resources spent on the asset are wasted, since the asset will yield d regardless of how much is spent on it while lending to entrepreneurs yields additional output. However, shifting resources to production would rob the current owners of the asset from resources they earn selling their assets. Redirecting all resources towards production is therefore not Pareto improving. In fact, the equilibrium is efficient. Intuitively, suppose current asset owners could destroy their assets so that they stop yielding any dividends going forward. Paying the old for their assets can thus be viewed as an investment to preserve the asset. Efficiency dictates the returns to all investments should be equal at the margin. In the next section, we show that returns will not be equated when assets are risky.

2 Risky Assets, Credit Booms, and Bubbles

We now consider the case where the asset pays stochastic dividends. We return to assuming only one asset. Let the dividend on this asset follow a regime-switching process such that the dividend d_t starts at $D > d$ when $t = 0$ and then switches to d with a constant probability $\pi \in (0, 1)$ in each period if it has yet to switch. Once the dividend falls to d, it will remain equal to d forever.

An equilibrium still consists of paths for asset prices $\{p_t\}_{t=0}^{\infty}$ and interest rates on loans $\{R_t\}_{t=0}^{\infty}$, but now also includes a path for the share of lending used to buy assets $\{\alpha_t\}_{t=0}^{\infty}$. These paths must be consistent with optimizing behavior by agents and must ensure asset and credit markets clear at all dates t for any d_t.

In what follows, it will prove convenient to distinguish for each date t whether d_t is D or d. If $d_t = D$, agents who buy the asset at date t will be unsure about the dividend d_{t+1} at $t+1$. If $d_t = d$, agents who buy the asset at date t know it will pay a dividend of d at date $t + 1$. Let $\left(p_t^D, R_t^D, \alpha_t^D\right)$ denote an equilibrium if $d_t = D$ and $\left(p_t^d, R_t^d, \alpha_t^d\right)$ denote an equilibrium if $d_t = d$. Once dividends fall, the equilibrium will be as in Section 1, with $p_t^d = p^d$, $R_t^d = R^d$, and $\alpha_t^d = 0$ for all t. We only need to solve for $\left\{\left(p_t^D, R_t^D, \alpha_t^D\right)\right\}_{t=0}^{\infty}$.

We first show that we can solve for the equilibrium price p_t^D and interest rate on loans R_t^D independently of α_t^D. As before, savers allocate all of their endowment e to either fund production or buy assets. The price p_t^D must thus continue to satisfy (3). Next, we argue that in equilibrium,

$$\left(1 + R_t^D\right) p_t^D = p_{t+1}^D + D \tag{6}$$

That is, the interest rate on loans $1 + R_t^D$ is equal to the return on the asset if $d_{t+1} = D$. We first argue that $p_{t+1}^D + D$ represents the maximum possible payoff to the asset by showing that

$$p_{t+1}^D + D > p^d + d$$

For suppose $p_t^D + D \leq p^d + d$ for some t. Since $D > d$, this implies $p_t^D < p^d$. From (3), we know the equilibrium interest rate on loans R_t must equal $\rho(p_t)$. If $p_t^D < p^d$, then since $\rho'(\cdot) > 0$, we have

$$R_t^D = \rho\left(p_t^D\right) < \rho\left(p^d\right) = R^d$$

But then we would have

$$\left(1 + R_t^D\right) p_t^D < \left(1 + R^d\right) p^d = p^d + d.$$

This means that an agent who borrows to buy assets can make positive profits after paying their debt obligation at date $t + 1$ if $d_{t+1} = d$. But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. It follows that $p_t^D + D > p^d + d$.

To show that $\left(1+R_t^D\right)p_t^D$ must equal the highest return on the asset, suppose $\left(1+R_t^D\right)p_t^D < p_{t+1}^D + D$. In this case, demand for borrowing would be infinite: Agents can earn positive profits if $d_{t+1} = D$ but default and earn zero if $d_{t+1} = d$. Since the supply of credit is finite, this cannot be an equilibrium. Next, suppose $\left(1+R_t^D\right)p_t^D > p_{t+1}^D + D$. In this case, no agent would borrow to buy the asset knowing they would surely default and incur the cost ϕ. Nor would any agent buy the asset given they can earn more making loans. In particular, since no agent borrows to buy assets, the only agents who borrow are entrepreneurs with productivity $y > R_t^D$, and they will repay for sure. The return to lending R_t would then exceed the return to buying the asset. If no agent buys the asset, the price of the asset p_t^D would have to be nonpositive to ensure the old don't want to sell the asset. But this cannot be an equilibrium price, since if $p_t^D \leq 0$ there would be infinite demand for the asset. The only remaining possibility is $\left(1+R_t^D\right)p_t^D = p_{t+1}^D + D$.

Condition (6) is identical to the condition for an asset that offers a constant dividend $d_t = D$ for all t. From the previous section, we know there is a unique path $\left\{p_t^D, R_t^D\right\}_{t=0}^{\infty}$ that satisfies both this condition and (3). The equilibrium price p_t^D is constant and equal to p^D, where p^D solves

$$\rho\left(p^D\right)p^D = D$$

Likewise, the interest rate on loans R_t^D is constant and equal to $R^D = \rho\left(p^D\right)$. The fact that the asset trades as if it delivers $d_t = D$ forever, even though the dividend can fall with a probability π that can be arbitrarily close to 1, suggests the asset is overvalued. We return to this point below.

The only part of the equilibrium we still need to solve for is α_t^D. We can do this using the expected returns to buying an asset $\bar{r}_t^D$ and lending $\overline{R}_t^D$. The expected return to buying the asset is given by

$$1+\bar{r}_t^D = (1-\pi)\left(1+\tfrac{D}{p^D}\right) + \pi\left(\tfrac{d+p^d}{p^D}\right) \equiv 1+\bar{r}^D \tag{7}$$

As for the expected return to lending, a fraction α_t^D of lending is used to buy assets and the rest finances production. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs, $1+\bar{r}^D - \pi\Phi$. The remaining loans that finance production will be repaid in full, so the return on those loans is $1+R^D$. This implies

$$\begin{aligned}
1+\bar{R}_t^D &= \left(1-\alpha_t^D\right)\left(1+R^D\right) + \alpha_t^D\left(1+\bar{r}^D - \pi\Phi\right) \\
&= \left(1-\alpha_t^D\right)\left(1+\tfrac{D}{p^D}\right) + \alpha_t^D\left(1+\bar{r}^D - \pi\Phi\right)
\end{aligned} \tag{8}$$

If $\overline{R}_t^D > \bar{r}^D$, savers would prefer lending over buying assets. The only agents who would buy assets would be those who borrow to do so, and so $\alpha_t^D = \frac{p^D}{e}$. If $\overline{R}_t^D = \bar{r}^D$, savers would be indifferent between buying assets and lending. This means α_t^D can assume any value between 0 and $\frac{p^D}{e}$. Finally, if $\overline{R}_t^D < \bar{r}^D$, savers would prefer buying assets over lending. No agent would borrow to buy assets, implying $\alpha_t^D = 0$. Hence, the expected return to lending $\overline{R}_t^D$ and the share of lending used to buy assets α_t^D are jointly determined.

To solve for $\overline{R}_t^D$ and α_t^D, consider first the case where $\alpha^D = \frac{p^D}{e}$. This can only be an equilibrium if $\overline{R}_t^D \geq \bar{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, i.e., only if

$$\left(1-\tfrac{p^D}{e}\right)\tfrac{D}{p^D} + \tfrac{p^D}{e}\left(\bar{r}^D - \pi\Phi\right) \geq \bar{r}^D$$

Rearranging this equation and substituting in for $\overline{r}^D$ implies $\alpha_t^D = \frac{p^D}{e}$ is an equilibrium only if

$$\Phi \leq \left(\frac{e}{p^D} - 1\right)\left(\frac{D+p^D-d-p^d}{p^D}\right) \equiv \Phi^* \tag{9}$$

Next, consider the case where $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$. This can only be an equilibrium if $\overline{R}_t^D = r^D$ when we evaluate $\overline{R}_t^D$ at the relevant α_t^D. Since $\overline{R}_t^D$ is decreasing in α_t^D, this requires that $\overline{R}_t^D < \overline{r}^D$ when $\alpha_t^D = \frac{p^D}{e}$, or

$$\Phi > \Phi^* \tag{10}$$

In this case, the equilibrium value of α_t^D is the one that equates $\overline{R}_t^D$ and $\overline{r}^D$, which implies

$$\alpha_t^D = \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} \tag{11}$$

Finally, there cannot be an equilibrium in which $\alpha_t^D = 0$. This would require $\overline{R}_t^D \leq \overline{r}^D$ when $\alpha_t^D = 0$. But $\alpha_t^D = 0$ implies $\overline{R}_t^D = \frac{D}{p^D} > \overline{r}^D$. Hence, the value of α_t^D is unique and is either equal to $\frac{p^D}{e}$ or some value between 0 and $\frac{p^D}{e}$, depending on the cost of default Φ. We can summarize this result as follows:

Proposition 2 *When the dividend process follows a regime-switching process, in the limit as $\phi \to 0$, the unique equilibrium is given by*

$$(p_t, R_t) = \begin{cases} \left(p^D, R^D\right) & \text{if } d_t = D \\ \left(p^d, R^d\right) & \text{if } d_t = d \end{cases}$$

The share of lending used to buy assets α_t when $d_t = d$ equals 0 *and when $d_t = D$ is given by*

$$\alpha_t = \alpha^D = \begin{cases} \frac{p^D}{e} & \text{if } \Phi \leq \Phi^* \\ \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} & \text{if } \Phi > \Phi^* \end{cases}$$

While dividends are high, some agents must borrow to bet on assets. Intuitively, if only entrepreneurs borrowed, lending would be riskless. But the interest rate on loans R^D is equal to the maximum return on the asset, so lending would be more profitable than buying the asset. No agent would then buy the asset, and the asset market would not clear. So some agents must speculate in equilibrium. Note that the model does not pin down which agents borrow to buy assets. Less productive entrepreneurs who do not produce would certainly be willing to speculate. But since agents can hide any wealth not associated with the particular project they borrow for, entrepreneurs who borrow to produce and savers who make loans would be equally willing to borrow through a shell entity and speculate.

Now that we have characterized the equilibrium when assets are risky, we argue it can capture many features of the episodes documented by Borio and Lowe (2002), Jorda, Schularick, and Taylor (2015), and Mian, Sufi, and Verner (2017). Specifically, we show that our equilibrium can be associated with asset price booms and credit booms, asset bubbles, high realized returns but cheap borrowing during the boom, and an eventual crisis when asset prices collapse, speculators default, and consumption falls.

Asset Price Booms: We begin with asset prices. The equilibrium price of the asset while $d_t = D$ will be the same as in an economy in which dividends remain equal to D forever. But as we noted earlier, the price of an asset with a fixed dividend is increasing in the value of the dividend. Hence, $p^D > p^d$. Our economy starts with a high asset price that collapses when dividends fall, which we can view as a boom.

Empirically, asset booms do not feature a high constant price and a high dividend yield, but rapid price growth that drives returns rather than dividends. We can capture this in our setup if instead of assuming $d_t = D$ throughout the high regime, we allow dividends to start at d and then jump if the initial regime survives long enough. This is consistent with the fact that new technologies typically promise eventual profits rather than deliver profits from the start, and rents typically lag house prices in hot housing markets. Formally, suppose there is some finite date T such that in the high regime, $d_t = d$ until date T and then $d_t = D$ for $t \geq T$ as long as we remain in this regime. If we switch to the low regime, dividends will equal d forever.[5] Conditional on remaining in the high regime, the equilibrium from date T on is as in Proposition 2. But between date 0 and T, the equilibrium path of prices $\left\{p_t^D\right\}_{t=0}^T$ will follow the law of motion

$$p_{t+1}^D = \left(1 + \rho\left(p_t^D\right)\right) p_t^D - d \equiv \psi^d\left(p_t^D\right)$$

with the boundary condition that $p_T^D = p^D$. We illustrate the dynamics of prices in this case in Figure 2. Since $p^D > p^d$, the price p_0^D must start above the steady state p^d to reach p^D at date T. It then follows an explosive trajectory that hits p^D at date T. This accords with what we typically see in asset booms: Dividends are constant while prices grow at an increasing rate that exceeds the expected return to saving $\overline{R}_t^D$. Assuming a more realistic path for dividends thus generates more realistic asset price dynamics. But it also requires us to solve for a path of prices rather than a single price. For analytical convenience, we will continue to assume $d_t = D$ for the entire duration of the high regime.

Our setup also abstracts from how asset booms start. In principle, we could try to posit that we start in a regime with low dividends but there is a chance we enter the temporary high regime associated with a boom. But in that case, agents in the initial low regime would have an incentive to gamble on a regime switch. Assuming that $d_t = D$ for all dates in the temporary high regime, one can show that the asset would trade at p^D in the initial regime. That is, the asset boom would be manifest even before we entered the high regime. This is reminiscent of the Diba and Grossman (1987) result that asset bubbles cannot suddenly appear but must be present from the inception of the asset. Martin and Ventura (2012) show how to get around the latter result by allowing for arrival of new assets that cannot be traded before they come into existence, and let bubbles emerge on these. We can similarly get around the related issue in our setup by allowing for new assets as per Remark 2. That is, new assets are always being generated. Most of these pay a predictable but decaying return, but periodically new assets arrive that start in a high regime before transition to offering the same predictable but decaying return that assets typically do.[6] With some

[5] This setup is related to Zeira (1999). He assumed dividends grow until a stochastic date. In both his setup and ours, dividends can rise during the initial regime. However, we assume dividends fall when the regime changes, while Zeira assumes dividends remain unchanged when the regime changes.

[6] To ensure the return on assets is riskless outside of booms may require one-off changes in the dividends of existing assets if the assets that arrive ar risky to ensure the return on existing assets is the same as it would be if the new assets were riskless.

modifications, then, our model can generate periodic booms. Moreover, these can feature explosive price growth and constant dividends if we posit rising dividends during the boom regime.

Credit Booms: Next, we show that the asset boom is associated with a lending boom. When $d_t = D$, the amount agents borrow to buy assets is given by

$$\frac{\alpha^D}{1-\alpha^D}\int_{R^D}^{\infty} n\left(y\right)dy \tag{12}$$

When $d_t = d$, by contrast, agents will not borrow to buy assets. The asset boom will be associated with a boom in borrowing against assets.

Since informational frictions imply it is hard to distinguish between borrowing to buy assets and borrowing for productive purposes, arguably the relevant empirical measure is not borrowing against assets but total borrowing. The total amount agents borrow to buy assets or produce is given by

$$\frac{1}{1-\alpha_t}\int_{R_t}^{\infty} n\left(y\right)dy$$

The term $\frac{1}{1-\alpha_t}$ exceeds 1 when $d_t = D$ and equals 1 when $d_t = d$. At the same time, the integral $\int_{R_t}^{\infty} n\left(y\right)y$ is smaller when $d_t = D$ than when $d_t = d$ given $R^D > R^d$. Total lending can therefore rise or fall when d_t falls to d. When $\Phi \leq \Phi^*$, savers lend out all of their endowment e when $d_t = D$ but lend less than e when $d_t = d$. Only when $\Phi \gg \Phi^*$ can total borrowing fall with dividends. The asset boom is thus associated with a boom in lending against assets, and with a boom in total lending if default costs are not too large.

Asset Bubbles: Next, we consider how assets prices compare to fundamental values, i.e., the present discounted value of the dividends assets are expected to generate. In practice, it is difficult to measure fundamental values. The reason booms are suspected to be bubbles is how fast asset prices rise without any clear changes in the expected flow of dividends to justify this increase. In the model, of course, we can exactly compute the fundamental value of an asset and determine if a boom coincides with a bubble.

We begin by defining the fundamental value of the asset in our model. For this, it will help to distinguish among several rates of return when $d_t = D$. The first is the interest rate on loans R^D that borrowers are asked to repay. Recall this rate is the maximal possible return on the asset, i.e.,

$$1 + R^D = 1 + \tfrac{D}{p^D} \tag{13}$$

During the boom, lenders will not expect to always collect this interest in full, since a fraction $\alpha^D > 0$ of lending is used to buy assets and may end in default. Lenders expect to earn $1 + \overline{R}^D$, given by

$$1 + \overline{R}^D = \left(1 - \alpha^D\pi\right)\left(1 + \tfrac{D}{p^D}\right) + \alpha^D\pi\left(\tfrac{d+p^d}{p^D} - \Phi\right) \tag{14}$$

Finally, the expected return to buying the asset is given by

$$1 + \overline{r}^D = \tfrac{(1-\pi)\left(D+p^D\right)+\pi\left(d+p^d\right)}{p^D} \tag{15}$$

These three returns can be ranked, with $R^D > \overline{R}^D \geq \overline{r}^D$. To derive the last inequality, note that if the expected return to buying the asset $1+\overline{r}^D$ exceeded $1+\overline{R}^D$, no agent would agree to lend given they can buy the asset. But at any finite interest rate, demand for credit from entrepreneurs will be positive.

We now need to take a stand on the rate at which to discount dividends when we define the fundamental value of an asset. Given the information frictions in our economy, the relevant discount rate is arguably the rate any agent with wealth would expect to earn in the absence of any information about who they might be lending to. The expected return such agents earn in our economy is the expected return to savers. Since savers always lend in our economy, this expected return must equal $\overline{R}^D$. Using the fact that the equilibrium is stationary, we can define the fundamental value of the asset f^D recursively as

$$f^D = \frac{\pi\left(d+p^d\right)+(1-\pi)\left(D+f^D\right)}{1+\bar{R}^D} \tag{16}$$

Equation (16) incorporates $1+\overline{R}^D$ as the discount rate and uses the fact there $p^d = f^d$, since recall from the previous section that with constant dividends the price p^d coincides with the fundamental value of the asset d/R^d. Rearranging (16) implies

$$1+\overline{R}^D = \frac{\pi\left(d+p^d\right)+(1-\pi)\left(D+f^D\right)}{f^D} \tag{17}$$

Comparing (17) with (15) shows that $p^D > f^D$ whenever $\overline{R}^D > \overline{r}^D$ and $p^D = f^D$ whenever $\overline{R}^D = \overline{r}^D$. From Proposition 2 we can infer that when $\Phi < \Phi^*$, the expected return on loans $\overline{R}^D$ exceeds the expected return on the asset $\overline{r}^D$. In this case the asset will exhibit a bubble. But when $\Phi \geq \Phi^*$, the expected return on loans $\overline{R}^D$ will equal the expected return on the asset $\overline{r}^D$. In this case the price of the asset coincides with fundamentals. As we summarize in the next proposition, whether a bubble exists depends on Φ:

Proposition 3 *Let f^D denote the value of dividends discounted at the expected return on loans $\bar{R}_t$. Then the difference between the price of the asset and its fundamental value $b^D = p^D - f^D$ is*

$$b^D = \left(\pi\left(d+p^d\right)+(1-\pi)D\right)\left[\frac{1}{\pi+\bar{r}^D} - \frac{1}{\pi+\bar{R}^D}\right] \tag{18}$$

Hence, there exists a bubble when $\Phi < \Phi^*$ but not when $\Phi \geq \Phi^*$.

Bubbles can arise in our setting because leveraged agents with the option to default only care about the upside potential of the asset and are willing to pay more than its expected value to buy it. When Φ is large, savers will be reluctant to lend against all available assets. This requires that savers buy some fraction of the assets. But since savers value an asset at its fundamentals, the asset cannot be a bubble in this case.

Although bubbles only arise when $\Phi < \Phi^*$, there is a sense in which agents spend too much on assets for all Φ. To see this, note that since $R^D > \overline{R}^D \geq \overline{r}^D$, the marginal entrepreneur during the boom can earn a higher return R^D than the expected return $\overline{r}^D$ an asset can generate regardless of whether $\overline{R}^D > \overline{r}^D$ or $\overline{R}^D = \overline{r}^D$. Hence, it is not bubbles that lead to misallocation in our model, but risk shifting. This suggests that the relevant question for whether to intervene during an asset boom is not whether the asset is a bubble but whether there is evidence of risk shifting. We return to this point in the next section.

Realized Returns and Interest Rates: We next consider rates of return during the boom. Since $R^D > R^d$, the high dividend regime will be associated with a higher realized return on investment, both for those who buy assets and for those who lend.[7] A boom will appear to be a good time for savers.

However, there are two important caveats to this result. First, even if *realized* returns are higher during the boom, *expected* returns may be lower. The expected return to lending during the boom is $\overline{R}^D$ and after the boom is R^d. The expected return $\overline{R}^D$ defined in (14) is a weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D}$ minus expected default costs. Since $D/p^D = R^D > R^d$ and $R^d = d/p^d > d/p^D$, we have

$$1 + \frac{D}{p^D} > 1 + R^d > \frac{d+p^d}{p^D}$$

If the weighted average of $1 + \frac{D}{p^D}$ and $\frac{d+p^d}{p^D}$ gives enough weight to the latter, the expected return to lending will be below $1 + R^d$ even before accounting for default costs. Asset booms can therefore be times of low expected returns even though realized returns while the boom continues are high.

Second, notwithstanding the high returns agents earn during the boom, there is a sense in which interest rates are too low during the boom. Consider an economy in which lenders could monitor borrowers. Lenders would charge borrowers different rates depending on whether they produce or buy assets. Those who borrow to buy assets would be charged an interest rate at least as high as the maximum return on the asset, $1+D/\widehat{p}^D$, where a hat denotes the asset price in the counterfactual economy with monitoring. If $\Phi > 0$, the only possible equilibrium with monitoring would involve no borrowing to buy the asset while $d_t = D$. Savers would instead have to buy the asset directly from the old. But they must also make loans to entrepreneurs. Hence, the expected return on the asset with monitoring, $(1-\pi)\left(1 + D/\widehat{p}^D\right) + \pi\left(d + p^d\right)/\widehat{p}^D$, must be the same as the interest rate on loans to those who produce and repay in full. We can show that this implies $\widehat{p}^D < p^D$. Intuitively, savers will lend more to entrepreneurs to produce when they can monitor borrowers, meaning fewer resources will be spent to buy the asset. This implies

$$R^D = \frac{D}{p^D} < \frac{D}{\widehat{p}^D}$$

The interest rate when lenders cannot monitor borrowers will be lower than the interest rate charged to those who buy assets with perfect monitoring. Lending against risky assets thus carries too low of an interest rate. Essentially, imperfect monitoring forces entrepreneurs to cross-subsidize speculators so the latter face interest rates that don't reflect the risk of the assets they buy.

Fallout from the Crash: Finally, we turn to how asset booms end in our model. When dividends fall, agents who previously borrowed to buy assets will be forced to default and will impose costs Φp^D on their

[7] Although the higher return is due to higher dividends, recall that we can get higher returns associated with capital gains by assuming dividends will be high only if the high regimes lasts sufficiently long.

lenders. The collapse in asset prices thus triggers a fall in the resources this cohort can consume above and beyond the decline in the dividend income they earn. By construction, the decline is proportional to the price of assets p^D during the boom. A larger boom thus implies a larger loss once the boom ends. In our model this is because recovery costs are larger the more resources were invested in assets. But, as we noted in the Introduction, we view this as a stand-in for other channels in which a fall in asset prices would lead to lower output, e.g. debt overhang and or deleveraging. These mechanisms also suggest the decline in output after a crash should depend on the magnitude of losses associated with the crash.

Our model can thus capture several features of asset booms. In the remainder of the paper, we examine whether in our model there is a reason to intervene against booms, and whether the particular interventions that are debated by policymakers can improve welfare.

3 Inefficiency of Equilibria

In this section, we argue that there are two distinct senses in which the equilibrium of our model is inefficient when $d_t = D$. The first concerns resource misallocation: The marginal return to production during the boom exceeds the expected return on assets, so there are potential gains to redirecting some of the resources spent on assets to production. The second concerns excessive leverage: Agents who borrow to buy assets ignore the default costs Φp^D they impose on others and so take on too much debt. A key result is that these inefficiencies arise independently of whether $\Phi < \Phi^*$, or, alternatively, regardless of whether asset prices exceed fundamentals. In that sense, evidence of risk-shifting would make it unnecessary for policymakers to determine whether asset prices represent a bubble to decide if intervention is warranted.

We begin with misallocation. As we noted above, when $d_t = D$, the productivity of the marginal entrepreneur is equal to the interest rate on loans R^D. But R^D exceeds the expected return on loans $\overline{R}^D$, since agents who borrow to buy assets can default. At the same time, $\overline{R}^D$ is at least as high as the expected return on the asset $\overline{r}^D$, or else savers would refuse to lend in order to buy assets, yet demand for credit from entrepreneurs is always positive. Hence, $R^D > \overline{r}^D$. This inequality implies young agents could achieve a higher return on their endowment if they could arrange to shift some of what they spend on assets to the marginal entrepreneur. Intuitively, agents who borrow to buy the asset ignore the losses their lenders incur. As a result, their private gain to buying the asset exceeds its social return, and too many resources are allocated to buying assets. This distortion only occurs during the boom. Once the boom ends, the return on the asset d/p^d will be the same as the productivity of the marginal entrepreneur R^d.

Although a given cohort could secure a higher return to their resources by coordinating to shift resources from buying assets to producing, doing so would hurt the previous cohort from whom they buy assets. Redirecting resources to production would therefore not constitute a Pareto improvement. This point was already made in Grossman and Yanagawa (1993). They also studied an overlapping generations economy in which agents use resources to produce and buy assets. While their model did not feature risk shifting, it did feature a production externality that implied the return to production is higher than the return on the asset. They emphasize that even though resources are misallocated, it will be impossible to make all agents better off by reallocating resources. We now argue that this impossibility hinges on assuming that assets are exogenously given. If agents must use resources to create assets, as is true for new technologies or housing, the resources buyers pay for assets no longer represent pure rents to sellers. In that case, shifting resources from asset creation to entrepreneurs can make all agents better off.

Formally, suppose the old at date 0 are endowed with neither goods nor assets, but they know how to convert goods into assets. For simplicity, suppose assets can only be created at date 0. The technology for producing assets is summarized by an increasing function $c(q)$ which denotes the amount of goods needed to produce the q-th asset. Since goods are endowed to the young, the old must obtain goods from them. We assume the old collect the revenue from selling their asset advance. They then use some of the goods they receive as payment to produce assets and consume any goods left over. Optimality dictates they should create assets up to the point q^* at which the marginal cost of producing assets $c(q^*)$ equals the price p_0. Hence, they will collect $p_0 q^*$. Since $c(q)$ is increasing, this amount exceeds the amount of goods $C(q^*) \equiv \int_0^{q^*} c(q)\, dq$ needed to produce q^* assets.

With endogenous asset creation, the equilibrium condition (6) remains unchanged. However, we need to replace p_t in (3) with $p_t q^* = p_t c^{-1}(p_0)$. This expression is increasing in p_0 for all t, and so we can show that the equilibrium remains unique and qualitatively similar to before. Suppose we intervene and marginally reduce the quantity of assets produced at date 0 from its equilibrium value q^*. Since $c(q^*) = p_0$, this intervention will leave the consumption of the old unchanged. Cohorts born at dates $t \geq 0$ can redirect the p_t resources they would have spent on the last asset to entrepreneurs. If $d_t = D$ at date t, the productivity of the marginal producer R^D exceeds the expected return on the asset $\overline{r}^D$. If $d_t = d$ at date t, the return from the marginal producer R^d would equal the return on the asset r^d. Since $\Pr(d_t = D) > 0$ for all t, a marginal reduction in q^* makes all cohorts better off ex ante. As long as the resources the young spend on the asset at date 0 are not pure rents to the old, there is room for a Pareto-improving intervention.

The inefficiency associated with misallocation we've described holds for all values of Φ. But when $\Phi > 0$, our model admits an additional inefficiency associated with excessive leverage. Even if we hold the quantity of assets created at date 0 fixed, we can make all agents better off if we let lenders directly buy the assets their borrowers would have purchased and reimburse borrowers for their forgone income. This avoids the default costs Φp^D lenders incur when dividends fall. Essentially, there is no socially useful purpose for agents to borrow and buy risky assets. Yet in equilibrium they do so because they don't bear the costs of their default. The same would be true in models in which instead of recovery costs, Φ represents the amount of forgone output when asset prices fall due to debt overhang or deleveraging.

Both of these inefficiencies hold for any $\Phi > 0$. However, Proposition 3 implies bubbles only arise when $\Phi < \Phi^*$. Thus, the fact that the equilibrium is inefficient is independent of whether the price of the asset is equal to fundamentals or not. Although our model can give rise to bubbles, these are not the reason there is a need to intervene. The inherent problem with risk-shifting is not that asset values exceed fundamentals but that asset prices do not reflect their social value due to externalities associated with leveraged asset purchases. The implication is that policymakers who have evidence or risk-shifting do not need to determine whether asset prices coincide with fundamentals to decide whether to intervene. The purpose of intervention is not to equate prices with fundamentals, but to remedy distortions due to speculation.

Our model suggests there is scope for intervention to reduce the amount of assets created during a boom and to discourage borrowing against any assets that are created. However, policymakers are likely to face the same difficulties as private agents in distinguishing between speculation and more productive uses of assets, and cannot design policies directed at those who create or purchase risky assets. Policymakers might still rely on blunt tools, e.g. changing interest rates using monetary policy or restricting leverage for all borrowers. The remainder of the paper considers whether these interventions improve welfare.

To study these interventions, however, we need to relax some of the simplifying assumptions we have relied on so far. First, to capture the effects of monetary policy, we need to relax our assumption that the amount of resources e each cohort can allocate to production and assets is fixed. While this assumption is convenient, models of how monetary policy affects interest rates often assume that price rigidities allow economic activity to expand or contract when the monetary authority moves. In the next section, we drop the assumption that agents are endowed with a fixed amount of goods to incorporate monetary policy.

To capture the effect of leverage restrictions, we need to relax our assumption that entrepreneurs are endowed with nothing. When borrowers lack any resources, there is no way to restrict leverage other than cutting off credit altogether. In Section 5 we return to assuming savers are endowed with a fixed amount of goods, but we assume entrepreneurs are also endowed with resources. Whereas entrepreneurs without wealth must take on infinite leverage, those with wealth face a choice of how much leverage to take on. This requires us to move from a single credit market to many markets that span all possible leverage choices. We can then study the effect of leverage restrictions that shut down markets with especially high leverage.

4 Monetary Policy

We begin with monetary policy. As we noted above, this requires us to abandon our assumption that savers are endowed with an exogenous amount of goods. We follow Galí (2014), who also considers monetary policy in an overlapping generations economy with assets. In particular, we introduce two modifications. First, we assume savers are endowed with labor that can be used to produce goods rather than goods themselves. Second, we introduce a monetary authority that moves after goods producers set their prices but before they hire labor. This allows the real wage – and consequently output – to respond to monetary policy.

We leave the formal details of the analysis to Appendix B and only sketch the results here. Our assumptions imply that labor supply only depends on the real wage. Under these assumptions, in the absence of money, the equilibrium real wage is constant over time and independent of d_t. Thus, in the absence of money, the reduced-form representation of our economy is the same as the model we have assumed up to now: Each cohort of savers has a constant budget e which it must allocate between entrepreneurial activity and purchasing assets. The difference is that e is now endogenous and can be influenced by policy.

We then introduce money. Each period, producers set the prices of their goods and the monetary authority sets a nominal interest rate. As in Galí (2014), we consider an equilibrium in which money doesn't circulate. This requires that inflation adjust so the real value of the nominal rate set by the monetary authority is equal to the real return agents earn elsewhere, leaving agents indifferent to holding money. If producers set prices after the monetary authority moves, or can perfectly anticipate what the monetary authority will do, changing the nominal interest rate will not affect the real wage or any other real variable: Producers will set their prices in a way that leaves the real wage unchanged. Monetary policy would not effect real variables, only goods prices. But if producers set their prices before the monetary authority moves and cannot perfectly anticipate what it will do, producers will set their price as a markup over the expected wage that will prevail after the monetary authority moves. If the nominal interest rate turns out to be higher (lower) than expected, the real wage can be higher (lower) than expected. Essentially, an unanticipated move by the monetary authority allows a self-fulfilling fall in demand for goods. Lower demand for goods means producers don't need to hire as much labor, the real wage falls, and since agents earn less, demand for goods will indeed be lower. A surprise move by the monetary authority will thus change earnings e, just as an income tax or subsidy would. We can deduce the implications of such a policy on asset prices and interest rates using comparative statics on e_0 in our original endowment economy. The next proposition, based on our analysis in Appendix B, summarizes these effects.

Proposition 4 *An unanticipated monetary intervention at date* 0 *that reduces earnings* e_0 *below the earnings* e *that would have prevailed absent any intervention leads to a lower asset price* p_0^D *and a higher real interest rate on loans* R_0^D *than would have prevailed absent any intervention.*

We next turn to the welfare implications of a contractionary intervention at date 0. Since sellers set prices each period, an intervention at date 0 will have no impact on any cohorts born at dates $t = 1, 2, 3, ...$ The cohort born at date 0 works less, while the expected amount of resources they can consume is

$$\left[(1-\pi)\left(D+p_1^D\right)+\pi\left(d+p_1^d\right)\right]-\pi\Phi p_0^D+\int_{R_0^D}^{\infty}(1+y)\,n(y)\,dy \tag{19}$$

The first term in (19) represents the expected payout on the asset at date 1 and is unaffected by what the monetary authority does at date 0. The next term represents expected default costs. A contractionary policy at date 0 drives down the price p_0^D and lowers the expected costs of default. The last term represents the proceeds from production by entrepreneurs in this cohort. Since tighter monetary policy increases R_0^D, fewer entrepreneurs produce. A contractionary monetary policy thus mitigates excessive borrowing against assets but exacerbates underproduction by entrepreneurs. The impact of the intervention on this cohort is ambiguous, but for sufficiently large Φ the first effect will dominate and this cohort can consume more and work less. Finally, the old at date 0 will be worse off, since their earnings p_0^D fall. However, since the effect of policy on p_0^D and R_0^D is independent of Φ, for sufficiently large Φ it should be possible for the cohort at date 0 to leave the old at date 0 whole and still be better off on account of the lower default costs. Although contractionary monetary policy has generally ambiguous effects on welfare, it can lead to a Pareto improvement. This result is reminiscent of Svensson (2017), who argues tighter monetary policy is generally costly but can lower the odds of a financial crisis. In our framework, the probability the boom ends is fixed at π, but tighter monetary policy mitigates the severity of the output decline if the boom ends.

Given that a contractionary intervention mitigates excessive borrowing during the boom but exacerbates underproduction by entrepreneurs, we consider an alternative in Appendix B that can avoid this tradeoff. Suppose the monetary authority did nothing at date 0 but credibly promised to be contractionary at date 1 if the boom continued. Since producers set prices anticipating the average outcome at date 1, this means the monetary authority will be expansionary at date 1 if the boom ends by date 1. This is equivalent to promising a temporarily high endowment $e_1^d > e$ at date 1 if $d_1 = d$ and a temporarily low endowment $e_1^D < e$ if $d_1 = D$. Per Proposition 4, the contractionary policy at date 1 will depress p_1^D and increase R_1^D. However, as the next result states, this will reduce both p_0^D and R_0^D at date 0.

Proposition 5 *A commitment by the monetary authority at date* 0 *to set* $e_1^d > e > e_1^D$ *leads to a lower asset price* p_0^D *and a lower interest rate on loans* R_0^D *at date* 0 *than would have prevailed absent any intervention.*

A promise to tighten if a boom continues (and consequently ease if the boom ends) mitigates both excessive leverage and insufficient entrepreneurial activity at date 0, in contrast to tightening immediately. Not surprisingly, this policy can raise welfare under more general circumstances than immediate tightening. Formally, cohorts born at $t = 2, 3, ...$ after the intervention will be unaffected, We show in Appendix B that the cohort born at date 1 will be better off if $d_1 = d$. Although they work more than without the intervention, the monopoly power we need to allow for price setting implies employment is too low in the absence of intervention, and so higher employment raises welfare. Whether this cohort will be better off if $d_1 = D$ is ambiguous, just as a direct intervention at date 0 was ambiguous: This cohort will fund less entrepreneurial activity given R_1^D is higher but will incur smaller default costs Φp_1^D. Even if Φ is small so default isn't very costly, as long as the probability π that dividends fall is close to 1, this cohort will be better off *ex ante*. The cohort born at date $t = 0$ will be strictly better off, since both expected default costs Φp_0^D are lower and more entrepreneurial activity is financed when R_0^D is lower. Finally, the old at date 0 will be worse off given the amount they earn from the assets they sell p_0^D will be lower. But the young at date 0 would be better off even if they had to fully compensate the old when $\Phi > 0$. Hence, this intervention can be Pareto improving even when Φ is small and tightening at date 0 is not Pareto improving.

The advantage of a commitment to tighten in the future is that it discourages speculation by punishing those who buy risky assets but not those who produce. Lenders would do the same if we allowed them to write contingent financial contracts. We ruled this out on the grounds that enforcing contingent contracting is too costly. But contingent monetary policy does not involve enforcement and may serve as a substitute.

5 Macroprudential Regulation

We now turn to interventions that involve credit regulation. As we already anticipated, this will require us to relax our assumption that entrepreneurs lack resources. When entrepreneurs have no wealth, any arbitrarily small down-payment requirement would shut down all credit. This would eliminate speculation, but it would also end all trade between savers and entrepreneurs. To analyze interventions that only restrict rather than eliminate leverage, we need borrowers to be able to produce and speculate even when leverage is restricted. We therefore allow for entrepreneurs to be endowed with some resources. But this modification introduces a complication. When agents have no wealth, they can only be infinitely levered. When they have wealth, they can choose how much leverage to take on. This requires multiple markets to accommodate leverage choice in lieu of a single market as we have analyzed so far. To simplify things, we return to assuming agents are endowed with goods rather than labor. We could combine both elements to incorporate both monetary and macroprudential policies, but we would have to analyze this case numerically.

Each cohort still consists of two types, savers endowed with e goods but can't produce and entrepreneurs who can convert goods at date t into goods at date $t+1$ but have limited resources. Up to now, we assumed entrepreneurs were endowed with no resources but varied in productivity y. We now consider the opposite case: Entrepreneurs vary in their endowment but all have the same productivity y^*. We discuss the case where entrepreneurs vary in both wealth and productivity at the end of this section.

We assume a uniform distribution for wealth w. Specifically, for each $w \in [0,1]$, the density of entrepreneurs with wealth w is equal to $2\varphi e$, where φ is a constant such that $0 < \varphi < 1$ and e is the endowment of savers. The total endowment of all entrepreneurs is therefore

$$\int_0^1 w\,(2\varphi e)\,dw = \varphi e$$

Together, savers and entrepreneurs are endowed with $(1+\varphi)\,e$. To produce at capacity, entrepreneurs need

$$\int_0^1 (1-w)\,(2\varphi e)\,dw = \varphi e$$

Since $\varphi < 1$, entrepreneurs require fewer resources than savers have, in contrast to what we assumed in (2).

As for the common productivity y^*, we assume it is large enough to exceed the maximal return on the asset. To see that the maximal return on the asset is finite, observe that the asset price p_t is bounded below by $(1-\varphi)\,e$, the amount of resources left to spend on the asset if all entrepreneurs produce at capacity, and is bounded above by $(1+\varphi)\,e$, the total resources each cohort is endowed with. The maximal return on the asset occurs when $d_{t+1} = D$, the price of the asset at date t assumes its lowest value $(1-\varphi)\,e$, and the price at $t+1$ assumes its maximum value $(1+\varphi)\,e$. We assume $1+y^*$ exceeds this return, i.e.,

$$1 + y^* > \frac{D + (1+\varphi)\,e - (1-\varphi)\,e}{(1-\varphi)\,e} = \frac{D + 2\varphi e}{(1-\varphi)\,e} \tag{20}$$

Assuming a large y^* implies all entrepreneurs will want to produce at capacity in equilibrium. This is in contrast to how before only an endogenously determined fraction of entrepreneurs produced in equilibrium. Assuming all entrepreneurs are fully funded allows us to avoid solving for the endogenous fraction of entrepreneurs funded in each of a continuum of markets, which greatly simplifies the analysis.

Now that entrepreneurs have positive wealth, they can help finance their own investments. We assume lenders can observe the resources borrowers use to finance their investment but not what borrowers choose to invest in. Verifying how much borrowers invest is different from understanding what they invest in. By paying for a share of their investment, the borrower commits resources that can be used to repay the lender in case of default, since a lender who knows about the borrower's share of the project can go after its proceeds. However, we continue to assume lenders cannot observe any resources the agent has beyond what she invests in her project, i.e., borrowers can use shell entities to hide any additional wealth they have.

Formally, borrowers choose the fraction $\lambda \in [0, 1)$ of their investment to finance. We model this as a continuum of markets indexed by $\lambda \in [0, 1)$. An agent who borrows in market λ can borrow $\frac{1-\lambda}{\lambda}$ units for each unit of her own wealth that she invests. She can thus leverage her endowment of w to finance an investment of size $\frac{w}{\lambda}$. When $w > 0$, the choice of leverage is non-trivial: By going to a market with a lower λ, an entrepreneur can borrow more and produce at a larger scale, but this will leave their lender with a smaller cushion to go after in case of default. Back when we assumed all entrepreneurs had no wealth, agents had no choice. They could only borrow in market $\lambda = 0$ and choose infinite leverage. Now that agents have wealth, we need a market for each $\lambda \in [0, 1)$ to accommodate any leverage they might choose. The reason we assumed entrepreneurs had no resources up to now is precisely to focus on a single market.

We now define and solve for an equilibrium when there is a continuum of markets. To anticipate our results, we describe an equilibrium in which entrepreneurs with wealth w go to market $\lambda = w$ and borrow $1 - w$ to produce at capacity. Thus, entrepreneurs sort into different markets. Intuitively, entrepreneurs are happy to invest all of their wealth in production and reassure their lender about any potential losses in case of default. At the same time, savers pretending to be entrepreneurs will put up some of their wealth to leverage and buy assets, but only in markets with low λ where the share they fund is small. This motivates us to consider the effect of leverage restrictions, i.e., shutting down markets where λ is below some floor $\underline{\lambda}$.

5.1 Equilibrium with Multiple Markets

Given we now have multiple markets, an equilibrium consists of a path of interest rates $\{R_t(\lambda)\}_{t=0}^{\infty}$ for each market $\lambda \in [0, 1)$, amounts borrowed in each market, and a path of asset prices $\{p_t\}_{t=0}^{\infty}$. We now need to keep track of the amount borrowed for each activity in each market. Let $f_t^a(\lambda)$ and $f_t^p(\lambda)$ denote the rate at which agents borrow in market λ to buy assets and to produce, respectively, and define $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ as total borrowing in market λ for any purpose. We can integrate these rates to obtain the total amounts borrowed in all markets, $\int_0^1 f_t^a(\lambda)\, d\lambda$ and $\int_0^1 f_t^p(\lambda)\, d\lambda$. Although we refer to borrowing rates, we are not imposing that agents must borrow infinitesimal amounts in all markets. Indeed, once we introduce leverage restrictions, there will be a market that will attract a mass of borrowers. We discuss how to deal with this formally in Appendix C, but, loosely, such markets effectively feature infinite borrowing rates. We will refer to market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$. The price p_t, interest rates $R_t(\lambda)$, and amounts borrowed $f_t^a(\lambda)$ and $f_t^p(\lambda)$ must ensure all markets clear when agents acts optimally.

To determine if lenders are optimizing, we need to know what they expect to earn from lending in any market $\lambda \in [0, 1)$. Building on our previous notation, let $\overline{R}_t(\lambda)$ denote the expected return to lending at date t in market λ. If market λ is active, the expected return $\overline{R}_t(\lambda)$ to lending in market λ must equal what lenders recover from the agents they lend to. We can thus deduce $\overline{R}_t(\lambda)$ from the interest rate $R_t(\lambda)$ and the amounts $f_t^a(\lambda)$ and $f_t^p(\lambda)$ agents borrow to buy assets and produce, respectively. But if market λ is inactive, there is nothing to guide lenders on what to expect if they were to lend to a market where no borrowers show up. Instead, we need to assign an expected return $\overline{R}_t(\lambda)$ to each inactive market as part of our definition of an equilibrium. In what follows, we first look for an equilibrium in which all markets are active to avoid the question of how to assign $\overline{R}_t(\lambda)$ in inactive markets. We then discuss equilibria in which markets can be inactive. This naturally leads into our analysis of regulatory interventions in which some markets are inactive by decree rather than because of what agents believe.

We begin with the case where $d_t = d$ for all t. As in Section 1, we proceed as if equilibrium prices are deterministic and verify this is the case in Appendix C. With no uncertainty there is no risk of default, so the expected return to lending $\overline{R}_t(\lambda)$ in each active market λ is equal to the interest rate on loans $R_t(\lambda)$. The expected return in all active markets must be the same for lenders to agree to lend in all markets, and hence the interest rate on loans $R_t(\lambda)$ is the same for all $\lambda \in [0, 1)$. As before, in the absence of risk the common interest rate on all loans must equal the return on the asset $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$ to ensure savers are willing to both buy assets and make loans. That is, if all markets are active in equilibrium, then $R_t(\lambda) = r_t$ for all $\lambda \in [0, 1)$. At these interest rates, no agent will borrow to buy assets in order to earn zero profits if $\phi > 0$. Since (20) ensures y^* exceeds r_t, all entrepreneurs regardless of wealth will want to borrow to produce at capacity. Since $R_t(\lambda)$ is the same for all λ, entrepreneurs will be indifferent as to which market they borrow in as long as they borrow enough to reach capacity. In particular, entrepreneurs with wealth w will be willing to borrow $1 - w$ in market $\lambda = w$. This arrangement ensures all markets will be active.

With all entrepreneurs producing at capacity, total resources invested in production equal $2\varphi e$. Since young agents want to consume when old, any of the total endowment $(1 + \varphi) e$ of each cohort that is not used to produce will be spent on the asset. This implies

$$p_t + 2\varphi e = (1 + \varphi) e \tag{21}$$

It follows that $p_t = (1 - \varphi) e$ for all t. The return to buying the asset r_t and the interest rate on loans $R_t(\lambda)$ in all markets λ will then be $\frac{d}{(1-\varphi)e}$. This leads to the following analog to our earlier Proposition 1:

Proposition 6 *When $d_t = d$ for all t, there exists an equilibrium in which all markets are active. In any such equilibrium, $p_t = (1-\varphi)e \equiv p^d$ for all t, $R_t(\lambda) = \frac{d}{(1-\varphi)e} \equiv R^d$ for all $\lambda \in [0, 1)$ and all t, entrepreneurs borrow and produce at capacity, and in the limit as $\phi \to 0$ no agents borrow to buy assets.*

Next, we turn to the regime-switching case where $d_t = D$ at date 0 and then switches to d with probability π each period. Once again, we use a superscript D to refer to an equilibrium object at date t when $d_t = D$ and a superscript d when $d_t = d$. We already characterized the equilibrium when $d_t = d$ in which all markets are active in Proposition 6. We only need to solve for the equilibrium when $d_t = D$.

We begin with interest rates. For each active market λ, either agents borrow to buy assets in that market, i.e., $f_t^a(\lambda) > 0$, or they do not, i.e., $f_t^a(\lambda) = 0$. In the latter case, there will be no default and the expected return to lending $\overline{R}_t^D(\lambda)$ will equal the interest rate on loans $R_t^D(\lambda)$. In equilibrium, $\overline{R}_t^D(\lambda)$ must be the same in all active markets for lenders to agree to lend in these markets. Denote this common expected return by $\overline{R}_t^D$. Then $R_t^D(\lambda) = \overline{R}_t^D$ in any active market λ in which $f_t^a(\lambda) = 0$.

Next, consider an active market λ in which agents do borrow to buy assets, i.e., $f_t^a(\lambda) > 0$. Agents would only choose to borrow to buy assets if the expected to default if the return on the asset is low. Intuitively, borrowing to buy assets and not defaulting cannot be profitable, since lenders would not be willing to lend at an interest rate below the expected return they could earn from buying the assets themselves. As long as $\phi > 0$, the only way we can get agents to borrow to buy assets is if they default when the returns to the asset are low. The expected payoff from this strategy per unit spent to buy assets is given by

$$(1-\pi)\left[\frac{p_{t+1}^D + D}{p_t^D} - (1-\lambda)\left(1 + R_t^D(\lambda)\right)\right]$$

Note that for each unit of resources these agents spend on assets, a fraction λ of the resources they spend comes from their own wealth. If they had lent out these resources instead of leveraging them to buy assets, they would heave earned $(1+\overline{R}_t^D)\lambda$. In equilibrium, if agents borrow to buy assets in market λ, these two returns must be equal. If lending offered a higher return than borrowing and buying assets, nobody would borrow to buy assets in market λ, contradicting the fact that $f_t^a(\lambda) > 0$. Conversely, if borrowing in market λ to buy assets offered a higher return, no agent would be willing to lend given in any market, including market λ, again contradicting the fact that $f_t^a(\lambda) > 0$. Taking the limit as $\phi \to 0$ and equating the two payoffs yields an expression for the interest rate on loans $R_t^D(\lambda)$ in any active market λ in which $f_t^a(\lambda) > 0$:

$$1 + R_t^D(\lambda) = \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] \tag{22}$$

Thus, we have expressions for the interest rate $R_t^D(\lambda)$ if $f_t^a(\lambda) = 0$ and if $f_t^a(\lambda) > 0$. The next lemma, derived in Appendix C, shows there exists a cutoff Λ_t^D such that $R_t(\lambda)$ is given by (22) in markets $\lambda < \Lambda_t^D$ but is equal to $\overline{R}_t^D$ in markets $\lambda \geq \Lambda_t^D$.

Lemma: *If all markets are active, then there exists a cutoff* $\Lambda_t^D \in [0,1)$ *such that*

$$1 + R_t^D(\lambda) = \begin{cases} \frac{1}{1-\lambda}\left[\frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right] & \text{if } \lambda \in [0, \Lambda_t^D) \\ 1 + \overline{R}_t^D & \text{if } \lambda \in [\Lambda_t^D, 1) \end{cases} \tag{23}$$

The lemma implies that for $\lambda \geq \Lambda_t^D$ agents do not borrow to buy assets. This is because $R_t^D(\lambda) = \overline{R}_t^D$ implies there is no risk of default for these values of λ, and above we argued agents will only borrow to buy assets if they can default. Intuitively, agents will not speculate when they have enough skin in the game. Markets can thus be segmented into those with $\lambda < \Lambda_t^D$ where agents borrow to speculate on assets and those with $\lambda \geq \Lambda_t^D$ in which all borrowing finances production.

Figure 3 graphically illustrates the schedule of interest rates from (23). In market $\lambda = 0$, where agents are infinitely levered, the interest rate $R_t^D(0)$ is equal to the maximal return on the asset, $\frac{p_{t+1}^D + D}{p_t^D}$. This is the same as we saw in Section 2, where $\lambda = 0$ was the only possible market. The logic is the same: When agents put no resources down, they must hand over all of the returns from the asset to the lender to ensure they earn no profits. For $0 < \lambda \leq \Lambda_t^D$, the interest rate on loans $R_t(\lambda)$ decreases with λ. We prove this formally in Appendix C, but intuitively, when the borrower pledges more of her own resources, the lender need not charge as high of an interest rate to cover shortfalls in case of default. For $\lambda \geq \Lambda_t^D$ once default is no longer a concern, the interest rate $R_t(\lambda)$ is constant and equal to $\overline{R}_t^D$ for all λ.

Given the schedule of interest rates in (23), what would entrepreneurs choose? Their productivity y^* exceeds the maximum return on the asset. Above, we argued that was equal to $R_t^D(0)$. We also know that $R_t^D(0)$ exceeds $\overline{R}_t^D$, the expected return to lending. Entrepreneurs can therefore earn a higher return investing in production than from buying assets or from lending. So they should use their endowment w to produce. But they must also choose whether to borrow in some market $\lambda \in [0, 1]$ to scale up their production, where we include the option $\lambda = 1$ to allow them not to borrow at all.

Consider first an entrepreneur with wealth $w > \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of $\overline{R}_t^D$, the lowest available interest rate on loans. If she borrowed in some market $\lambda < w$, she would be able to borrow more than $1 - w$. But she has no use for this extra borrowing given her capacity. Moreover, the interest rate in this market would be the same or higher than $\overline{R}_t^D$. So there is no benefit to going to markets $\lambda < w$ over going to market $\lambda = w$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$, and she would face the same interest rate $\overline{R}_t^D$. This too offers no benefit over going to market $\lambda = w$. The best this entrepreneur can do is go to market $\lambda = w$ to borrow $1 - w$, although she can achieve the same payoff going to any market $\lambda \in \left[\Lambda_t^D, w\right]$.

Next, consider an entrepreneur with wealth $w \leq \Lambda_t^D$. If she borrowed in market $\lambda = w$, she could borrow up to $1 - w$ at an interest rate of $R_t^D(w)$. If she borrowed in some market $\lambda < w$, she would be able to borrow more than $1 - w$, but she has no use for this extra borrowing. Moreover, the interest rate in this market would be higher than $R_t^D(w)$. If she borrowed in some market $\lambda > w$, she would have to borrow less than $1 - w$. But she would face a lower interest rate. The question is whether it is worth reducing capacity to obtain a lower rate. Her payoff from borrowing in market $\lambda \in [w, \Lambda_t^D]$ would be $\frac{w}{\lambda}\left[1 + y^* - (1 - \lambda)(1 + R_t(\lambda))\right]$. Substituting in from (23), this is equal to

$$\frac{w}{\lambda}\left[1 + y^* - \frac{p_{t+1}^D + D}{p_t^D} + \frac{\lambda\left(1 + \overline{R}_t^D\right)}{1 - \pi}\right]$$

This payoff is decreasing in λ, so there is no advantage to borrowing in these markets instead of $\lambda = w$. Borrowing in any market $\lambda \in (\Lambda_t^D, 1)$ is dominated by borrowing in market $\lambda = \Lambda_t^D$, which we already argued was worse than borrowing in $\lambda = w$. So borrowing $1 - w$ in market $\lambda = w$ is uniquely optimal.

In any equilibrium where all markets are active, then, entrepreneurs with wealth $w \in [0, \Lambda_t^D)$ will choose market $\lambda = w$ while those with wealth $w \geq \Lambda_t^D$ will borrow in some market $\lambda \geq \Lambda_t^D$. This implies $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda_t^D)$ while $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$ is indeterminate. But this indeterminacy is thus irrelevant for allocations or welfare, since in any equilibrium where all markets are active agents with wealth $w \geq \Lambda_t^D$ will borrow $1 - w$ at an interest rate of $1 + \overline{R}_t^D$. Just as before, we can ensure all markets are active by assuming entrepreneurs with wealth $w \geq \Lambda_t^D$ borrow in market $\lambda = w$. In that case, $f_t(\lambda) = 2\varphi e$ for all $\lambda \in [0, 1)$ and not just for $\lambda \in [0, \Lambda_t^D)$.

Once again, any resources the young do not use to produce will be spent on the asset. This implies

$$p_t^D + 2\varphi e = (1 + \varphi) e \tag{24}$$

It follows that $p_t^D = (1 - \varphi) e$ for all t. This is the same price as when $d_t = d$. Although the price is the same, the expected return to buying the asset when $d_t = D$ is higher at $1 + \overline{r}^D = \frac{(1-\pi)D + \pi d}{(1-\varphi)e}$.

We now turn to the amount agents borrow to buy assets, $f_t^a(\lambda)$. Recall that the expected return to lending $\overline{R}_t^D(\lambda) = \overline{R}_t^D$ for all λ. Let $\alpha_t(\lambda) \equiv f_t^a(\lambda) / f_t(\lambda)$ denote the fraction of lending in market λ that is used to buy assets, which is well defined given all markets are active. Equating $\overline{R}_t^D(\lambda)$ with $\overline{R}_t^D$ implies

$$(1 - \pi\alpha_t(\lambda)) R_t^D(\lambda) + \pi\alpha_t(\lambda) \left[\frac{d}{(1 - \varphi) e} - \Phi \right] = \overline{R}_t^D \tag{25}$$

For markets $\lambda < \Lambda_t^D$, the fact that $R_t^D(\lambda) > \overline{R}_t^D$ implies $\alpha_t(\lambda) > 0$. Thus, agents must speculate in these markets. Using the value of $R_t(\lambda)$ in (23), we can solve for $\alpha_t(\lambda)$ and then for $f_t^a(\lambda)$ using the fact that $f_t^p(\lambda) = 2\varphi e$. Since the interest rate on loans $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0, \Lambda_t^D)$, then $\alpha_t(\lambda)$ and $f_t^a(\lambda)$ must be decreasing in λ for $\lambda < \Lambda_t^D$. That is, there will be more borrowing for speculation in markets with more leverage. This is not because leverage makes speculation more attractive, but because there has to be just enough speculation in equilibrium to ensure the return to lending is equal to $\overline{R}_t^D$ in all markets. Speculators borrow more to speculate in markets where lending to entrepreneurs is more profitable.

Next, we turn to markets $\lambda \geq \Lambda_t^D$. Since $R_t(\lambda) = \overline{R}_t$ for these λ, we know there is no default in these markets. Given agents only borrow to buy assets if they can default, we have $f_t^a(\lambda) = 0$ for $\lambda \in [\Lambda_t^D, 1)$. Thus, $f_t^a(\lambda)$ is uniquely determined for all $\lambda \in [0, 1)$ in any equilibrium in which all markets are active. We can now also say something about who engages in speculation. When we previously assumed entrepreneurs had no wealth, we argued both entrepreneurs and savers could speculate. This remains true here for market $\lambda = 0$. But in markets $\lambda > 0$, borrowers must invest their own wealth to speculate. Since entrepreneurs invest all of their resources in production, only savers will borrow to buy assets in markets $\lambda > 0$. In equilibrium, some savers lend and some borrow in markets $\lambda \in (0, \Lambda_t^D)$ in order to speculate.

So far, we have solved for the equilibrium price p_t^D, the amounts $f_t^a(\lambda)$ and $f_t^p(\lambda)$ agents borrow to buy assets and produce, and the interest rates on loans $R_t^D(\lambda)$ for all λ. However, some of these objects are defined in terms of the expected return to lending $\overline{R}_t^D$, which we have yet to derive. To solve for $\overline{R}_t^D$, we can track what savers earn in equilibrium. First, they lend to entrepreneurs, from which they earn

$$\int_0^1 \left(1 + R_t^D(w)\right)(1-w)(2\varphi e)\,dw$$

Second, all of the funds used to purchase assets comes from savers, either as lenders or as speculators who partly finance their asset purchases. This means they must also collect all the returns on these assets, which equal $\left(1+\overline{r}_t^D\right)p_t^D$. However, we need to net out expected default costs. Let γ_t^D denote the fraction of spending on assets that is financed with some debt. The agents who buy these assets will default if returns are low. Since default is proportional to the size of the borrower's project, expected default costs are $\gamma_t^D \Phi p_t^D$. Substituting for the price $p_t^D = (1-\varphi)\,e$ and equating what savers earn with $(1+\overline{R}_t^D)e$ yields

$$(1+\overline{R}_t^D)e = \left[1+\overline{r}^D - \pi\gamma_t^D\Phi\right](1-\varphi)\,e + \int_0^1 \left(1+R_t^D(w)\right)(1-w)(2\varphi e)\,dw \tag{26}$$

We need an additional equation to characterize γ_t^D. When the expected return to lending $\overline{R}_t^D$ exceeds the expected return to buying the asset $\overline{r}^D$, only agents who borrow will buy the asset. In that case, $\gamma_t^D = 1$. When $\overline{R}_t^D = \overline{r}^D$, then γ_t^D would have to ensure that $\overline{R}_t^D$ is indeed equal to $\overline{r}^D$. We can combine the two conditions into a single equation:

$$1+\overline{R}_t^D = \max\left\{1+\overline{r}^D, \left[1+\overline{r}^D - \pi\Phi\right](1-\varphi) + \int_0^1 \left(1+R_t^D(w)\right)(1-w)(2\varphi)\,dw\right\} \tag{27}$$

It is easy to verify that when $\overline{R}_t^D > \overline{r}^D$, equations (26) and (27) imply $\gamma_t^D = 1$, and when $\overline{R}_t^D = \overline{r}^D$ we can find a unique value of γ_t^D that will equate the two. Since $\overline{r}^D$ is time invariant, the solutions to these equations, $\overline{R}^D$ and γ^D, are also time invariant. Given $\overline{R}^D$, we can solve for the time invariant cutoff Λ^D as the smallest value of λ for which $R^D(\lambda) = \overline{R}^D$. This completes the characterization of an equilibrium when all markets are active, which we can summarize as follows.

Proposition 7 *There exists an equilibrium in which all markets are active while $d_t = D$. In any such equilibrium, the asset price is given by*

$$p_t^D = (1-\varphi)e \equiv p^D$$

and, in the limit as $\phi \to 0$, the interest rates on loans in different markets are given by

$$1 + R_t^D(\lambda) = \max\left\{1+\bar{R}^D, \tfrac{1}{1-\lambda}\left[1+\tfrac{D}{p^D} - \tfrac{\lambda(1+\bar{R}^D)}{1-\pi}\right]\right\}$$

where $\bar{R}^D$ is the value that solves (26) and (27) together with γ^D. Borrowing for production is given by $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0, \Lambda^D)$ and for $\lambda \in [\Lambda^D, 1)$ is any distribution for which $\int_{\Lambda^D}^1 f_t^p(\lambda)d\lambda = (1-(\Lambda^D)^2)\varphi e$. Borrowing for buying assets $f_t^a(\lambda)$ ensures $\bar{R}^D(\lambda) = \bar{R}^D$ for $\lambda \in [0, \Lambda^D)$ and $f_t^a(\lambda) = 0$ for $\lambda \in [\Lambda^D, 1)$.

The equilibrium above is qualitatively similar to the one in Proposition 2 when entrepreneurs had no endowment. Poor entrepreneurs still take on high leverage, but now this is by choice. In equilibrium agents borrow to buy assets in markets with high leverage, just as when all entrepreneurs lacked resources. A new feature is that rich entrepreneurs can avoid being lumped in with speculators by restricting their leverage. As before, the high dividend regime gives rise to credit booms and, if Φ isn't too large, bubbles. One difference from the previous case worth noting is that since now all entrepreneurs produce at capacity, there is no sense in which resources spent on assets could have been better deployed in production. In other words, there is no misallocation during the boom. However, if $\Phi > 0$, borrowing to buy assets remains socially wasteful, and an intervention might still raise welfare by curbing excessive leverage.

So far, we have only considered equilibria where all markets are active. But we can always construct an equilibrium in which, for any λ, the interest rate on loans $R_t(\lambda)$ exceeds y^* to ensure no agent would want to borrow in that market, and then set the expected return $\overline{R}_t(\lambda)$ arbitrarily low to ensure no one would want to lend in market λ. Such equilibria are essentially coordination failures where markets that could sustain trade are instead inactive. Inactivity in some markets can affect prices and interest rates in remaining active markets, and so characterizing equilibria with inactive markets requires us to solve again for interest rates, asset prices, and amounts borrowed. In what follows, we consider interventions that shut down markets with low λ. This is equivalent to equilibria in which these markets are inactive because of what agents believe, since the reason markets are inactive is irrelevant for how inactivity affects remaining markets. Given our interest is in the effect of policy interventions that shut down markets that would have otherwise been open, it seems natural to focus on an equilibrium where all markets are active as the benchmark and limit our analysis of inactivity to markets with low λ.

5.2 Leverage Restrictions

Consider an intervention that shuts down all markets λ below some floor $\underline{\lambda}$. This is equivalent to a cap on leverage. Agents with wealth $w < \underline{\lambda}$ could only undertake projects of size $w/\underline{\lambda} < 1$, although entrepreneurs with wealth $w \geq \underline{\lambda}$ could still operate at full capacity. For simplicity we consider a permanent floor, although one could equally consider a floor that is only in effect while $d_t = D$.

We consider equilibria in which all markets $\lambda \geq \underline{\lambda}$ are active to focus on the effect of restricting leverage. The equilibrium where all markets are active can then be viewed as a special case in which $\underline{\lambda} = 0$. When $\underline{\lambda} > 0$, the same logic as above implies that interest rates $R_t(\lambda)$ will be given by (23) when $d_t = D$, although the expected return to lending $\overline{R}_t^D$ and Λ_t^D will vary with $\underline{\lambda}$. Given interest rates still satisfy (23), entrepreneurs will want to produce at full capacity. But entrepreneurs with $w < \underline{\lambda}$ can no longer do so. Since their profits decrease with λ for $\lambda > w$, these entrepreneurs will all flock to $\underline{\lambda}$ and produce at scale $w/\underline{\lambda}$. The total inputs entrepreneurs will use to produce is then

$$\begin{aligned}\int_{w=0}^{\underline{\lambda}} 2\varphi e\left(\frac{w}{\underline{\lambda}}\right) dw + \int_{w=\underline{\lambda}}^{1} (2\varphi e)\, dw &= \frac{\varphi e}{\underline{\lambda}} w^2 \Big/_{0}^{\underline{\lambda}} + 2\varphi e\,(1-\underline{\lambda}) \\ &= \underline{\lambda}\varphi e + (1-\underline{\lambda})\, 2\varphi e\end{aligned}$$

The amount that remains to spend on the asset is $(1+\varphi)\,e$ minus the above, which pins down its price:

$$p_t^D = (1-\varphi\,(1-\underline{\lambda}))\,e \tag{28}$$

Increasing $\underline{\lambda}$ will lead to a higher asset price. Intuitively, leverage restrictions force poor entrepreneurs to operate at a smaller scale. Since savers want to save a fixed amount e regardless of $\underline{\lambda}$, the decline in production will release resources to buy assets, pushing p_t^D up. When $\underline{\lambda}$ is imposed permanently, the same logic implies $p_t^d = (1-\varphi\,(1-\underline{\lambda}))\,e$. The expected return on the asset when $d_t = D$ is thus

$$1+\overline{r}_t^D = \frac{(1-\pi)\left(D+p_{t+1}^D\right)+\pi\left(d+p_{t+1}^d\right)}{p_t^D} = 1+\frac{(1-\pi)\,D+\pi d}{(1-\varphi\,(1-\underline{\lambda}))\,e}$$

Increasing $\underline{\lambda}$ thus reduces the expected return to buying the asset. While it is hard to describe the effects of increasing $\underline{\lambda}$ on the entire schedule of interest rates $R_t(\lambda)$, we show in Appendix C that the expected return to lending $\overline{R}^D$ declines with $\underline{\lambda}$. We summarize the effects of raising $\underline{\lambda}$ as follows.

Proposition 8 *The asset price $p_t^D = (1-\varphi(1-\underline{\lambda}))e$ is increasing in $\underline{\lambda}$, while the expected returns on the asset $\bar{r}^D = \frac{(1-\pi)D+\pi d}{(1-\varphi(1-\underline{\lambda}))e}$ and from lending $\bar{R}^D$ when $d_t = D$ are decreasing in $\underline{\lambda}$ for a permanent floor $\underline{\lambda}$.*

Note the contrast between the effects of contractionary monetary policy we discussed in the previous section and the effects of restrictions on leverage above. Both policies reduce output: Monetary policy reduces what is produced today, while leverage restrictions reduce the amount entrepreneurs today produce for next period. However, tighter monetary policy dampens asset prices and raises the returns to saving while leverage restrictions increase asset prices and lower the return to saving. This suggests leverage restrictions may be counterproductive, stoking asset prices rather than dampening them. This counterproductive aspect of leverage restrictions is new as far as we know. Stein (2013) argues leverage restrictions may be limited and ineffective, but his point was that borrowers can often circumvent them, not that regulations might contribute to more speculation. The logic for our result is that risk-shifting models require an additional investment activity to cross-subsidize speculation. If this other investment is particularly sensitive to leverage restrictions, restricting leverage may redirect resources toward speculation. We anticipate that the same would hold true in risk-shifting models of housing in which speculators buy the same asset that liquidity constrained households buy. If the demand for housing by constrained households is particularly sensitive to leverage restrictions but the amount of funds available for mortgage lending is relatively inelastic with respect to interest rates, leverage restrictions could end up encouraging speculation on housing.

Proposition 8 establishes that tighter leverage restrictions drive up asset prices. But that does not necessarily mean that total borrowing against assets must rise. Even if a higher $\underline{\lambda}$ increases p^D, it could still lower the share of assets purchased with debt γ^D. Our next result shows that when $\underline{\lambda}$ is low and all assets are purchased with debt, or when $\underline{\lambda}$ is high enough to discourage speculation altogether, increasing $\underline{\lambda}$ will increase p^D without changing γ^D. In these cases, raising $\underline{\lambda}$ will make agents worse off. For intermediate values, increasing $\underline{\lambda}$ at some point must reduce both the share of assets purchased with debt and expected default costs. On its own this would make society better off, but increasing $\underline{\lambda}$ also reduces what poor entrepreneurs can produce. For large Φ, however, reducing default costs would be paramount.

Proposition 9 *There exist cutoffs $\Lambda_0 \leq \Lambda_1 < 1$ in $[0, 1)$ such that*

1. *If $\underline{\lambda} < \Lambda_0$, increasing $\underline{\lambda}$ leaves $\gamma^D = 1$, increases expected default costs $\pi\gamma^D\Phi p^D$, and leaves fewer goods for cohorts to consume from date $t = 1$ on.*

2. *If $\underline{\lambda} \geq \Lambda_1$, $\gamma^D = 0$ and there is no default. Increasing $\underline{\lambda}$ then leaves fewer goods for cohorts to consume from date $t = 1$ on.*

3. *If $\Lambda_0 < \underline{\lambda} < \Lambda_1$, increasing $\underline{\lambda}$ there exist values of $\underline{\lambda}$ at which increasing $\underline{\lambda}$ lowers γ^D and expected default costs $\pi\gamma^D\Phi p^D$. In this case, increasing $\underline{\lambda}$ while $d_t = D$ can be Pareto improving for large Φ.*

In contrast to monetary policy, a threat to tighten credit conditions in the future rather than tighten them immediately will not mitigate this counterproductive aspect of our model. Raising $\underline{\lambda}$ next period will increase p^D_{t+1}, and regardless of how it affects γ^D_{t+1}, a higher p^D_{t+1} at date $t+1$ makes speculation at date t more attractive. However, our finding that increasing $\underline{\lambda}$ raises asset prices contemporaneously will not hold more generally. Suppose that instead of assuming entrepreneurs are identical in either wealth or productivity, we allow both wealth and productivity of entrepreneurs to follow some general distribution $n(w, y)$. Entrepreneurs with positive wealth and low productivity would behave like savers. An increase in $\underline{\lambda}$ that lowers the return to saving could induce some of the entrepreneurs who are on the margin between lending and producing to switch from lending to borrowing in order to produce. If enough entrepreneurs switch from lending to producing, the fall in lending and the increase in demand for borrowing to produce could leave fewer resources to spend on the asset, and its price will fall. We confirm numerically that there exist distributions $n(w, y)$ for which increasing $\underline{\lambda}$ reduces p^D_t.[8]

While increasing $\underline{\lambda}$ can lower asset prices when $d_t = D$ under some circumstances, this will only be true when there is risk-shifting. When $d_t = d$, increasing $\underline{\lambda}$ will raise p^d_t regardless of the distribution $n(w, y)$. To see this, note that when $d_t = d$ there is no default. $R^d_t(\lambda)$ is then equal to a constant R^d_t for all λ. This common rate R^d_t and the asset price p^d_t satisfy two equilibrium conditions similar to (3) and (4). First, since all the resources of savers and entrepreneurs will be used to produce or buy the asset, we have

$$\int_{R^d}^{\infty}\int_0^1 \min\left\{1, \frac{w}{\underline{\lambda}}\right\} n(w, y)\; dw\, dy + p^d_t = \int_0^{\infty}\int_0^1 wn(w, y)\; dw\, dy + e \tag{29}$$

This defines R^d_t as a function $\rho_{\underline{\lambda}}\left(p^d_t\right)$ of the price p^d_t which is increasing in p^d_t for a fixed $\underline{\lambda}$ and decreasing in $\underline{\lambda}$ for a fixed p^d_t. Second, the interest rate on loans must equal the return on the asset, and so

$$\left(1 + R^d_t\right) p^d_t = d + p^d_{t+1} \tag{30}$$

Substituting in $R^d_t = \rho_{\underline{\lambda}}\left(p^d_t\right)$ implies $p^d_{t+1} = \rho_{\underline{\lambda}}\left(p^d_t\right) - d$. Figure 4 illustrates the effect of increasing $\underline{\lambda}$ graphically. Since $\rho_{\underline{\lambda}}\left(p^d_t\right)$ is decreasing in $\underline{\lambda}$ for a fixed p_t, the curve that plots p^d_{t+1} as a function of p^d_t is

[8] Even without relying on a more general distribution $n(w, y)$, our results are in part due to our assumption that savers only like to consume when old. This means their saving is inelastic to the interest rate. If we modified this, tighter leverage constraints that reduce the returns to savings could lead them to save less, which might lead to lower asset prices.

lower for all $p_t^d > 0$, which implies a higher steady state p^d. Intuitively, increasing $\underline{\lambda}$ requires the interest rate on loans to fall so that credit markets clear even after demand for borrowing by poor entrepreneurs falls. In the absence of risk-shifting, the interest rate on loans is equal to the return on the asset, so the latter must fall. But the return on the asset is negatively related to its price. Hence, increasing $\underline{\lambda}$ must increase the price of the asset. By contrast, with risk-shifting, the interest rate on loans and the return on the asset generally differ, so it will be possible for interest rates on loans to fall but the return on the asset to rise. The fact that tighter leverage restrictions only reduce asset prices when there is risk-shifting suggests a possible way for policymakers to indirectly detect risk-shifting.

6 Conclusion

This paper examines the role of policy in a risk shifting model of asset pricing. As in previous work on risk-shifting, we show that our model can capture many observable features of asset booms, including high asset prices that may exceed fundamentals, accompanying credit booms, and an eventual crisis. The general equilibrium framework we use allows us to go beyond this and analyze policy and welfare. We show that risk-shifting leads to misallocation and excessive leverage, opening up a possible role for intervention. We then look at whether the leading policy proposals of contractionary monetary policy and leverage restrictions can help mitigate these distortions. In our model, tighter monetary policy increases interest rates and lowers asset prices, which reduces excessive leverage but further inhibits investment that is already underfunded. Leverage restrictions have the opposite effect, lowering interest rates and, at least under certain conditions, increasing asset prices. But they also discourage borrowing against assets. Both policies turn out to have ambiguous welfare implications. Whether a policy improves welfare depends on how it affects speculators vis-a-vis the productive activity that cross-subsidize them. It will also depend on how it is implemented; credibly promising to tighten if a boom persists may improve welfare even when tightening immediately does not. Although for analytical tractability we analyze these two policies separately, the model suggests the most effective policy is the one that disproportionately discourages speculation, since this is what policymakers would like to do. Which policy is more effective depends not only on how policy is implemented, but how production and speculation respond to the policy. Finally, we find that when default costs are large, risk shifting can occur without giving rise to bubbles, something previous work has overlooked. This implies that policymakers contemplating intervening against asset booms might not need to determine if asset prices exceed fundamentals to justify their intervention if given evidence of risk-shifting.

We focus on risk shifting because asset booms tend to be associated with opaque assets that make it difficult for lenders to judge the risks they face with any given borrower. There is a large literature on bubbles that seeks to explain asset booms without risk shifting. These models should not be viewed as competing explanations, since the mechanisms they consider are complementary to the risk-shifting we study. For example, there is a large literature which shows how bubbles can arise with fully rational agents because of dynamic inefficiency as in Galí (2014, 2017) or binding credit market frictions as in Martin and Ventura (2012), Hirano and Yanagawa (2017), and Miao and Wang (forthcoming). Recent work by Bengui and Phan (2018) showed it is possible combine risk-shifting and dynamic inefficiency. One can similarly combine risk-shifting and borrowing constraints by replacing our assumption that entrepreneurs have a finite capacity with the assumption that an entrepreneur's scale is bounded by how much they can borrow. In this case, the distortions from risk-shifting that we emphasize would have to be balanced against the fact that overvalued assets may help entrepreneurs relax their borrowing constraints. There is also a literature that shows how disagreement can give rise to bubbles, e.g. Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), Simsek (2013), and Barberis, Greenwood, Jin, and Shleifer (2018). This too can be combined with risk-shifting. Recall that in the last section of the paper, we had savers lending to otherwise identical savers who then speculate on assets. Differences in beliefs can help pin down who lends to whom, since savers who are pessimistic about assets will presumably lend to savers who are optimistic about them. Whether risk-shifting interacts with disagreement in interesting ways remains an open question.

Our model also suggests directions for future research on risk-shifting models of asset prices. For example, we assumed lenders suffer a cost Φ when their borrowers default. In practice, the main costs associated with the collapse of asset prices involve a decline in output due to the way agents respond when asset prices fall. To get at these channels would require introducing financial intermediaries or borrowing constraints for individual households. These may have important implications for what type of interventions are best during booms, since how interventions affect outcomes once asset prices collapse will likely matter for welfare. In terms of applications, we have described the analog between our setup and the housing market. However, cross-subsidization in the housing market works differently, since there both types of agents buy the same asset. By contrast, in our model only speculators buy an asset. This raises the question of whether an intervention that shifts resources from illiquid home buyers to speculators still drives house prices up as in our setting. It is also not obvious whether the policy implications we deduce in our model would hold in open economy settings. For example, we argued that a contractionary monetary policy raises interest rates and dampens asset prices. But if contractionary monetary policy leads to higher real rates that attract capital inflows, it is not clear whether asset prices will still fall. Extending our framework to deal with these issues is essential for figuring out its relevance and limitations for real world scenarios.

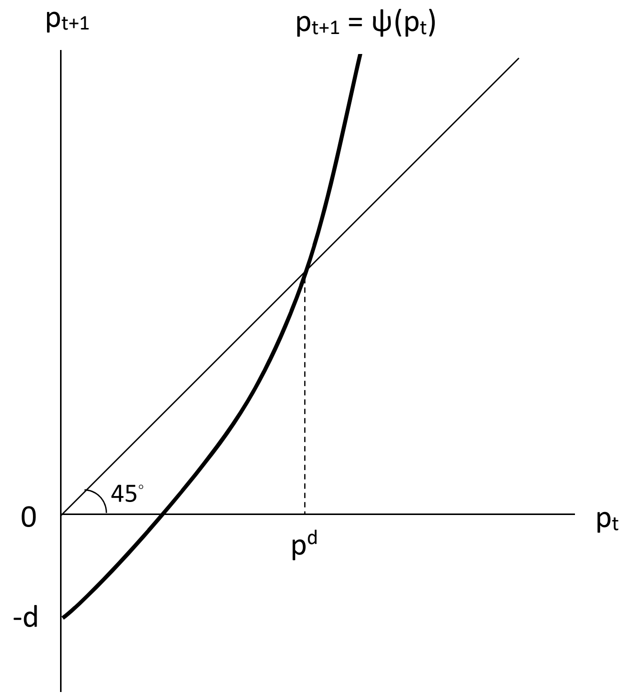

Figure 1: Determination of equilibrium price p^d with deterministic dividends

The value p^d denotes the steady state for the dynamical system $p_{t+1} = \psi(p_t)$. Any path which begins away from p^d leads either to a negative price or a price above e, neither of which can occur in equilibrium. Hence, the unique equilibrium is for the price to equal the steady state value p^d at all dates.

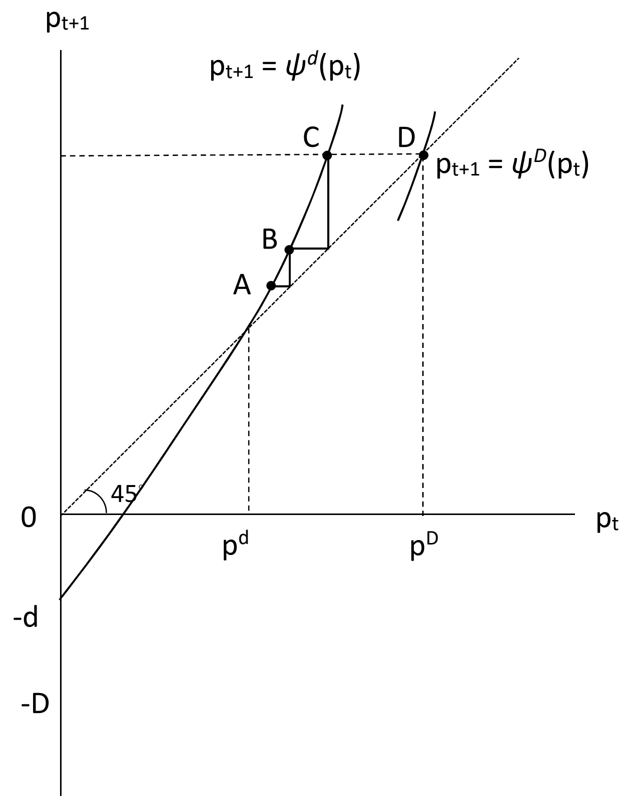

Figure 2: Equilibrium prices p_t^D with delayed dividend increase in high regime

The figure depicts the case where, during the high regime, dividends started out equal to d until some date T, at which point dividends would rise to D and remain there until the regime switched to equaling d forever. Before date T, prices obey the law of motion $p_{t+1}^D = \psi^d(p_t^D)$ and so follow an explosive path that reaches p^D at date T. From date T on, the path of prices follows the law of motion $p_{t+1}^D = \psi^D(p_t^D)$, and at this point the price remains constant at p^D. The path of prices up to date T would include the points A,B,C, and D.

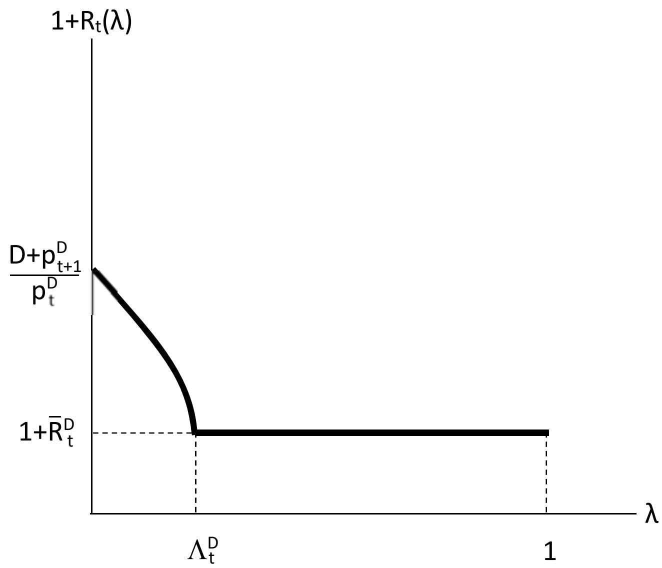

Figure 3: Interest rates as a function of share λ of investment that borrowers pay

The figure depicts the equilibrium schedule of interest rates across different markets. Interest rates are declining in the share λ of their projects that borrowers finance. For $\lambda < \Lambda_t^D$ the interest rate is falling in λ, and for $\lambda \geq \Lambda_t^D$ it is constant.

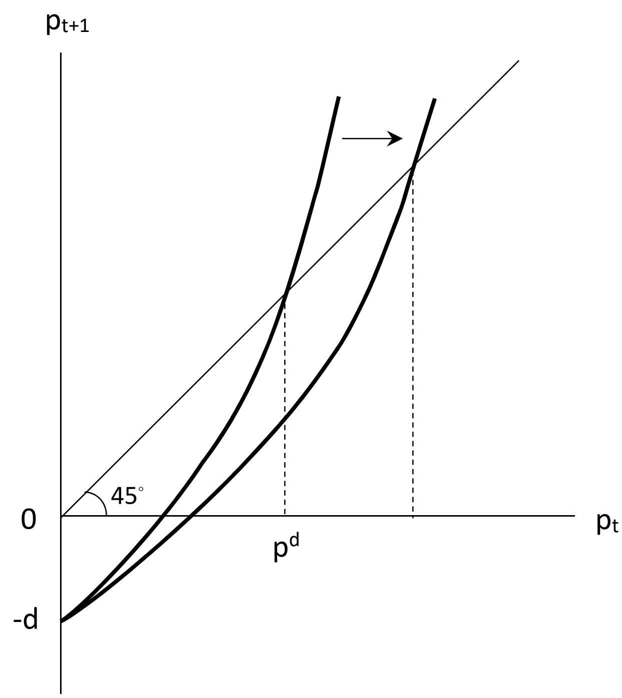

Figure 4: Effect of increasing floor $\underline{\lambda}$ with deterministic dividends

Appendix A: Proof of Proposition 1

Proof of Proposition 1: In the text, we showed there is a unique deterministic equilibrium. Here we allow for stochastic equilibrium paths for $\{p_t, R_t\}_{t=0}^{\infty}$ and confirm that the equilibrium is in fact deterministic.

First, note that for any date t, in equilibrium it must be the case that $0 < p_t \leq e$. If the price $p_t \leq 0$ there would be infinite demand for the asset given its dividend $d > 0$ and there is free disposal. But the supply of assets is finite, so this cannot be an equilibrium. At the same time, the most any cohort can spend to buy the assets is e. Let z_t denote the return to buying the asset, i.e., $z_t \equiv \frac{d+p_{t+1}}{p_t}$. This can be random if p_{t+1} is random. Let $G_t(z)$ denote the (possibly degenerate) distribution of the return z_t. Since $0 < p_t \leq e$ for all t, the maximum return $z_t^{\max}$ is finite, since $z_t^{\max} = \frac{d+p_{t+1}^{\max}}{p_t} \leq \frac{d+e}{p_t} < \infty$, where $p_{t+1}^{\max}$ is the maximum possible realization of the price at date $t+1$.

The equilibrium satisfies two conditions. First, as in (3), all resources will be used either to buy assets or to initiate production:

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e \tag{31}$$

The implies $R_t = \rho(p_t)$ where $\rho'(\cdot) > 0$. Second, the interest rate on loans R_t must satisfy

$$(1 + R_t)\, p_t = d + p_{t+1}^{\max} \tag{32}$$

If the interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, no agent would want to buy assets, which cannot be an equilibrium. If interest rate on loans $1 + R_t$ exceeded $\frac{d+p_{t+1}^{\max}}{p_t}$, agents could earn positive profits from borrowing, so demand for credit would be infinite. Substituting $R_t = \rho(p_t)$ into (32) implies

$$p_{t+1}^{\max} = (1 + \rho(p_t))\, p_t - d$$

Suppose $p_t > p^d$. Consider the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. Formally, set $\widetilde{p}_t = p_t$ and define

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t > p^d$, the sequence $\widetilde{p}_t$ would shoot off to infinity and would exceed e in finite time. This means there is a state of the world in which the price exceeds e, which cannot be an equilibrium. So $p_t \leq p^d$.

Next, suppose $p_t < p^d$. Again, we can construct the sequence $\{\widetilde{p}_\tau\}_{\tau=t}^{\infty}$ that comprises the upper support of prices at each date given the history of previous prices, starting from p_t. That is, we set $\widetilde{p}_t = p_t$ and then

$$\widetilde{p}_{\tau+1} = (1 + \rho(\widetilde{p}_\tau))\, \widetilde{p}_\tau - d$$

Since $p_t < p^d$, the sequence $\widetilde{p}_t$ would turn negative. Hence, there is a state of the world in which the price is negative, which cannot be an equilibrium. The distribution of the price at date t is degenerate with full support at p^d. From (31), $R_t = \rho(p_t)$ is uniquely determined as well. ■

Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup as in our discussion in Section 4. We set up the model and derive the results that underlie Propositions 4 and 5 in the text.

B.1 Agent Types and Endowments

Our approach largely follows Galí (2014) in how we incorporate production, nominal price rigidity, and monetary policy into an overlapping generations economy with assets. As in our benchmark model, agents live two periods and care only about consumption when old. Each cohort still consists of two types – savers who are endowed with resources but cannot produce intertemporally and entrepreneurs endowed with no resources who can convert goods at date t into goods at date $t+1$. We continue to model entrepreneurs as in the benchmark model, but we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each who must choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.[9] Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date t convert labor into goods at date t. Entrepreneurs then convert the goods producers created at date t into goods at date $t+1$. Producers operate within the period; entrepreneurs operate across periods.

B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor to home and market production. Home production yields the same good as the market, but using a technology $h(\ell)$ that is concave in the amount of labor ℓ allocated to home production. We assume $h'(0) = 1$ and $h'(1) = 0$ for reasons that will become clear below.

Workers who sell their labor on the market earn a wage W_t per unit labor. Their labor services are hired by producers, whom we index by $i \in [0,1]$. Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer i hires n_{it} units of labor, she will produce $x_{it} = n_{it}$ units of intermediate good i. The different intermediate goods can then be combined to form final

[9] This setup borrows from Adam (2003) rather than Galí (2014). The latter assumes agents are homogeneous, selling labor when young and hiring labor when old. We want income to only accrue to the young as in our benchmark model.

consumption goods according to a constant elasticity of substitution (CES) production function available to all agents. That is, given x_{it} of each $i \in [0,1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{33}$$

Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. At these prices, the x_{it} that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields

$$x_{it} = X_t \left(\frac{P_{it}}{P_t} \right)^{-\frac{1}{\sigma}} \tag{34}$$

If we set $X_t = 1$, we can compute the price of the cost of the optimal bundle of intermediate goods $x_{it} = \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma}$ needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{35}$$

Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (34) and wage W_t. To allow producers to move either before or after the monetary authority, we condition producer i's choice on their information Ω_{it} when choosing their price. Each producer will set P_{it} to solve

$$\max_{P_{it}} E \left[\left(P_{it} - W_t \right) X_t \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is then

$$P_{it} = \frac{E \left[W_t X_t | \Omega_t \right]}{(1-\sigma) E \left[X_t | \Omega_t \right]} \tag{36}$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$X_t = \left(\int_0^1 n_t^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive $(W_t/P_t)\, n_t$ of these goods and producers get the remaining $(1 - W_t/P_t)\, n_t$. Workers also produce goods at home. Their income is thus $(W_t/P_t)\, n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = W_t/P_t \tag{37}$$

By contrast, the total resources available to young agents is $e_t = n_t + h(1-n_t)$, which is maximized at

$$h'(1-n_t) = 1$$

Our assumption that $h'(0) = 1$ implies total resources are maximized when $n_t = 0$ and all goods are produced in the market, and $e_t = n_t + h(1-n_t)$ is increasing in n_t for all $n_t \in [0,1]$.

B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings $e_t = n_t + h(1 - n_t)$. As in the benchmark model, they can buy assets and make loans. Without money, this specification would be equivalent to our benchmark model, the only difference being that the income of savers e_t which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y)\, dy + p_t = e_t$$

where p_t is the real price of the asset and R_t is the real interest rate on loans. The interest rate R_t must still ensure agents cannot earn profits by borrowing and buying assets. When $d_t = d$, this requires

$$\left(1 + R_t^d\right) p_t^d = d + p_{t+1}^d$$

and when $d_t = D$, this requires

$$\left(1 + R_t^D\right) p_t^D = D + p_{t+1}^D$$

We can then use R_t and p_t to solve for the expected return on loans:

$$\overline{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max\left\{\overline{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t}\left(\overline{r}_t^D - \pi\Phi\right)\right\} & \text{if } d_t = D \end{cases} \tag{38}$$

where $\overline{r}_t^D$ is the expected real return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage W_t/P_t will be constant over time. Employment n_t and total earnings of all savers $e_t = n_t + h(1 - n_t)$ will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and P_t and W_t denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. The monetary authority announces a nominal interest rate i_t at each date t. The monetary authority commits to pay this rate at date $t + 1$ to those who lend to it (with money it can always issue), and will charge i_t to those who borrow from it with full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t + 1$. Since agents always lend to entrepreneurs, the expected return on savings will equal $\overline{R}_t$, the expected return on loans. This implies

$$1 + i_t = \left(1 + \overline{R}_t\right) \Pi_t \tag{39}$$

When the monetary authority changes the nominal interest rate i_t, either inflation Π_t or the expected return $1 + \overline{R}_t$ or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

B.4 Defining an Equilibrium

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

$$
\begin{array}{ll}
\text{(i) Optimal pricing:} & P_t = \dfrac{E\left[W_t X_t | \Omega_t\right]}{(1-\sigma)\, E\left[X_t|\Omega_t\right]} \\
\text{(ii) Optimal labor supply:} & h'\left(1-n_t\right) = W_t / P_t \\
\text{(iii) Optimal saving:} & \int_{R_t}^{\infty} n\left(y\right) dy + p_t = e_t \\
\text{(iv) Credit market clearing:} & 1 + R_t = \begin{cases} \frac{D + p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d + p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases} \\
\text{(v) Money market clearing:} & \Pi_t = \dfrac{1+i_t}{1+\overline{R}_t}
\end{array}
$$

where the expected return on loans $\overline{R}_t$ in the last condition is given by (38).

B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices P_{it} after observing the wage W_t. This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output X_t. Hence, their information set $\Omega_t = \{W_t, X_t\}$. It follows that $E\left[W_t X_t|\Omega_t\right] = W_t X_t$ and $E\left[X_t|\Omega_t\right] = X_t$. The optimal pricing rule (i) then implies

$$P_t = \frac{W_t}{1-\sigma}$$

The real wage is thus constant and equal to $1-\sigma$. Substituting this into (ii) yields

$$h'\left(1-n_t\right) = 1-\sigma \tag{40}$$

Since $h\left(\cdot\right)$ is concave, n_t is equal to some constant n^* for all t. It follows that $e_t = n^* + h\left(1-n^*\right)$ is also constant for all t. We can then use (iii) and (iv) to solve for p_t and R_t as in the benchmark model, and then use (38) to compute $\overline{R}_t$. Finally, given $\overline{R}_t$ we can use the implied Π_t from (v) to derive $\{P_t\}_{t=1}^{\infty}$ for any initial value for P_0. The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage $W_t = (1-\sigma)\, P_t$.

B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good P_{it} before the monetary authority moves. That is, producers set prices, the monetary authority sets $1+i_t$, and then producers hire workers at a nominal wage W_t. This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage W_t, and so $\Omega_t = \{W_t, X_t\}$ and $W_t/P_t = 1-\sigma$ as before.

Next, suppose monetary policy is contingent on some random variable, i.e., $i_t = i(\xi_t)$ where $\{\xi_t\}_{t=0}^{\infty}$ is a sequence of random variables. For simplicity, consider the case where ξ_t is only random at $t=0$, i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1-\chi \end{cases}$$
$$\xi_t \text{ is deterministic for } t = 1, 2, \ldots$$

From date $t=1$ on, we know from the optimal price-setting condition (i) that $W_t/P_t = 1-\sigma$. It then follows that $n_t = n^*$ and $e_t = e^* \equiv n^* + h(1-n^*)$ for all $t \geq 1$, and we can determine p_t, R_t, and $\overline{R}_t$ for $t \geq 1$ just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as for a given realization of ξ_0. Assume wlog that $i_0^H > i_0^L$. The optimal price setting condition (1) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1-\chi) n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1-\chi) n_0^L} = 1 - \sigma \tag{41}$$

That is, the output-weighted average real wage over the two values of ξ is equal to $1-\sigma$. Optimal labor supply (ii) then implies

$$\begin{aligned} h'\left(1-n_0^H\right) &= \min\left\{\frac{W_0^H}{P_0}, 1\right\} \\ h'\left(1-n_0^L\right) &= \min\left\{\frac{W_0^L}{P_0}, 1\right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we choose the real wage when $\xi = H$ to be this parameter. The three equations above yield values for W_0^L/P_0, n_0^H, and n_0^L given W_0^H/P_0. From these, we can deduce earnings $e_0^\xi = n_0^\xi + h\left(1-n_0^\xi\right)$ for each $\xi \in \{H, L\}$. We can then use (iii) and (iv) to derive p_0^ξ and R_0^ξ by solving

$$\begin{aligned} \int_{R_0^\xi}^{\infty} n(y)\,dy + p_0^\xi &= e_0^\xi & (42) \\ \left(1+R_0^\xi\right) p_0^\xi &= D + p^D & (43) \end{aligned}$$

and then compute the expected return on loans $\overline{R}_0^\xi$ using (38), and, via (v), the inflation rate Π_0^ξ for each $\xi \in \{H, L\}$. As before, the price level P_0 is indeterminate. Optimal pricing only restricts the average real wage across states but not the real wage for each realization of ξ_0, introducing an indeterminacy. The equilibrium real wage can exceed $1-\sigma$ for one realization of ξ_0 if it falls below $1-\sigma$ for the other realization.

There case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case, $W_0^H/P_0 = W_0^L/P_0 = 1-\sigma$. But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal to coordinate real activity rather but does not directly affect it, there are equilibria in which $W_0^H > W_0^L$ as well as equilibria in which $W_0^H < W_0^L$.[10] Since higher nominal interest rates seem to be contractionary in practice, we focus on equilibria in which $W_0^H/P_0 < 1-\sigma < W_0^L/P_0$, i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from condition (ii) we know that a higher nominal interest rate will be associated with lower employment ($n_0^H < n^* < n_0^L$) and hence lower earnings ($e_0^H < e^* < e_0^L$). From (42), we can infer that $R_0^\xi = \rho^\xi \left(p_0^\xi\right)$ where $\rho^H(x) > \rho^L(x)$ for the same value x. As is clear from Figure 1, this implies a higher nominal interest rate will be associated with a lower real asset price ($p_0^H < p^D < p_0^L$). This also implies a higher real interest rate on loans ($R_0^H > R^D > R_0^L$). The real expected return to buying assets will also be higher ($\overline{r}_0^H > \overline{r}^D > \overline{r}_0^L$). However, whether the real expected return to lending $\overline{R}_0^H$ will be higher is ambiguous. (38) implies $\overline{R}_0^\xi$ is either equal to $\overline{r}_0^\xi$ or to a weighted average of R_0^ξ and $\overline{r}_0^\xi$. In the latter case, although both terms are higher when $\xi = H$ the weight on $\overline{r}_0^\xi$, which is p_0^ξ/e_0^ξ, can be higher or lower for $\xi = H$. These results are summarized in Proposition 4 in the paper.

B.7 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case, ξ_0 and ξ_t for $t \geq 2$ are deterministic, while $\xi_1 = d_1 \in \{d, D\}$. That is, we assume producers set prices each period before d_t is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed ξ_0 was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1-\sigma < W_1^d/P_1.$$

This implies $n_1^D < n^* < n_1^d$ and so $e_1^D < e < e_1^d$. In other words, if dividends fall and the boom ends, monetary policy must be expansionary. By the same logic as above, such a policy would imply $p_1^D < p^D$ and $p_1^d > p^d$, as well as $R_1^D < R^D$ and $R_1^d > R^d$. Turning back to date 0, conditions (iii) and (iv) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n(y)\,dy + p_0^D &= e \\ \left(1+R_0^D\right) p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to p_1^D reveals that $p_0^D < p^D$ and $R_0^D < R^D$. That is, while contractionary monetary policy at date 0 dampens p_0^D but raises R_0^D at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both p_0^D and R_0^Dat date 0. These results are summarized in Proposition 5 in the paper.

[10] One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to take actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

Appendix C: Macroprudential Regulation

In this appendix, we define an equilibrium for an economy with multiple markets as in Section 5. We then show that for an equilibrium in which all markets are active, various aspects of the equilibrium are uniquely determined. We then discuss some comparative static results with respect to the set of active markets.

C.1 Defining an Equilibrium

We begin with some notation. Let p_t denote the price of the asset at date t. Given asset prices, we can define the return to buying the asset at date t as

$$z_t \equiv \frac{d_{t+1}+p_{t+1}}{p_t}$$

The return z_t can be random both because d_{t+1} might be uncertain (if $d_t = D$) and because p_{t+1} might in principle be stochastic. Let $G_t(z)$ denote the (possibly degenerate) cumulative distribution of the return z_t, i.e., $G(z) \equiv \Pr(z_t \leq z)$. Let $1 + r_t^{\max}$ denote the maximum possible return on the asset. As discussed in the text, $1 + r_t^{\max}$ is finite, since $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e}$. We will use $\overline{r}_t$ to denote the expected return to buying the asset at date t, i.e.,

$$1 + \overline{r}_t \equiv \int_0^{1+r_t^{\max}} z_t dG_t(z)$$

We now define variables for the different markets $\lambda \in [0, 1)$ agents can borrow in. Let $R_t(\lambda)$ denote the interest rate on loans in market λ, so an agent who agrees to pay a share λ of the project she undertakes will promise to pay back $1 + R_t(\lambda)$ for each unit she borrows. Since agents may default, let $\overline{R}_t(\lambda)$ denote what lenders expect to earn from lending in market λ given the possibility of default. Finally, we represent borrowing in markets with density functions $f_t^a(\lambda)$ and $f_t^p(\lambda)$ for all $\lambda \in [0, 1)$ such that the total amount of resources borrowed to buy assets and produce are given by $\int_A f_t^a(\lambda)\, d\lambda$ and $\int_A f_t^p(\lambda)\, d\lambda$, respectively. Let $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$ denote the density of borrowing for any purpose in market λ.

Representing the quantities agents borrow in each market as a density function ignores the possibility that there may be equilibria in which agents borrow a positive mass of resources in certain markets. More generally, we can allow for a set $\Delta \subset [0, 1)$ with countably many elements such that each market $\lambda \in \Delta$ is associated with a positive mass of borrowing $m_t^x(\lambda) > 0$. The amount borrowed in any market $\lambda \in [0, 1)\backslash\Delta$ can still be represented with a density function. Heuristically, we can appeal to the Dirac-delta construction and represent the amount borrowed in any market as if it were a density. That is, for any $\lambda \in \Delta$, we set the density $f_t^x(\lambda) = m_t^x(\lambda)\, \delta_\lambda(\lambda)$, where $\delta_\lambda(q)$ is the Dirac-delta function defined so that $\delta_\lambda(q) = 0$ for $q \neq 0$ and $\int_0^1 \delta_\lambda(q)\, dq = 1$. This convention treats markets $\lambda \in \Delta$ as essentially having an infinite density. We will refer to a market λ as *inactive* if $f_t(\lambda) = 0$ and *active* if $f_t(\lambda) > 0$ or if $\lambda \in \Delta$.

Given these preliminaries, we define an equilibrium as a path $\left\{p_t, f_t^p(\lambda), f_t^a(\lambda), R_t(\lambda), \overline{R}_t(\lambda)\right\}_{t=0}^{\infty}$ that satisfies conditions (44)-(49) below to ensure that all markets clear when agents are optimizing.

Our first three conditions stipulate that agents act optimally. We begin with lenders. Optimality requires that agents will only invest their wealth where the expected return is highest. Let $\overline{R}_t$ denote the maximal expected return to lending in any market λ, i.e.,

$$\overline{R}_t \equiv \sup_{\lambda \in [0,1)} \overline{R}_t(\lambda)$$

Optimal lending requires that agents lend in market λ' only if it they expect to earn $\overline{R}_t$ and if this rate exceeds the expected return to buying the asset, i.e.,

$$f_t(\lambda') > 0 \text{ only if } \overline{R}_t(\lambda') = \overline{R}_t \text{ and } \overline{R}_t \geq \overline{r}_t \tag{44}$$

Next, entrepreneurs must act optimally. We first argue this means they should use their endowment to produce. Recall entrepreneurs have productivity y^* where $y^* > r_t^{\max} \geq \overline{r}_t$ from (20), so producing is better than buying assets. But y^* must also exceed the expected return to lending $\overline{R}_t$. For suppose $\overline{R}_t$ were higher than y^*. Since $y^* > r_t^{\max}$, then $\overline{R}_t$ must also exceed $r_t^{\max}$. In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least $\overline{R}_t$. Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since production offers the highest return, entrepreneurs should use their endowment w to produce.

Since entrepreneurs can leverage their endowment to produce at a larger capacity, we also need to characterize their borrowing. If they borrow in market λ where $\lambda < w$, they can borrow enough to reach full capacity. Optimality requires that there will be borrowing to produce in market λ' only if some entrepreneur finds it optimal to borrow in that market from all $\lambda \in [0,1]$, including $\lambda = 1$ for no borrowing. This implies

$$f_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg\max_{\lambda \in [0,1]} \left\{ \begin{array}{ll} [1 + y - (1-w)(1 + R_t(\lambda))] & \text{if } \lambda \leq w \\ \dfrac{w}{\lambda}[1 + y - (1-\lambda)(1 + R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } w \tag{45}$$

Third, agents who borrow to buy assets must act optimally. They will agree to borrow in market $\lambda \in [0,1)$ to buy assets only if doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1-\lambda)$$

The expected profits from borrowing in market λ to buy one consumption unit's worth of assets is

$$\int_{x_t(\lambda)}^{\infty} (z_t - x_t(\lambda))\, dG(z_t) \tag{46}$$

Agents will borrow in market λ to buy assets only if (46) equal $\left(1 + \overline{R}_t\right)\lambda$, the return on what they must spend on assets. If (46) were lower than $\left(1 + \overline{R}_t\right)\lambda$, no agent would borrow to buy assets. If (46) were higher than $\left(1 + \overline{R}_t\right)\lambda$, then no one would ever lend given they can borrow in market λ', and so $f_t(\lambda') = 0$. But this contradicts the fact that $f_t^a(\lambda) > 0$. Optimality implies

$$f_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda'))\, dG(z_t) = \left(1 + \overline{R}_t\right)\lambda' \tag{47}$$

Fourth, savers will not waste any resources. Since entrepreneurs use their endowment to produce, all the resources spent to buy the asset must come from savers. This implies that e must be either lent to entrepreneurs to produce or be spent on assets:

$$\int_0^1 f_t^p(\lambda)\, d\lambda + p_t = e \tag{48}$$

Finally, we turn to equilibrium beliefs. In any active market λ', lenders must expect the return on lending $\overline{R}_t(\lambda')$ to conform with the actual fraction of borrowers who borrow in market λ' with the intent to produce and to buy assets, respectively. That is,

$$\overline{R}_t(\lambda') = \frac{f_t^p(\lambda')}{f_t(\lambda')} R_t(\lambda') + \frac{f_t^a(\lambda')}{f_t(\lambda')} E_t \max\left\{R_t(\lambda'), \frac{d_{t+1}+p_{t+1}}{p_t} - 1\right\} \text{ if } f_t(\lambda') > 0 \tag{49}$$

In a market $\lambda \in \Delta$ with a positive mass of borrowing, the expression $\frac{f_t^x(\lambda')}{f_t(\lambda')}$ will be replaced by $\frac{m_t^x(\lambda)}{m_t(\lambda)}$. Condition (49) does not impose any restrictions on expectations in inactive markets where $f_t(\lambda') = 0$.

C.2 Solving for Equilibrium

We now proceed to solve for an equilibrium. As in the text, we restrict attention to equilibria in which all markets $\lambda \in [0,1)$ are active. Such equilibria are natural given we focus on the effects of interventions to shut down markets. Our first result characterizes the schedule of interest rates in such an equilibrium.

Proposition C1: *In an equilibrium where all markets are active, there exists a value $\Lambda_t \in [0,1]$ such that the equilibrium interest rate schedule will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\widetilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \Lambda_t) \\ 1 + \overline{R}_t & \text{if } \lambda \in [\Lambda_t, 1) \end{cases} \tag{50}$$

where $\widetilde{x}_t(\lambda)$ is the value of x that solves

$$\int_{z=x}^{1+r_t^{\max}} (z - x)\, dG_t(z) = \left(1 + \overline{R}_t\right)\lambda \tag{51}$$

The schedule of interest rates $R_t(\lambda)$ is a decreasing and continuous function of λ for $\lambda \in [0, \Lambda_t]$.

Proof of Proposition C1: Our proof relies proceeds as two lemmas.

Lemma C1: In an equilibrium where all markets are active, $1 + R_t(\lambda) = \max\left\{\frac{\widetilde{x}_t(\lambda)}{1-\lambda}, 1 + \overline{R}_t\right\}$, where $\widetilde{x}_t(\lambda)$ equals the x that solves (51) and $\overline{R}_t$ is the expected return to lending in any market λ.

Proof of Lemma C1: Recall we defined $x_t(\lambda) \equiv (1 + R_t(\lambda))(1 - \lambda)$ as the equilibrium debt obligation for an agent who invests one unit of resources in assets. As we argued above, for all λ we have

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) \leq \left(1 + \overline{R}_t\right)\lambda \tag{52}$$

since otherwise agents would refuse to lend, which is incompatible with $f_t(\lambda) > 0$ for all $\lambda \in [0,1)$. The expression $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is strictly decreasing in x. It also tends to $+\infty$ as $x \to -\infty$ and to 0 as $x \to 1 + r_t^{\max}$. Hence, for any $\lambda \in [0,1)$ and any $\overline{R}_t \geq 0$, there exists a unique $x \in (-\infty, 1 + r_t^{\max}]$ for which

$$\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z) = \left(1 + \overline{R}_t\right)\lambda \tag{53}$$

Denote $\widetilde{x}_t(\lambda)$ as the unique solution to equation (53). By contrast, $x_t(\lambda)$ refers to the value of $(1 + R_t(\lambda))(1-\lambda)$ evaluated at the equilibrium interest rate $R_t(\lambda)$.

For any $\lambda' \in [0,1)$ in which (52) holds with equality, we have $\widetilde{x}_t(\lambda') = x_t(\lambda')$,and so

$$1 + R_t(\lambda') = \frac{\widetilde{x}_t(\lambda')}{1-\lambda'}$$

For any remaining values of $\lambda' \in [0,1)$, condition (52) holds as a strict inequality. This means borrowing in market λ' and buying assets yields a lower payoff than lending out the resources needed to borrow in market λ'. Hence, no agent will borrow in market λ' to buy assets, implying $f_t^a(\lambda') = 0$. In an equilibrium where all markets are active, $f_t^p(\lambda') > 0$. From (44) we know that $\overline{R}_t(\lambda') = \overline{R}_t$, and from (49) we know that since $f_t^a(\lambda') = 0$ then $\overline{R}_t(\lambda') = R_t(\lambda')$. Combining these implies $R_t(\lambda') = \overline{R}_t$.

Hence, in an equilibrium where all markets are active, we must have either $R_t(\lambda) = \overline{R}_t$ or $R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ for all $\lambda \in [0,1)$. To further show that $1 + R_t(\lambda) = \max\left\{1 + \overline{R}_t, \frac{\widetilde{x}_t(\lambda)}{1-\lambda}\right\}$, consider a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} > 1 + \overline{R}_t$, i.e., for which $\widetilde{x}_t(\lambda) > x_t(\lambda)$. Since $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is decreasing in x, this means

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) > \int_{z=\widetilde{x}(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}(\lambda))\, dG_t(z) = \left(1 + \overline{R}_t\right)\lambda$$

Since in equilibrium we must satisfy (52), it follows that in this case we have $1 + R_t(\lambda) = \frac{\widetilde{x}_t(\lambda)}{1-\lambda}$.

Next, consider a value of λ for which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} < 1 + \overline{R}_t$, i.e., for which $\widetilde{x}_t(\lambda) < x_t(\lambda)$. Then we would have

$$\left(1 + \overline{R}_t\right)\lambda = \int_{z=\widetilde{x}(\lambda)}^{1+r_t^{\max}} (z - \widetilde{x}(\lambda))\, dG_t(z) > \int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z)$$

In this case, (52) can only hold as a strict inequality. But we already know that in this case $R_t(\lambda) = \overline{R}_t$. This establishes the lemma. ■

Our next lemma establishes that $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is a weakly decreasing and continuous function of λ. Combined with Lemma C1, this implies there exists a cutoff Λ_t such that $R_t(\lambda) = \overline{R}_t$ for $\lambda \geq \Lambda_t$.

Lemma C2: In any equilibrium where all markets are active, $\frac{\widetilde{x}_t(\lambda)}{1-\lambda}$ is nonincreasing and continuous in λ.

Proof of Lemma C2: The function $\widetilde{x}_t(\lambda)$ corresponds to the value of x which solves (51). Although the distribution $G_t(z)$ can contain mass points, the integral $\int_{z=x}^{1+r_t^{\max}} (z-x)\, dG_t(z)$ is still continuous in x.

This implies $\widetilde{x}_t(\lambda)$ is a continuous function of λ. However, $\widetilde{x}_t(\lambda)$ may exhibit kinks, meaning its directional derivatives need not be equal at all values. To show that $\widetilde{x}_t(\lambda)$ is decreasing, it will suffice to show that all of its directional derivatives are nonpositive for all $\lambda \in [0,1)$. Totally differentiating (51) implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -\frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

For any λ where $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$, $\lim_{\lambda'\to\lambda^+} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda'\to\lambda^-} \int_{\widetilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z)$. Nevertheless, both $\lim_{\lambda'\to\lambda^+} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ and $\lim_{\lambda'\to\lambda^-} \frac{d\widetilde{x}_t(\lambda')}{d\lambda'}$ are negative, so $\widetilde{x}_t(\lambda)$ is strictly decreasing in λ.

Next, define $\widetilde{R}_t(\lambda) \equiv \frac{\widetilde{x}_t(\lambda)}{1-\lambda} - 1$. The function $\widetilde{R}_t(\lambda)$ is also continuous in λ with possible kink-points. Differentiating the equation $\widetilde{x}_t(\lambda) = (1-\lambda)(1+\widetilde{R}_t(\lambda))$ implies

$$\frac{d\widetilde{x}_t(\lambda)}{d\lambda} = -(1+\widetilde{R}_t(\lambda)) + (1-\lambda)\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for $\frac{d\widetilde{x}_t(\lambda)}{d\lambda}$ above yields

$$\begin{aligned}
\frac{d\widetilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) + \frac{d\widetilde{x}_t(\lambda)}{d\lambda}\right] \\
&= \frac{1}{1-\lambda}\left[1+\widetilde{R}_t(\lambda) - \frac{1+\overline{R}_t}{\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\right] \\
&= \frac{1}{(1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}\left[(1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - \left(1+\overline{R}_t\right)\right] \qquad (54)
\end{aligned}$$

Once again, $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$ is discontinuous at λ where $\widetilde{x}_t(\lambda)$ is a mass point of $G_t(z)$.

To evaluate the sign of $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$, we must consider two cases. First, suppose $\widetilde{R}_t(\lambda) < \overline{R}_t$. Then

$$\begin{aligned}
(1+\widetilde{R}_t(\lambda))\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) &< \left(1+\overline{R}_t\right)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \\
&\leq 1+\overline{R}_t
\end{aligned}$$

In that case, we have $\frac{d\widetilde{R}_t(\lambda)}{d\lambda} < 0$ from (54) regardless of the direction we take the derivative. Next, suppose $\widetilde{R}_t(\lambda) \geq \overline{R}_t$. From Lemma C1, in this case (52) holds with equality. Rearranging this equation, we get

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z - \left(1+\widetilde{R}_t(\lambda)\right)\right] dG_t(z) = \lambda\left[\left(1+\overline{R}_t\right) - \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left(1+\widetilde{R}_t(\lambda)\right) dG_t(z)\right]$$

We can establish that $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$ in (54) is nonnegative for $\lambda > 0$ if we can show that

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[z_t - \left(1+\widetilde{R}_t(\lambda)\right)\right] dG_t(z) \geq 0$$

Towards this, observe that the expected profits from borrowing in market λ to buy assets are given by

$$\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left(z_t-(1+R_t)\right) dG\left(z\right)+\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG\left(z\right)$$

Since these are equal to $\left(1+\overline{R}_t\right)\lambda$ when $\widetilde{R}_t\left(\lambda\right)\geq\overline{R}_t$, we have

$$\begin{aligned}\left(1+\overline{R}_t\right)\lambda &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left(z_t-(1+R_t)\right) dG\left(z\right)+\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG\left(z\right)\\ &\leq \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left(z_t-(1+R_t)\right) dG\left(z\right)+\int_{0}^{1+r_t^{\max}} \lambda z_t dG\left(z\right)\\ &= \int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)\left(z_t-(1+R_t)\right) dG\left(z\right)+\left(1+\overline{r}_t^D\right)\lambda \end{aligned} \tag{55}$$

But in an equilibrium where all markets are active, we must have $\overline{R}_t^D\geq\overline{r}_t^D$. This implies

$$0\leq\left(\overline{R}_t-\overline{r}_t^D\right)\lambda\leq(1-\lambda)\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}}\left(z_t-(1+R_t)\right)dG\left(z\right)$$

This confirms $\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}}\left(z_t-(1+R_t)\right)dG\left(z\right)\geq 0$. All directional derivatives $\frac{d\widetilde{R}_t(\lambda)}{d\lambda}$ are nonnegative. ■

From Lemmas C1 and C2, set Λ_t to be either 1 or the minimum value in $[0,1]$ for which $R_t\left(\lambda\right)=\overline{R}_t$. It follows that $R_t\left(\lambda\right)>\overline{R}_t$ for $\lambda<\Lambda_t$ and $R_t\left(\lambda\right)=\overline{R}_t$ for all $\lambda\geq\Lambda_t$. This establishes the proposition. ■

We can use the schedule of interest rates in Proposition C1 to determine how much entrepreneurs should produce and in which markets to borrow if they do.

Proposition C2: *In an equilibrium where all markets are active, entrepreneurs with wealth w will borrow $1-w$ units to produce, in a market with an interest rate equal to $R_t\left(w\right)$.*

Proof of Proposition C2: Consider an entrepreneur with wealth w. If she borrows in a market λ where $\lambda\leq w$, she can produce at full capacity and would only need to put down $\lambda\left(\frac{1-w}{1-\lambda}\right)$ resources to borrow $1-w$ to reach full capacity. This would earn her an expected profit of

$$1+y^*-\left(1+R_t\left(\lambda\right)\right)(1-w)$$

This value is maximized by choosing λ to minimize $R_t\left(\lambda\right)$. From Proposition C1, we know $R_t\left(\lambda\right)$ is weakly decreasing in λ and is therefore maximized at $\lambda=w$.

Next, suppose she borrows in a market λ where $\lambda>w$. In that case, she could not produce at full capacity. Since $y^*>r_t^{\max}=R_t\left(0\right)\geq R_t\left(\lambda\right)$ for all $\lambda\in[0,1)$, it will be optimal to borrow enough to produce at the maximal capacity possible. For $\lambda>w$, this maximum is $\frac{w}{\lambda}$. Her profits would thus equal

$$\frac{w}{\lambda}\left(1+y^*-x_t\left(\lambda\right)\right) \tag{56}$$

where recall $x_t(\lambda) = (1-\lambda)(1+R_t(\lambda))$ is the amount a borrower is required to repay per each unit of resource she borrows. Since $R_t(\lambda) = \overline{R}_t$ for all $\lambda \in (\Lambda_t, 1)$, there would be no benefit to going to market $\lambda > \Lambda_t$: She would have to produce less at the same interest rate as in market Λ_t. The only case that remains is the interval of markets $\lambda \in [w, \Lambda_t]$. In that case, we can differentiate profits in (56) to get

$$\begin{aligned}
\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1+y^* - x_t(\lambda))\right) &= -\frac{w}{\lambda^2}\left[(1+y^* - x_t(\lambda)) + \lambda\frac{dx_t(\lambda)}{d\lambda}\right] \\
&= -\frac{w}{\lambda^2}\left[(1+y^* - x(\lambda)) - \frac{\lambda\left(1+\overline{R}_t\right)}{\int_x^{1+r_t^{\max}} dG_t(z)}\right] \\
&= -\frac{w}{\lambda^2\int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (1+y^* - x_t(\lambda))\, dG_t(z) - \lambda\left(1+\overline{R}_t\right)\right]
\end{aligned}$$

Since $y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$, we have

$$\frac{d}{d\lambda}\left(\frac{w}{\lambda}(1+y^* - x_t(\lambda))\right) < -\frac{w}{\lambda^2\int_x^{1+r_t^{\max}} dG_t(z)}\left[\int_x^{1+r_t^{\max}} (z - x_t(\lambda))\, dG_t(z) - \lambda\left(1+\overline{R}_t\right)\right]$$

But for $\lambda \leq \Lambda_t$, the expression in brackets is equal to 0. Hence, borrowing in a market with $\lambda > w$ will be strictly dominated by borrowing in the market with $\lambda = w$. At the optimum, each entrepreneur borrow $1-w$ at a rate of $R_t(w)$. ■

Proposition C3: *In an equilibrium where all markets are active, the equilibrium price of the asset will be given by* $p_t = (1-\varphi)e$

Proof of Proposition C3: Condition (48) implies that all the resources of the young in cohort t will be used to either produce or to buy assets. From Proposition C2, we know that all entrepreneurs will produce at capacity, so the total amount used to produce is given by

$$\int_0^1 (2\varphi e)\, dw = 2\varphi e$$

This implies

$$p_t + 2\varphi e = (1+\varphi)e$$

and so $p_t = (1-\varphi)e$ as claimed. ■

Propositions C1-C3 do not require any restrictions on the distribution of d_t. When $d_t = d$, the return on the asset $1+r_t$ will have a degenerate distribution with full mass at $\frac{d}{(1-\varphi)e}$. Substituting this into (51) reveals that $\widetilde{x}(\lambda) = (1-\lambda)\left(1+\frac{d}{(1-\varphi)e}\right)$ for all λ, that $\frac{d\widetilde{R}(\lambda)}{d\lambda} = 0$ for all λ, and the cutoff $\Lambda_t = 0$. Hence, when all markets are active, $R_t(\lambda) = \overline{R}_t = \frac{d}{(1-\varphi)e}$ for all $\lambda \in [0,1)$ as described in the text. One equilibrium in which all markets are active if it entrepreneurs with wealth w borrow in market $\lambda = w$. But other equilibria in which all markets are active also exist.

When dividends follow a regime-switching process, then if $d_t = D$ at date t, z_t would be distributed as

$$z_t = \begin{cases} \frac{D}{(1-\varphi)e} & \text{w/prob } 1-\pi \\ \frac{d}{(1-\varphi)e} & \text{w/prob } \pi \end{cases}$$

We can verify that this distribution implies that $\frac{dR_t(\lambda)}{d\lambda} < 0$ when $\lambda \leq \Lambda_t^D$. In particular, observe that (55) in Lemma C2 relies on the fact that $\int_{\widetilde{x}_t(\lambda)}^{1+r_t^{\max}} z_t dG(z) \leq \int_0^{1+r_t^{\max}} z_t dG(z)$. But for the above distribution, the first expression is equal to $(1-\pi)\left(1+\frac{D}{(1-\varphi)e}\right)$, which is strictly less than $1+\frac{(1-\pi)D+\pi d}{(1-\varphi)e}$ which corresponds to the second expression. Hence, we can replace (55) with a strictly inequality, implying $\frac{dR_t(\lambda)}{d\lambda}$ is strictly negative for $\lambda \leq \Lambda_t^D$. This is in line with what we discuss in the text and depict in Figure 3.

Since Λ_t^D is the minimum value of λ at which $\frac{\widetilde{x}_t(\lambda)}{1-\lambda} = 1+\overline{R}_t^D$, we have

$$\frac{1}{1-\Lambda_t}\left[1+\frac{D}{(1-\varphi)e}-\left(1+\overline{R}_t^D\right)\frac{\Lambda_t}{1-\pi}\right] = 1+\overline{R}_t^D$$

which, upon rearranging, yields

$$\Lambda_t^D = \frac{1-\pi}{\pi\left(1+\overline{R}_t^D\right)}\left(\frac{D}{(1-\varphi)e}-\overline{R}_t^D\right)$$

Since $R_t^D(\lambda)$ is decreasing in λ for $\lambda \in [0,\Lambda_t^D)$, Proposition C2 implies only borrowers with wealth w borrow in market $\lambda = w$ for $w \in [0,\Lambda_t^D)$. Hence, $f_t^p(\lambda) = 2\varphi e$ for $\lambda \in [0,\Lambda_t^D)$. By contrast, $f_t^p(\lambda)$ is indeterminate for $\lambda \in [\Lambda_t^D, 1)$. However, we know that $f_t^p(\Lambda_t) > 0$, since borrowers with wealth $w = \Lambda_t^D$ will have to borrow in this market to borrow $1-w$. As for the amount borrowed to buy assets, $f_t^a(\lambda)$, we can deduce $f_t^a(\lambda)$ for $\lambda \in [0,\Lambda_t^D]$ from $R_t^D(\lambda)$, $\overline{R}_t^D$, and $f_t^p(\lambda)$ using (49). For $\lambda > \Lambda_t^D$, the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ at $\lambda = \Lambda_t^D$, combined with the fact that $\frac{dR_t^D(\lambda)}{d\lambda} < 0$ for $\lambda > \Lambda_t^D$ from Lemma C2, implies that no agent would want to borrow to buy assets. So $f_t^a(\lambda) = 0$ for all $\lambda \geq \Lambda_t^D$. We can solve for $\overline{R}_t^D$ as in the text.

C.3 Comparative Statics

Next, we consider equilibria where all markets above some floor $\underline{\lambda}$ are active. These results correspond to Propositions 8 and 9 in the text. The first result concerns how the equilibrium changes with $\underline{\lambda}$.

Proof of Proposition 8: In the text, we show that p^D and $\overline{r}^D$ are increasing and decreasing in $\underline{\lambda}$, respectively. Here, we show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$. For any $\underline{\lambda}$, either $\overline{R}^D$ equals $\overline{r}^D$ or exceeds $\overline{r}^D$. Since the expected return on loans $\overline{R}^D$ is continuous in $\underline{\lambda}$, it will suffice to show that $\overline{R}^D$ is decreasing in $\underline{\lambda}$ when $\overline{R}^D > \overline{r}^D$.

When $\overline{R}^D > \overline{r}^D$, we have $\gamma^D = 1$, and the equilibrium conditions for R^D and Λ^D are given by

$$\begin{aligned}(1-\Lambda^D)\overline{R}^D &= \left[\frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \Lambda^D\left(\frac{1+\overline{R}^D}{1-\pi}-1\right)\right] && (57)\\ 1+\overline{R}^D &= (1-\varphi(1-\underline{\lambda}))\left[1+\overline{r}^D-\pi\Phi\right]+ \\ & \quad 2\varphi\int_0^1\left[\min\left\{\frac{w}{\underline{\lambda}},1\right\}-w\right]\left[1+R^D(\max\{w,\underline{\lambda}\})\right]dw && (58)\end{aligned}$$

If $\overline{R}^D > \overline{r}^D$, the floor $\underline{\lambda}$ must be below the cutoff Λ^D. For suppose $\underline{\lambda} \geq \Lambda^D$. Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected return on the asset must be equal to ensure both the credit market and asset market clear. Since $\underline{\lambda} < \Lambda^D$, we can expand the integral term in (58) to obtain

$$\begin{aligned}\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\} - w\right]\left[1+R^D\left(\max\left\{w,\underline{\lambda}\right\}\right)\right] &= \left(1+R\left(\underline{\lambda}\right)\right)\left(\frac{1}{\underline{\lambda}}-1\right)\int_0^{\underline{\lambda}} wdw + \\ &\quad \int_{\underline{\lambda}}^{\Lambda^D}\left(1+R\left(w\right)\right)\left(1-w\right)dw + \left(1+\overline{R}^D\right)\int_{\Lambda^D}^1 \left(1-w\right)dw\end{aligned}$$

We use the fact that $1+R^D\left(\lambda\right) = \frac{1}{1-\lambda}\left[1+\frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \frac{\lambda\left(1+\overline{R}_t^D\right)}{1-\pi}\right]$ to express the three integrals above as

$$\left(1+R\left(\underline{\lambda}\right)\right)\left(\frac{1}{\underline{\lambda}}-1\right)\int_0^{\underline{\lambda}} wdw = \left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \underline{\lambda}\left(\tfrac{1+\overline{R}^D}{1-\pi}-1\right)\right]\frac{\underline{\lambda}}{2} \tag{59}$$

$$\int_{\underline{\lambda}}^{\Lambda^D}\left(1+R\left(w\right)\right)\left(1-w\right)dw = \int_{\underline{\lambda}}^{\Lambda^D}\left[1+\tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{w\left(1+\overline{R}_t^D\right)}{1-\pi}\right]dw \tag{60}$$

$$\left(1+\overline{R}^D\right)\int_{\Lambda^D}^1\left(1-w\right)dw = \frac{1}{2}\left(1+\overline{R}^D\right)\left(1-\Lambda^D\right)^2 \tag{61}$$

We can write (57) and (58) more compactly as

$$\begin{aligned} h_1\left(\overline{R}^D,\Lambda^D\right) &= 0\\ h_2\left(\overline{R}^D,\Lambda^D\right) &= 0\end{aligned}$$

Totally differentiating this system of equations gives us the comparative statics of the equilibrium $\overline{R}^D$ and Λ^D with respect to any variable a as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D}\\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D}\end{bmatrix}\begin{bmatrix} d\overline{R}^D/da \\ d\Lambda^D/da\end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a}\end{bmatrix}$$

Differentiating (57) and (58) using expressions (59)-(61) yields

$$\begin{array}{ll} \frac{\partial h_1}{\partial \overline{R}^D} = 1-\Lambda^D + \frac{\Lambda^D}{1-\pi} & \frac{\partial h_1}{\partial \Lambda^D} = \frac{\pi(1+\overline{R}^D)}{1-\pi}\\ \frac{\partial h_2}{\partial \overline{R}^D} = 1+\varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right] & \frac{\partial h_2}{\partial \Lambda^D} = 0\end{array}$$

When we evaluate comparative statics with respect to $\underline{\lambda}$, we now have

$$\begin{aligned}\begin{bmatrix} d\overline{R}^D/d\underline{\lambda} \\ d\lambda^D/d\underline{\lambda}\end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \overline{R}^D} & \frac{\partial h_1}{\partial \Lambda^D}\\ \frac{\partial h_2}{\partial \overline{R}^D} & \frac{\partial h_2}{\partial \Lambda^D}\end{bmatrix}^{-1}\begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}}\end{bmatrix}\\ &= \frac{\varphi}{\kappa}\begin{bmatrix} 0 & \frac{\pi}{1-\pi}\left(1+\overline{R}^D\right)\\ 1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2 & -\left(1-\Lambda^D+\frac{\Lambda^D}{1-\pi}\right)\end{bmatrix}\begin{bmatrix} -\frac{D}{(1-(1-\underline{\lambda})\varphi)^2 e} \\ -\frac{2D\left(1+\Lambda^D\varphi\right)}{(1-(1-\underline{\lambda})\varphi)^2 e} - (1+\pi\Phi)\end{bmatrix}\end{aligned}$$

where $\kappa = \frac{\pi(1+\overline{R}^D)}{1-\pi}\left(1+\varphi\frac{\left(\Lambda^D\right)^2}{1-\pi} - \varphi\left(1-\Lambda^D\right)^2\right) > 0$. It follows that

$$\frac{d\overline{R}^D}{d\underline{\lambda}} = -\varphi\left(1+\varphi\left[\frac{1}{1-\pi}\left(\Lambda^D\right)^2 - \left(1-\Lambda^D\right)^2\right]\right)^{-1}\left[\frac{2D\left(1+\Lambda^D\varphi\right)}{\left(1-\left(1-\underline{\lambda}\right)\varphi\right)^2 e} + \left(1+\pi\Phi\right)\right] < 0$$

Since $\overline{R}^D$ is decreasing in $\underline{\lambda}$ whether $\overline{R}^D > \overline{r}^D$ or $\overline{R}^D = \overline{r}^D$, the claim follows. ■

Proposition 9 concerns how changing $\underline{\lambda}$ affects the expected costs of default $\gamma^D \Phi p^D$. Since we already know p^D is increasing in $\underline{\lambda}$, any changes in expected default costs occur entirely through γ^D. Our next result argues that there exists cutoffs Λ_0 and Λ_1 such that $d\gamma^D/d\underline{\lambda} = 0$ when $\underline{\lambda} < \Lambda_0$ or $\underline{\lambda} > \Lambda_1$. When $\Lambda_0 < \underline{\lambda} < \Lambda_1$, we only claim it must be decreasing for some $\underline{\lambda}$ in this interval.

Proof of Proposition 9: Define

$$\rho(\underline{\lambda}) = \frac{\overline{R}^D}{(1-(1-\underline{\lambda})\varphi)}$$

Using the fact that $\frac{d\overline{R}^D}{d\underline{\lambda}} < 0$, we have

$$\frac{d\rho(\underline{\lambda})}{d\underline{\lambda}} = \frac{d\overline{R}^D/d\underline{\lambda} - \varphi\rho(\underline{\lambda})}{1-(1-\underline{\lambda})\varphi} < 0$$

Since

$$\overline{R}^D/\overline{r}^D = [(1-\pi) D + \pi d]\, \rho(\underline{\lambda})$$

it follows that the ratio $\overline{R}^D/\overline{r}^D$ is decreasing in $\underline{\lambda}$. Hence, there exists a value $\Lambda_0 \geq 0$ such that $\overline{R}^D > \overline{r}^D$ for $\underline{\lambda} < \Lambda_0$ and $\overline{R}^D = \overline{r}^D$ for $\underline{\lambda} \geq \Lambda_0$. Since $\overline{R}^D > \overline{r}^D$ when $\underline{\lambda} < \Lambda_0$, then $\gamma^D = 1$ for $\underline{\lambda} < \Lambda_0$. It follows that expected default costs $\pi\gamma^D \Phi p^D = \pi\Phi p^D$ are increasing in $\underline{\lambda}$ in this region. A higher $\underline{\lambda}$ for $\lambda < \Lambda_0$ reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where $\underline{\lambda} \geq \Lambda_0$. Here, we know $\overline{R}^D = \overline{r}^D$. Substituting this into (57) yields

$$\left(1-\Lambda^D\right)\left(1+\overline{r}^D\right) = \left[1 + \tfrac{D}{(1-(1-\underline{\lambda})\varphi)e} - \tfrac{\Lambda^D}{1-\pi}\left(1+\overline{r}^D\right)\right]$$

which, upon rearranging,

$$\Lambda^D = \tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e + (1-\pi)D + \pi d}$$

From this, we can conclude that $\Lambda^D \geq \underline{\lambda}$ if

$$\tfrac{(1-\pi)(D-d)}{(1-(1-\underline{\lambda})\varphi)e + (1-\pi)D + \pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1-\pi)(D-d) \geq \underline{\lambda}\left[(1-(1-\underline{\lambda})\varphi)e + (1-\pi)D + \pi d\right] \tag{62}$$

The RHS of (62) is a quadratic in $\underline{\lambda}$ with a positive coefficient on the quadratic term. The inequality is satisfied when $\underline{\lambda} = 0$ and violated when $\underline{\lambda} = 1$. This implies there exists a cutoff $\Lambda_1 \in (0,1)$ such that $\Lambda^D > \underline{\lambda}$ if $\underline{\lambda} \in [0, \Lambda_1)$ and $\Lambda^D < \underline{\lambda}$ if $\underline{\lambda} \in (\Lambda_1, 1)$. We can deduce that $\Lambda_1 \geq \Lambda_0$ since by definition Λ_0 is the cutoff such that $\overline{R}^D = \overline{r}^D$ when $\underline{\lambda} \geq \Lambda_0$, yet at $\underline{\lambda} = \Lambda_1$ we have

$$\overline{R}^D = R\left(\Lambda^D\right) = R(\underline{\lambda}) = R(\Lambda_1)$$

By construction, we know that $R(\underline{\lambda})$ when $\underline{\lambda} = \Lambda_1$ is equal to $\overline{r}^D$. This implies $\Lambda_1 \geq \Lambda_0$.

When $\underline{\lambda} > \Lambda_1$ no agent will borrow to buy the asset, so $\gamma^D = 0$. Expected default costs are 0, and so the only effect of increasing $\underline{\lambda}$ is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where $\Lambda_0 < \underline{\lambda} < \Lambda_1$. We do not analyze this case in general. However, when $\Lambda^D = \underline{\lambda}$, the interest rate in all active markets would equal $\overline{R}^D$, since the only active markets are those with $\lambda \geq \underline{\lambda} = \Lambda^D$. Since $\underline{\lambda} \geq \Lambda_0$, we know that $\overline{R}^D = \overline{r}^D$ and so the interest rate in all active markets is $\overline{r}^D$. The equilibrium condition that determines γ^D is given by

$$\begin{aligned}
\left(1+\overline{r}^D\right) &= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]\left[1+R^D\left(\max\{w,\underline{\lambda}\}\right)\right]dw \\
&= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\int_0^1 \left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]dw \\
&= \left(1-(1-\underline{\lambda})\varphi\right)\left[1+\overline{r}^D-\gamma^D\pi\Phi\right] + 2\varphi\left(1+\overline{r}^D\right)\left[\underline{\lambda}/2+(1-\underline{\lambda})-1/2\right] \\
&= 1+\overline{r}^D-\gamma^D\left(1-(1-\underline{\lambda})\varphi\right)\pi\Phi
\end{aligned}$$

Hence, when $\underline{\lambda} = \Lambda_1$, we have $\gamma^D = 0$. For $\underline{\lambda} < \Lambda_1$, however, $\gamma^D > 0$, since

$$\int_0^1 \left[1+R^D\left(\max\{w,\underline{\lambda}\}\right)\right]\left[\min\left\{\tfrac{w}{\underline{\lambda}},1\right\}-w\right]dw$$

will be strictly greater than $\frac{1}{2}\left(1+\overline{r}^D\right)(1-\underline{\lambda})$. Hence, in the limit as $\underline{\lambda} \uparrow \Lambda_1$, we have $d\gamma^D/d\underline{\lambda} < 0$ expected default costs $\pi\gamma^D\Phi p^D$ must be decreasing in $\underline{\lambda}$ since this expression goes from a positive value to 0.

Finally, to show that this can generate a Pareto improvement, observe that increasing $\underline{\lambda}$ while dividends are high will make the initial old at date 0 better off given p_0^D increases. Cohorts born after dividends have fallen will be unaffected if $\underline{\lambda}$ is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs $E\left[d_{t+1}\right] - \Phi\pi\gamma^D p_t^D$ as well as the output produced by entrepreneurs. If Φ is sufficiently large and φ is small, we can promise these agents a higher expected consumption. ■

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