# Learning in Financial Markets: Implications for Debt-Equity Conflicts[*]

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## Abstract

Financial markets reveal information which firm managers can utilize when making equity value-enhancing investment decisions. However, in the presence of risky debt, such investments are not necessarily socially efficient. We show that learning from prices eliminates some inefficient decisions. However, while investors' endogenous learning further *attenuates* risk-shifting, it *amplifies* debt overhang. With risk-shifting, investors acquire more information about riskier projects: thus, the price reveals more information to the manager, attenuating risk-shifting. With debt overhang, investors acquire less information about risk-reducing projects, worsening the problem. Our model provides a novel channel through which financial markets impact agency frictions between firm stakeholders.

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# 1  Introduction

Our paper is motivated by two well-known observations. First, investors' incentive to acquire information generally increases with the volatility of the asset's underlying cash flows.[1] Second, in the presence of risky debt, firm managers prefer more volatile cash flows, ceteris paribus; however, such preferences may lead to socially inefficient investment decisions.[2,3] We argue that the growing feedback effect literature provides a novel connection between these two observations: investors' private information, contained in secondary market prices, can serve as a valuable source of information for firm managers.[4] As a result, the riskiness of the firm's cash flows is endogenously determined: the managers' decision to invest (which *alters* cash flow volatility) depends upon investors' decision to acquire information (which is *contingent upon* cash flow volatility). Thus, understanding how investors' endogenous learning affects investment efficiency and the agency conflicts between firms' stakeholders is both a natural and important issue for study.

The main challenge in studying such an interplay is that most noisy rational expectation equilibrium (REE) models, which are instrumental in analyzing this effect, rely on a linear pricing function. To accommodate the non-linearity introduced by debt, the first part of the paper develops a novel, non-linear REE which incorporates a feedback loop between the equity price and the firm's investment decision. We then utilize this setting to study how managerial learning from prices and investors' information choice affects the agency conflicts between stakeholders. Our paper has two main results. We begin by demonstrating that, in the presence of risky debt, learning from prices generically eliminates some inefficient investment decisions. However, we then show that investors' endogenous learning creates an asymmetric effect which depends upon the type of investment opportunity. In particular, we show that while the *most inefficient risk-shifting projects are least likely to be adopted* after observing prices, the opposite is true *when debt overhang is feasible: the most*

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[1]There is a large literature consistent with this general observation, starting with Grossman and Stiglitz (1980), Hellwig (1980) and corroborated by more recent work including Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016).

[2]This statement assumes, as we do throughout the paper, that the firm manager is incented to act in equity holders' best interests, i.e., no agency conflict exists between the firm manager and equity holders.

[3]Both the theory of risk-shifting (Jensen and Meckling (1976)) and debt overhang (Myers (1977)) are consistent with the latter observation.

[4]See Bond, Edmans, and Goldstein (2012) for a survey of this literature.

*efficient investments are most likely to be abandoned.* Consistent with these predictions, the empirical literature has thus far found evidence consistent with debt overhang (e.g., Moyen (2007)) but little support for risk-shifting (e.g., Gilje (2016)).[5] Our paper provides a single, novel channel through which such a disparity arises. Interestingly, we show that this asymmetry does not arise when managers can choose how much information to acquire. Finally, we show that when the feedback channel is present, whether strategic complementarity in information acquisition arises depends upon the project's ex-ante prospects *as well as* its correlation with the firm's existing assets.

We consider a three-date (two-period) model. At date zero, the firm owns an existing asset and has access to a potential investment. While the firm manager and investors share common prior beliefs about the investment, each (competitive) investor can acquire costly, private information about the project's likelihood of success. At date zero, each investor chooses how much information to acquire in anticipation of trading an equity claim in the next period. At date one, the firm manager must decide whether or not to invest in the new project, and can use the information contained in the price of equity when doing so. Investors incorporate this "feedback channel" into their demand schedules and the manager's decision is ultimately reflected in the price. At date two, the cash flows of any assets owned by the firm are realized and the proceeds are paid to existing debt and equity investors.

The extent to which the firm manager's investment decision relies on the price depends upon the quality of the information contained therein: as investors acquire more information, the manager conditions more heavily on the price. Note, though, that investors' incentive to acquire information increases when the value of the traded claim is more sensitive to the signal they receive. We consider investment projects which can amplify or attenuate the information sensitivity of equity, depending upon the investment's payoff distribution. This proves to be the crucial distinction between risk-shifting and debt overhang. Projects subject to risk-shifting (which are positively correlated with assets-in-place) increase the information sensitivity of equity. Projects which can induce debt overhang (negatively correlated with assets-in-place) cause it to fall. Ex-ante, this leads to endogenous variation in investors' private information which, in turn, generates ex-post variation in the likelihood

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[5]The following section provides a more detailed exploration of this literature.

that the manager makes the investment.

We show that, all else equal, the riskiest and most inefficient forms of risk-shifting are least likely to be chosen inefficiently *after the manager conditions on prices*.[6] A risk-shifting project transfers cash flows from bad to good states of the world, which increases the information sensitivity of equity. Moreover, the more ex-ante inefficient the project, the larger this change in information sensitivity. This increases the marginal value of acquiring information for equity holders, leading to more informative prices. As a result, the firm manager conditions more heavily on the price, which increases the variance of his posterior beliefs.[7] We show that as the variance of the manager's beliefs grows, investments that meet the manager's break-even threshold are also more likely to be ex-post efficient. Thus, more inefficient projects have a lower likelihood of being chosen after the manager conditions on prices.

On the other hand, when faced with the prospect of debt overhang, the manager is most likely to forgo the most efficient, risk-reducing investments. The argument closely follows the logic above. Conditional on investment, a project which exhibits the potential for debt overhang decreases the information sensitivity of equity. The more efficient (and risk-reducing) the project is ex-ante, the larger the fall in both information sensitivity and investor information acquisition. As a result, even after conditioning on prices, the manager is more likely to inefficiently opt out of investment: lower-quality information implies that the manager is more likely to stick with his ex-ante decision. In short, this suggests that endogenous information acquisition increases the likelihood that the worst examples of debt overhang persist.

Our model suggests that this difference in the prevalence of risk-shifting and debt overhang is more likely to arise when the firm has publicly-, not privately-held equity. Further, our results will be more pronounced in settings where investors have access to payoff-relevant information that managers do not possess. For instance, Luo (2005) provides evidence that an acquisition is more likely to be canceled if the market reacts negatively, particularly in cases where learning is more probable. The

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[6]We define efficiency with respect to the ex-ante net present value of the investment.

[7]If the manager could not condition on prices, he would invest in these ex-ante inefficient projects with certainty: doing so increases the expected value of equity.

model also implies that investment-to-price sensitivity, a measure of managerial learning, should be higher (lower) when firm managers have the opportunity to indulge in risk-shifting (debt overhang).

To highlight the unique asymmetry introduced by investor learning, we introduce managerial information acquisition (while shutting down investor learning) to the benchmark model. We show that managerial information acquisition increases with the information sensitivity of *the project*: as a result, managerial learning reduces the most inefficient examples of both risk-shifting *and* debt overhang.[8] The contrast in this result, relative to our benchmark, is driven by the investment's correlation with existing assets. With risk-shifting, a risky project increases both the information sensitivity of investment and equity, whereas with debt overhang (where the project is negatively correlated with existing assets), a more information-sensitive project lowers the information sensitivity of equity.

Finally, we note two additional contributions of our model to the theoretical literature. First, our model generalizes the analysis of information acquisition found in Dow, Goldstein, and Guembel (2017). In addition to the standard strategic substitutability, such as that found in Grossman and Stiglitz (1980), Dow et al. (2017) show that the ex-ante likelihood of investment success can generate strategic complementarity. In their setting, when the ex-ante fundamentals of a project are weak, the firm only invests if the information in prices suggests that it is profitable to do so; as a result, the marginal value of learning can increase when other investors produce information, as this increases the chance that the firm will make the investment. Hence, strategic complementarity can arise across investors. Our model generalizes this result but provides an important counterpoint. By incorporating existing assets, we are able to show that this result depends upon the sign of the correlation between the return of the investment and the cash flows generated by assets-in-place. When the investment return is negatively correlated with that of assets-in-place, strategic complementarity can only arise with ex-ante stronger, not weaker, fundamentals. In this case, as more investors learn, it becomes less likely that the manager chooses to invest, which increases the information sensitivity of equity and generates complementarity.

Second, as noted earlier, solving the model required the development of a new non-linear rational

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[8]Note that investor information acquisition is increasing in the information sensitivity of *equity*.

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expectations equilibrium. To do so, the first part of the paper extends the model of Davis (2019) and Albagli, Hellwig, and Tsyvinski (2015) to create a novel, tractable, non-linear REE with debt, equity and a feedback loop between security prices and the firm's investment decision. The flexibility and tractability of this model provides a foundation from which to answer research questions in which the presence of risky debt is an essential ingredient.

## 1.1 Related Literature

At its core, our model emphasizes the role played by financial markets in aggregating and disseminating information, following Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). Recently, a theoretical literature has emerged which studies the role of secondary financial markets as an important source of information for decision makers, including firm managers (as in our model).[9],[10] Bond et al. (2012) provide a comprehensive survey of this "feedback effect" literature: below, we highlight those papers which most closely resemble our own.

As in Bond, Goldstein, and Prescott (2009), Goldstein et al. (2013), Bond and Goldstein (2015) and Dow et al. (2017), investors in our model act competitively; the private information they possess is impounded into the price through their trading activity in a non-strategic manner.[11] Similar to the analysis of Bond et al. (2009), we show that our rational expectations pricing function has the potential to exhibit non-montonicity: the existence, therefore, of a feedback equilibrium requires a restriction on the project characteristics. We show that, like Goldstein et al. (2013) and Dow et al. (2017), the feedback effect has the potential to create strategic complementarities across investors. In Goldstein et al. (2013), this complementarity arises through trading behavior, whereas in our model

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[9]For theoretical papers, see, for example, Goldstein and Guembel (2008), Goldstein, Ozdenoren, and Yuan (2013), Edmans, Goldstein, and Jiang (2015), Hirshleifer, Subrahmanyam, and Titman (2006), Bond and Eraslan (2010), Sockin and Xiong (2015), Sockin (2015), Schneemeier (2015), Yang (2019) and Benhabib, Liu, and Wang (2019) among many others.

[10]For empirical evidence, see, for example, Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Frésard (2012). Relatedly, Ozoguz and Rebello (2013), Foucault and Fresard (2014); Foucault and Frésard (2018), and Dessaint, Foucault, Frésard, and Matray (2018) provide evidence that firms learn from the stock price of their product-market peers.

[11]In contrast, investors have price impact and act strategically in Goldstein and Guembel (2008), Edmans et al. (2015) and Boleslavsky, Kelly, and Taylor (2017).

(and in Dow et al. (2017)), this arises through the information acquisition decision of investors. Unlike Dow et al. (2017), however, we assume the firm has existing assets, which we show is crucial in determining under what conditions complementarity arises.

In the presence of risky debt, Myers (1977) argued that equity holders may forego investing in projects with positive returns (when gains accrue to debt holders) while (Jensen and Meckling (1976)) suggested that equity holders may prefer to invest in risky projects, even if inefficient (when losses are borne by debt holders). We show that when firm managers learn from prices both activities are reduced; however, once we account for investors' endogenous learning, we show that this should affect risk-shifting more than debt overhang. The latter pattern is consistent with the empirical literature, including Mello and Parsons (1992), Parrino and Weisbach (1999), and Moyen (2007), who find evidence of debt overhang, while Andrade and Kaplan (1998), Rauh (2008) and Gilje (2016) find little evidence for risk-shifting. In a related paper, which combines both the feedback effect and measures of investment efficiency, Strobl (2014) shows that when investor's information choice is endogenously determined, the use of stock price-based incentive contracts can lead to over-investment. As a result, committing to invest in a negative-NPV project may be ex-ante optimal if it incents investors to acquire more information. In our paper, we do not solve for the optimal contract between the manager and firm and assume that the manager uses the information in prices to maximize the payoff to equity.

The existing theoretical literature has suggested other explanations for why risk-shifting may not arise. In dynamic settings, both Diamond (1989) and Hirshleifer and Thakor (1992) consider the impact of reputational concerns on investment decisions. Similarly, Almeida, Campello, and Weisbach (2011) suggests that firms may reduce risk today so that positive-NPV projects can be funded in the future. We analyze a static setting and emphasize the role that contemporaneous prices (instead of access to future opportunities) can play in affecting agency problems; in contrast to the existing literature, however, our model also predicts an asymmetric impact on risk-shifting relative to debt overhang.

Finally, our model focuses on the conflict between bond holders and equity holders; as a result,

and unlike standard REE models in which prices and cash flows are linear, our framework must allow for non-linear claims (i.e., debt and equity). As such, it is most closely related to Davis (2019), Albagli et al. (2015); Albagli, Hellwig, and Tsyvinski (2017) and Chabakauri, Yuan, and Zachariadis (2016). In this paper, we extend the model of Davis (2019). While both papers emphasize the importance of endogenous investor information acquisition, the focus of Davis (2019) is the firm's optimal issuance policy (post-investment) while we examine the firm's investment decision. Moreover, our extension allows for feedback between the manger's investment decision and the price, a feature Davis (2019) does not consider. Albagli et al. (2017) does explore the implications of a systematic wedge between prices and cash flow expectations for corporate risk-taking and investment. While our paper also features a similar wedge, our main result is not driven by this feature: instead, it arises from investor's endogenous learning.

## 2 The Model

### 2.1 Model Setup

There are three dates, $t \in \{0, 1, 2\}$, and two states of the world, $s \in \{L, H\}$. A firm owns a risky asset which generates a payoff, $x$, at date 2; this asset represents the firm's assets in place. The distribution of this payoff is state-dependent: $x \sim G_H$ (in the high state) or $x \sim G_L$ (in the low state), where (i) both $G_s$ are non-degenerate distributions with continuous support and (ii) $G_s(x) = 0$ for all $x < 0$.[12] Agents in the model do not know $q \equiv \mathbb{P}[s = H]$ with certainty, but at date zero know that

$$q = \Phi[z] \quad \text{where} \quad z \sim \mathcal{N}(\mu_z, \tau_z^{-1}) \tag{1}$$

and $\Phi$ is the CDF of a standard normal distribution.

At date one, the firm also has access to a zero-cost, risky investment. At date two, the investment

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[12]The assumptions of limited liability ($G_s(x) = 0$ for all $x < 0$) and continuity are without loss of generality and greatly simplify the expressions in our proofs.

generates a state-dependent cash flow of $y_s$ which is known to all agents.[13] We assume that the total distribution of cash flows in the high state first-order stochastically dominates the total distribution of cash flows in the low state, with or without investment.[14] As a result, given an agent's information set, $\mathcal{F}$, the NPV of the project can be written:

$$NPV(\mathcal{F}) = \mathbb{E}[q|\mathcal{F}]y_H + (1 - \mathbb{E}[q|\mathcal{F}])y_L. \tag{2}$$

If the payoffs in both states $y_H$ and $y_L$ are positive (negative), it is always (never) optimal to invest, eliminating any potential feedback effect. This leaves two non-trivial cases to consider.

**Case 1** $(y_H > 0 > y_L)$: In this case, the payoff to investment is positively correlated with the cash flows generated by the assets in place.[15] Such an investment could be viewed as an amplifying investment on the payoff of the firm's assets-in-place. Under this assumption, investment is efficient (i.e., $NPV(\mathcal{F}) > 0$) if and only if

$$\mathbb{E}[q|\mathcal{F}] > \frac{-y_L}{y_H - y_L} \equiv K_0. \tag{3}$$

**Case 2** $(y_H < 0 < y_L)$: In this case, there is a negative correlation between the cash flows of the project and those generated by the existing asset. Such an investment could be viewed as a corrective action taken by the firm, similar to that described in Bond et al. (2009). In particular, the benefit of this corrective action is high when the firm's fundamentals are low (similar to a standard insurance claim). Under this assumption, investment is efficient if and only if $\mathbb{E}[q|\mathcal{F}] < K_0$, i.e., if the likelihood of the low state is sufficiently high.

In a first-best world, the firm would follow the decision rules above. We assume, however, that the

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[13]The assumption of zero cost is isomorphic to a setting in which the required investment (denoted $I_y$) is non-zero but can be made using the firm's existing cash (contained in the assets-in-place), i.e., it does not require equity holders to contribute additional capital. In Section 5, we discuss the implications of relaxing this assumption. We assume that $y_s$ is constant for tractability.

[14]Specifically, we assume that $G_L(x + y_L) > G_H(x + y_H)$ and $G_L(x) > G_H(x)$, which ensures the existence of a financial market equilibrium.

[15]Alternatively, it could be that the degree of correlation here represents the extent to which the *information* about the existing asset's payoff is correlated with the investment.

investment decision is made by a risk-neutral manager who maximizes the equity payoff.[16] As a result, he makes his investment decision based upon its impact on the expected value of equity which will not, in general, follow the first-best policy. The manager takes as given the firm's capital structure: specifically, outstanding equity and any previously issued debt.[17] Without loss of generality, we assume this outstanding liability is zero-coupon debt with face value $D$ due at date two.

In addition to the manager, there exists a unit-measure continuum of risk-neutral investors who trade equity at date one.[18] At date zero, investors share the manager's prior beliefs about all variables in the economy. Investors are subject to position limits; specifically, they can buy no more than one share and cannot short.[19] Finally, each investor also has access to a private signal about the payoff's expected value. Specifically, investor $i \in [0,1]$ observes

$$ s_i = z + \varepsilon_i \quad \varepsilon_i \sim \mathcal{N}(0, \tau_i^{-1}). $$

Each investor can choose the precision of his signal ($\tau_i > 0$) subject to a cost function, $C(\tau_i)$. We assume only that the cost function possesses standard characteristics: $C$ is continuous, $C(0) = C'(0) = 0$, and $C', C'' > 0$ for all $\tau_i$. The cost function is identical across investors. Figure 1 summarizes the evolution of the model.

Figure 1: Time-line of events

| Date 0 | Date 1 | Date 2 |
|---|---|---|
| Investors choose signal precisions subject to C(.) | (i) Investors privately observe signals $s_i$ and trade (ii) Manager observes price and makes investment decision | Assets-in-place and investment (if made) pay off and distributed to stakeholders |

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[16]This is equivalent to assuming that (i) the manager's payoff is an affine function of the date two payoff of equity and that (ii) any principal-agent conflicts between equity holders and the manager are eliminated by the manager's contract with the firm. We explore how the manager's incentives to invest change when his compensation is more short-term (dependent upon the date one price) in section 5.

[17]This debt may have been previously issued to finance the existing cash flow. In Davis (2019), the issuance of such risky debt is generically optimal.

[18]We assume that the debt is held privately, for instance, by a bank. As we show in the next section, the existence of a financial market equilibrium requires monotonicity in security prices which is violated by a debt claim in the presence of agency conflicts. In section 5, we argue that the presence of traded debt would serve to amplify our results.

[19]Both assumptions are without loss of generality in terms of our main comparative static: the effect of investment on information acquisition. We formally relax the assumption of short sale constraints in the appendix.

To simplify our notation, we define

$$E_s(\delta) \equiv \int_{D-\delta}^{\infty} (x + \delta - D)dG_s(x)$$

for $s \in \{H, L\}$, which denotes the expected value of equity in state $s$, conditional on the manager's investment decision. Specifically, $E_s(0)$ denotes the state-dependent value of equity absent investment, while $E_s(y_s)$ denotes the state-dependent value of equity after the investment has been made. For tractability, we also rewrite the impact of investment on the value of equity. Specifically, given any pair $y_L$ and $y_H$, it is without loss of generality to define a pair $\alpha$ and $\gamma$ which satisfies the following equations:

$$E_H(y_H) - E_H(0) \equiv \alpha(1 + \gamma) \tag{4}$$

$$E_L(y_L) - E_L(0) \equiv -\alpha. \tag{5}$$

It is straightforward to show that these equations imply that $\gamma > -1$ while (i) $\alpha > 0$ in case 1 and (ii) $\alpha < 0$ in case 2. Going forward, we will use this notation when making reference to each case.

## 2.2 Financial Market Equilibrium Absent Feedback

For intuition, we begin by analyzing the financial market equilibrium while shutting down the feedback effect. This implies that, for investor $i$ (with filtration $\mathcal{F}_i$ ), the value of equity can be expressed as

$$V_E(\mathcal{F}_i) = \begin{cases} E_L(0) + \mathbb{E}[q|\mathcal{F}_i] \left( \underbrace{E_H(0) - E_L(0)}_{\equiv \Delta E(\mathbf{0})} \right) & \text{absent investment} \\[4mm] E_L(y_L) + \mathbb{E}[q|\mathcal{F}_i] \left( \underbrace{E_H(y_H) - E_L(y_L)}_{\equiv \Delta E(\mathbf{y})} \right) & \text{with investment.} \end{cases} \tag{6}$$

The sensitivity of each investor's valuation with respect to their private information, $\mathbb{E}[q|\mathcal{F}_i]$, is captured by $\Delta E$. As a result, we will henceforth refer to $\Delta E$ as the **information sensitivity of**

**equity**. By analogy, we define the **information sensitivity of investment** as $|y_H - y_L|$.[20] With this definition in mind, it is straightforward to show the following.

**Lemma 2.1.** *The information sensitivity of equity $(\Delta E(\mathbf{y}))$ is increasing in $y_H$, decreasing in $y_L$ and hence is increasing in $\alpha$.*

It is easy to see that we can write the information-sensitivity of equity with investment as

$$\Delta E(\mathbf{y}) = \Delta E(\mathbf{0}) + \alpha \underbrace{(2 + \gamma)}_{>0}. \tag{7}$$

As a result, the information sensitivity of equity increases with case 1 investment (i.e., $\alpha > 0$): the expected value of equity is increasingly dependent upon the likelihood of the high state about which investors have information. For similar reasons, a case 2 investment (i.e., $\alpha < 0$) necessarily reduces the information sensitivity of equity. We emphasize that it is this relationship between investment and information sensitivity that is crucial in generating our predictions regarding the relative prevalence of different agency conflicts. We will return to this intuition in section 4.

In order to keep the price of equity from being fully revealing, we assume that there are also noise traders in the market who demand a fraction $\Phi(u)$ units of the outstanding equity; their demand is price-independent. We assume that $u \sim \mathcal{N}(0, \tau_n^{-1})$ and is independent of all other random variables. We focus on a symmetric equilibrium in which all investors choose the same signal precision, i.e., $\tau_i = \tau_e$ for all $i \in [0, 1]$.

We will conjecture and verify that investors can construct a signal $s_p$ of precision $\tau_p$ from the price of equity, and that this signal will be normally-distributed and independent of $s_i$, conditional upon the true value, $z$. Under this conjecture, each investor believes:

$$z | s_i, s_p \sim \mathcal{N}\left( \frac{\tau_z \mu_z + \tau_e s_i + \tau_p s_p}{\tau_z + \tau_e + \tau_p}, \frac{1}{\tau_z + \tau_e + \tau_p} \right). \tag{8}$$

Investor beliefs can be ordered by their private signals and so we posit a threshold strategy: an

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[20]The NPV of the investment is $y_L + \mathbb{E}[q|\mathcal{F}](y_H - y_L)$. Under FOSD, $E_H > E_L \implies |\Delta E| = \Delta E$.

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investor purchases one unit of equity if $s_i \geq h(z,u)$; otherwise, they hold only the risk-free security (with return normalized to one). Note that the threshold is a function of both fundamentals ($z$) as well as the realized liquidity shock ($u$). We normalize the outstanding supply of equity to one and impose market-clearing:

$$1 = \underbrace{[1 - \Phi\left(\sqrt{\tau_e}\left(h(z,u) - z\right)\right)]}_{\text{total demand by investors}} + \underbrace{\Phi(u)}_{\text{liquidity demand}}.$$

Rewriting this expression shows that markets clear if and only if $h(z,u) = z + \frac{u}{\sqrt{\tau_e}}$. Moreover, the marginal investor, whose information set is $\{s_i = h(z,u), p_E\}$, sets the price equal to his conditional expectation, given the investment decision:

$$p_E = \begin{cases} E_L(0) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{0}) & \text{absent investment} \\ \\ E_L(y_L) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{y}) & \text{with investment.} \end{cases} \tag{9}$$

Given that $\Delta E > 0$, the above price function is monotonic in $h(z,u)$. It is clear, therefore, that $h(z,u)$ is recoverable from the price, and so we write $s_p \equiv h(z,u)$. Moreover, $s_p$ is normally-distributed, with precision $\tau_p = \tau_e \tau_n$ and mean $z$. This verifies our conjecture. This also implies that price informativeness, $\tau_p$, increases when investors have more precise information (high $\tau_e$) and when noise trading intensity is low (high $\tau_n$).[21]

# 3 Feedback Effect Equilibrium

If the manager is able to condition on the price prior to making his investment decision, then a feedback loop is generated. Specifically, the information sensitivity of the security is a function of the manager's investment decision, which depends upon the information contained in the price.

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[21]The existence of this equilibrium is established in Davis (2019) which utilizes the more general existence result established in Albagli et al. (2017).

## 3.1  Financial Market Equilibrium

We begin by analyzing the manager's investment decision, taking the investors' information acquisitions decision as given. The net present value of investment for equity holders is denoted by

$$NPV_E(\mathcal{F}) \equiv \mathbb{E}[q|\mathcal{F}](E_H(y_H) - E_H(0)) + (1 - \mathbb{E}[q|\mathcal{F}])(E_L(y_L) - E_L(0)). \tag{10}$$

Given that the manager maximizes the value of equity, he invests when $NPV_E(\mathcal{F}_m) > 0$. This leads to the following result:

**Lemma 3.1.** *In case 1 (i.e., $\alpha > 0$), the manager with information set $\mathcal{F}_m$ invests iff*

$$\mathbb{E}[q|\mathcal{F}_m] > \frac{E_L(0) - E_L(y_L)}{\Delta E(\boldsymbol{y}) - \Delta E(\boldsymbol{0})} = \frac{1}{2 + \gamma} \equiv K. \tag{11}$$

*Similarly, in case 2 (i.e., $\alpha < 0$), the manager with information set $\mathcal{F}_m$ invests iff $\mathbb{E}[q|\mathcal{F}_m] < K$.*

In case 1, $\alpha > 0$: as a result, the value of equity increases in the high state and decreases in the low state, and the manager invests if and only if the high state is sufficiently likely. On the other hand, a case 2 investment (in which $\alpha < 0$) yields positive returns in the low state only. As a result, the manager must be sufficiently pessimistic about the likelihood of the high state to invest. This reverses the cutoff for investment: $\mathbb{E}[q|\mathcal{F}_m] < K$. Finally, we note that the manager's threshold belief (K) is common knowledge amongst all agents in the economy.[22] We conjecture that the manager can extract a signal $s_p \sim \mathcal{N}\left(z, \tau_p^{-1}\right)$ from the price. Under this assumption, and using lemma 7.1 (found in the appendix), the manager's belief about the likelihood of the high state, given $s_p$, can be written:

$$\mathbb{E}[q|s_p] = \Phi\left(\frac{\tau_z \mu_z + \tau_p s_p}{\sqrt{(\tau_z + \tau_p)(1 + \tau_z + \tau_p)}}\right). \tag{12}$$

This implies that in case 1, for instance, the manager only invests if the signal he obtains from the

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[22]This holds because the manager and investors share common prior beliefs. In section 8.2, we relax this assumption and assume that the manager has private information about the asset's payoff in each state.

price is sufficiently optimistic; specifically, it must be that

$$s_p > \frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z + \tau_p)(1 + \tau_z + \tau_p)}\right] - \tau_z\mu_z}{\tau_p} \equiv f(K, \tau_p). \tag{13}$$

We then conjecture that investors are able to condition on the same information as the manager - they, too, can extract $s_p$ from the price. As a result, they *know with certainty above what price the manager will choose to invest* when they submit their demand schedules. For instance, investment will only occur in case 1 if the belief of the marginal investor is sufficiently optimistic, i.e., if

$$q_E > \underline{q_E} \equiv \Phi\left(\frac{\tau_z\mu_z + (\tau_e + \tau_p)f(K, \tau_p)}{\sqrt{(\tau_z + \tau_e + \tau_p)(1 + \tau_z + \tau_e + \tau_p)}}\right), \tag{14}$$

where $q_E \equiv \mathbb{E}[q|s_i = h(z, u), p_E]$.

Note that each investors' conditional valuation of the equity claim remains monotonic in their belief about the true value of $q$. In case 1, as $\mathbb{E}[q|s_i, p_E]$ increases, the expected value of the assets-in-place increases; moreover, if $p_E$ is sufficiently high (equivalently, if $s_p$ is sufficiently high), the manager invests, further increasing both the expected value of equity as well as the information sensitivity. In case 2, as $\mathbb{E}[q|s_i, p_E]$ decreases, the expected value of the assets in place decreases; however, when $p_E$ is sufficiently low (equivalently, if $s_p$ is sufficiently low), the manager invests, which increases the expected value of equity, *relative to the value absent investment*. However, such investment also *lowers* the information sensitivity of equity.

As above, we posit a threshold strategy in which investor $i$ purchases equity if and only if $s_i \geq h(z, u)$. Following the same steps, the price of equity is again the marginal investor's conditional value (whose belief about the likelihood of the high state is $q_E$), only now his valuation accounts for the manager's endogenous investment decision. For instance, in case 1, we write

$$p_E = \begin{cases} E_L(0) + q_E\Delta E(\mathbf{0}) & \text{if } q_E \leq \underline{q_E} \\ E_L(y_L) + q_E\Delta E(\mathbf{y}) & \text{if } q_E > \underline{q_E}. \end{cases} \tag{15}$$

As we show in the proof of Proposition 3.2, $q_E$ (and hence $s_p \equiv h(z, u)$) remains recoverable, verifying our conjecture regarding the information contained in the price.

**Definition 1.** *A Perfect Bayesian Equilibrium (PBE) with feedback consists of functions $d(s_i, p_E)$, $p_E(z, u)$, an optimal investment decision for firm managers such that (i) $d(s_i, p_E)$ is optimal given posterior beliefs; (ii) firm managers decision to invest is optimal given information in prices; (iii) the asset market clears for all $(z, u)$; and (iv) posterior beliefs satisfies Bayes rule whenever applicable.*

**Proposition 3.2.** *In case 1 and 2, a unique PBE with feedback exists when $\mu_z < \bar{\mu}_z$, defined in equation 28. In equilibrium, price informativeness is given by $\tau_p = \tau_e \tau_n$.*

Equilibrium existence requires a monotonic price function so that agents can invert the price function and extract $s_p$. Equation 28 (found in the Appendix) ensures that the price weakly increases at the cutoff point which is necessary for the price function to be monotonic (and hence invertible). Such a condition is necessary because the manager's investment decision is made to maximize his expectation of the *terminal payoff* to equity, not the date one price.[23] As in Bond et al. (2009), the manager and investors possess different information sets: in addition to the signal $s_p$, observed by the manager, the marginal investor also possesses a private signal $s_i$. Thus, $\mathbb{E}[q|s_p]$ will not generally equal $q_E = \mathbb{E}[q|s_i = h(z, u), s_p]$ and as a result, while the manager is indifferent between investing and abstaining at the cutoff, $\underline{q_E}$, the marginal investor will not be.[24] We show in the proof of the proposition that, despite this asymmetry, conditions exist on the primitives of the model to ensure equilibrium existence.
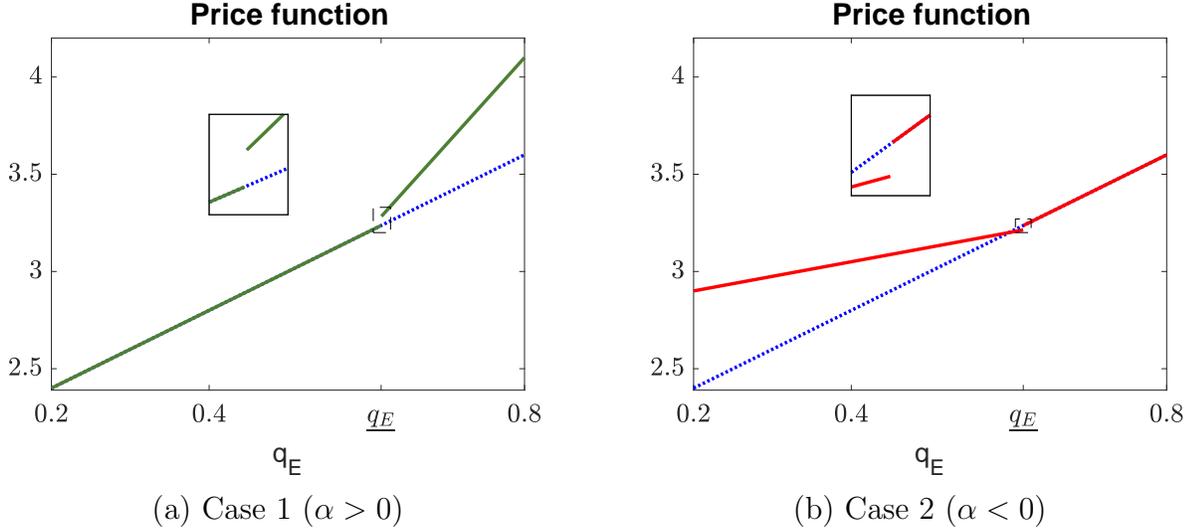
Figure 2 plots the price function in both cases. In both panels, the price function is monotonic in $q_E$, but exhibits a discontinuity at the threshold belief $(\underline{q_E})$ for the reasons described above. In the first panel (case 1), the price steepens above $\underline{q_E}$, i.e., it becomes more sensitive to investors' private information because the manager invests in this region. On the contrary, in the second case (panel

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[23]We show in section 5 that our key comparative statics are preserved in a setting where the manager instead maximizes the price of equity at date one. However, we show that the price function in that setting is always continuous.

[24]This feature is not generated by the particular distributional assumptions of our model but follows from the difference in the information sets.

(b)), the manager invests when $q_E < \underline{q_E}$. Such investment decreases information sensitivity and so the price function flattens in this region.

Figure 2: **Equity Price**



(a) Case 1 ($\alpha > 0$)

(b) Case 2 ($\alpha < 0$)

The figure plots the price of equity as a function of the marginal investor's belief for both cases. The solid line indicates the price path with the feedback effect. The dotted line indicates the hypothetical price absent the feedback effect, i.e., without investment. The relevant parameter values are $\tau_e = \tau_Z = \tau_n = 1$, $\mu_z = 0.2$, $E_L(0) = 2; E_H(0) = 4$. In case 1, $E_L(y_L) = 0.5; E_H(y_H) = 5$; in case 2, $E_L(y_L) = 2.75; E_H(y_H) = 3.5$.

## 3.2 Endogenous Information Equilibrium

Having established the existence of a financial market equilibrium at date one, we now analyze the investor's incentive to acquire information at date zero. The conditional expectation of an investor who observes private signal $s_i$, with precision $\tau_i$, is given by

$$q_i = \mathbb{E}[\Phi(z)|s_i, s_p] = \Phi\left(\frac{\tau_z\mu_z + \tau_i s_i + \tau_p s_p}{\sqrt{\psi_i(1 + \psi_i)}}\right)$$

where $\psi_i = \tau_z + \tau_i + \tau_p$.[25] Recall that investors (i) differ only in their beliefs about $q$ (through $s_i$) and (ii) purchase the asset only if their expectation of $q$ exceeds that of the marginal investor. Using

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[25]This expression is derived using Lemma 7.1, found in the appendix.

these observations, we can write the investor's expected utility from trade; for instance, in case 1,

$$EU(\tau_i, \tau_p) = \mathbb{E}\left( (q_i - q_E) \underbrace{\mathbb{1}_{q_i > q_E}}_{\substack{\text{buy if} \\ q_i > q_E}} \left[ \Delta E(\mathbf{0}) \overbrace{\mathbb{1}_{q_E < \underline{q_E}}}^{\substack{\text{Do not} \\ \text{Invest}}} + \Delta E(\mathbf{y}) \overbrace{\mathbb{1}_{q_E > \underline{q_E}}}^{\text{Invest}} \right] \right). \tag{16}$$

An investors' trading gains can be decomposed into the difference between his beliefs and those of the marginal investor (the first term of equation 16) and the information sensitivity, with and without investment (the term in square brackets).

**Proposition 3.3.** *The marginal value of acquiring information is positive and increasing in $\alpha$.*

Learning is always valuable for investors. Moreover, the value of private information for each investor is increasing in the information sensitivity of equity. Consider the first case, in which the investment payoff is positively correlated with assets-in-place. As $\alpha$ increases, the information sensitivity of equity (conditional on investment) increases which, in equilibrium, increases the marginal value of learning. On the other hand, in the second case, the investment payoff is negatively correlated with assets-in-place. As $\alpha$ decreases, the information sensitivity of equity (conditional on investment) falls, lowering the marginal value of information acquisition for investors.

We now establish the existence of an information acquisition equilibrium. Each investor chooses $\tau_i$ to maximize $EU(\tau_i, \tau_p) - C(\tau_i)$, taking the precision chosen by all other investors choices as given.[26] We will solve for a symmetric equilibrium in which all investors acquire signals of the same precision, i.e. $\tau_i = \tau_e$ for all $i \in [0, 1]$.

**Proposition 3.4.** *There is a unique, symmetric equilibrium in information acquisition as long as $\frac{\partial^2 EU}{\partial \tau_i \partial \tau_p} < 0$, i.e., as long as information acquisition exhibits substitutability aross investors.*

In the appendix, we prove formally that settings in which agency problems can arise necessarily exhibit substitutability and therefore there is a unique information acquisition equilibrium. In Section 6, however, we analyze those settings in which complementarity can arise and consider its implications.

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[26]Specifically, as shown in proposition 3.2, $\tau_p = \tau_e \tau_n$, where $\tau_e$ is the precision chosen by all other investors.

# 4 Learning and Debt-Equity Conflicts

We turn now to our main analysis: the effect of endogenous information acquisition, in combination with managerial learning from prices, on investment efficiency. Our focus is on two oft-studied settings which arise in the presence of risky debt: risk-shifting, as in Jensen and Meckling (1976) and debt overhang, as in Myers (1977). We follow the conventions of the literature in defining both.

**Definition 2. Risk-shifting** *arises when an inefficient investment increases the value of equity ($NPV_E(\mathcal{F}) > 0 > NPV(\mathcal{F})$), while* **debt overhang** *arises when an efficient investment lowers the value of equity ($NPV_E(\mathcal{F}) < 0 < NPV(\mathcal{F})$).*

We begin by establishing under what assumptions such agency conflicts can arise in our model.

**Lemma 4.1.** *Risk-shifting is only possible in case 1. Debt overhang is only possible in case 2.*

When investment success is positively correlated with the value of existing assets (case 1), equity holders earn a larger share of the payoff contingent upon success but, in the presence of risky debt, absorb a lower share of the loss if the project fails. As a result, a project may be viewed favorably by equity holders but not debt holders (or a social planner): risk-shifting but not debt overhang can arise. On the other hand, when investment success is negatively correlated with the value of assets in place (case 2), the holders of risky debt may be able to claim a larger share of the payoff when the project succeeds, while absorbing a smaller share of the loss. As a result, a project which is viewed favorably by debt holders (or a social planner) may not be chosen by the manager, who holds equity, i.e. the firm may exhibit debt overhang.

Finally, in what follows, we aim to understand how managerial learning from prices affects the likelihood of investment when projects differ in their riskiness and efficiency. In our setting, $\alpha$ is a natural choice. As $|\alpha|$ increases, project uncertainty rises, while an increase in $\alpha$ effectively transfers cash flows from the low state to the high state, capturing the project's relative efficiency from the perspective of an equity holder.[27],[28] Moreover, if $\gamma$ is chosen appropriately, it is straightforward to

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[27]In an ideal world, we would be able to do comparative statics on the $NPV$ or $NPV_E$ of the potential investment; however, both objects are functions of many variables, and taking the partial derivative with respect to a function of many variables is not a well-defined object.

[28]In what follows, a change in $\alpha$ leaves the investment threshold, $K$, fixed but changes both the $NPV$ and $NPV_E$

show that an increase in $\alpha$ also lowers investment efficiency. We establish this mapping formally in the following lemma.

**Lemma 4.2.** *If $\underline{\gamma} < \gamma < \bar{\gamma}$, then (i) $NPV(\mathcal{F}_0) < 0 < NPV_E(\mathcal{F}_0)$ when $\alpha > 0$, and (ii) $NPV(\mathcal{F}_0) > 0 > NPV_E(\mathcal{F}_0)$ when $\alpha < 0$. Moreover,*

$$\frac{\partial NPV(\mathcal{F}_0)}{\partial \alpha} < 0 \quad and \quad \frac{\partial NPV_E(\mathcal{F}_0)}{\partial \alpha} > 0. \tag{17}$$

Going forward, we will restrict our analysis such that the lemma's conditions hold.[29] Intuitively, in case 1 (i.e., $\alpha > 0$), while equity holders benefit from successful outcomes of high-risk (high $\alpha$) projects, the losses from unsuccessful outcomes are borne by debt holders. Furthermore, not only is there a transfer of wealth from debt holders to equity holders but there is a reduction in enterprise value - as $\alpha$ increases, these projects becomes increasingly more socially inefficient. Thus, when $\alpha > 0$, **higher $\alpha$ denotes increasingly inefficient examples of risk-shifting.**

Similarly, in case 2 (i.e., $\alpha < 0$), while debt holders benefit from successful outcomes of high-risk (low $\alpha$) projects, the losses from unsuccessful outcomes are borne by equity holders. Furthermore, not only is there a transfer of wealth from equity holders to debt holders but there is a increase in enterprise value - as $\alpha$ decreases, these projects becomes increasingly more socially efficient. As a result, when $\alpha < 0$, **lower $\alpha$ denotes increasingly inefficient examples of debt overhang.**

## 4.1 Feedback Effect & Efficiency Metrics

The following corollary to Lemma 4.1 establishes that managerial learning from prices (i.e., the feedback effect) generically reduces the extent of the agency conflict by providing useful information to the manager.

**Corollary 4.3.** *If $\tau_p > 0$, the feedback effect eliminates some inefficient investment decisions.*

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of the project. In the online appendix, we conduct a similar analysis which fixes the $NPV_E$ and alters the $NPV$ (and $K$). Our results are qualitatively unchanged.

[29]We relax this assumption in section 6 when we analyze complementarity in information acquisition.

Allowing the manager to condition on the price (in case 1) can eliminate some cases of risk-shifting by providing information which discourages the manger from making the investment. Similarly, the feedback effect can eliminate some examples of debt overhang (in case 2) by providing sufficiently positive information such that the manager chooses to invest. The extent to which such projects are eliminated depends upon the quality of the information in the price which, as we analyze below, depends upon the project's characteristics.

The proof of Lemma 4.1 shows that in the presence of risk-shifting opportunities (case 1), any time the manager chooses not to invest, it is the socially efficient decision. Therefore, in determining the degree of inefficiency generated by risk-shifting, we want a measure which captures how often the manager *knowingly* chooses an inefficient investment. In case 2, however, when debt overhang is possible, the proof indicates that any investment chosen by the manager must also be socially efficient. In determining the inefficiency generated by debt overhang, we need a measure which captures how often an efficient investment is foregone knowingly. In both cases, our metric should respect the segmentation of information in the economy - it should only be based on what *firm managers* know when they make the investment decision.

We propose two such metrics. The first is the ex-ante likelihood that the manager's investment decision (with information set $\mathcal{F}_m$) is socially inefficient:

$$P1 \equiv \mathbb{P}\left(NPV_E(\mathcal{F}_m) > 0 \text{ and } NPV(\mathcal{F}_m) < 0\right) \tag{18}$$

$$= \mathbb{P}\left(K_0 > E(q|s_p) > K\right). \tag{19}$$

Note that $K_0$ is the socially efficient threshold derived in Section 2.[30] In the presence of risk-shifting opportunities (case 1), $P1$ is the ex-ante likelihood that the investment undertaken by the firm manager is inefficient: the manager invests when the social planner would not. When debt overhang is possible (case 2), $P1$ is the ex-ante likelihood that a foregone investment is efficient: the manager abstains when the social planner would invest.[31]

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[30]The proof of Lemma 4.1 shows that $K < K_0$ and, as a result, this probability is always non-zero.

[31]An econometrician can also observe prices (and therefore infer the manager's beliefs), suggesting that the empir-

The second metric is the ex-ante *conditional* probability that an investment decision is inefficient:

$$P2 \equiv \frac{\mathbb{P}\left(K_0 > \mathbb{E}\left[q|s_p\right] > K\right)}{\mathbb{P}(\mathbb{E}\left[q|s_p\right] > K)}. \tag{20}$$

When the firm has access to a risk-shifting investment (case 1), $P2$ is the probability that a chosen investment is socially inefficient, given that an investment was made. When the opportunity for debt overhang arises (case 2), $P2$ is the probability that a foregone investment is socially efficient, given that the manager chose not to invest.

## 4.2   Investor Learning & Agency Problems

To determine the impact of the feedback channel on the equilibrium incidence of agency conflicts, we must account for investors' endogenous learning decision.[32] As 3.3 makes clear, investors acquire more information (i.e. $\tau_e$ increases) as the information sensitivity of equity increases (i.e., as $\alpha$ increases). As a result, the manager has access to more precise information through prices about the likelihood the investment pays off. As a result, the quality of this information (i.e., $\tau_p = \tau_e \tau_n$) directly impacts investment inefficiency, as summarized in the following proposition.

**Proposition 4.4.** *The ex-ante likelihood of an inefficient investment decision (both P1 and P2) decreases when investors acquire more information, i.e. as $\tau_p$ increases, if $\mu_z \in \left[\underline{\mu}, \overline{\mu}\right]$, where $\underline{\mu}, \overline{\mu}$ are defined in the appendix.*

We consider each case in turn. If the investment is positively correlated with assets-in-place (case 1), lemma 4.2 implies that the manager will surely invest absent the information contained in the price: with probability one, the manager's belief lies above his investment threshold ($K$), but below

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ically relevant measure should incorporate this conditioning information. However, for completeness, we also analyze an efficiency metric based on the ex-post realization of $z$:

$$P3 = \mathbb{P}\left(E\left(q|s_p\right) > K \text{ and } \Phi\left(z\right) < K_0\right).$$

This metric qualitatively follows the patterns we describe below.

[32]Proposition 6.1 shows that we are guaranteed a unique information acquisition equilibrium by focusing on the settings established in Lemma 4.2.

the efficiency threshold ($K_0$). As the price becomes more informative, the manager conditions more heavily on the price, which increases the variance of his posterior beliefs; this, in turn, decreases the probability that the firm manager's posterior belief falls in the range $[K, K_0]$. Further, as prices become more informative, such an investment is also more likely to be viewed as socially efficient so that, conditional on investment, it is less likely that the investment is inefficient. Similar logic applies in the second case. Now, absent the price signal, lemma 4.2 implies that the firm manager would never invest. As prices become more informative, managers are less likely to exhibit such debt overhang (because it less likely that $K < \mathbb{E}[q|s_p] < K_0$) and when investments are foregone they are more likely to be viewed as socially inefficient projects.

This logic implies that, regardless of the correlation between the investment and assets-in-place, the investment inefficiency (as measured by $P1$ and $P2$) decreases as $\tau_p$ increases. Moreover, consistent with Corollary 4.3, this proposition also implies that any managerial learning from prices reduces investment inefficiency. Finally, we emphasize that the restrictions on $\mu_z$ in Proposition 4.4 arise only because of the non-linear relationship between the information acquired and the expected payoff of the asset.[33]

With this last result in place, we can now state our main result.

**Proposition 4.5.** *Relative to a setting in which the precision of investors' signals is exogenously specified, when investors can choose how much information to acquire and*
*(1) as the opportunity for inefficient risk-shifting grows (an increase in $\alpha > 0$), the larger the reduction in observed investment inefficiency ($P1$ and $P2$);*
*(2) as the opportunity for inefficient debt overhang grows (a decrease in $\alpha < 0$), the smaller the reduction in observed investment inefficiency ($P1$ and $P2$).*

As described above, when the investment is positively correlated with assets-in-place (case 1), an increase in investment inefficiency *increases* the information sensitivity of equity. This, in turn,

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[33]Specifically, these restrictions ensure that the impact of the non-linearity, which manifests itself through Jensen's inequality as a change in the average conditional expectation, does not swamp the impact of learning, which increases the variation in conditional beliefs. Importantly, this is a restriction which arises due to the specific functional form of the $q$ and is not a restriction driven by the underlying economic mechanism.

increases the precision of the information acquired by investors, whose trade then increases the informativeness of priors. Finally, as Proposition 4.4 makes clear, when managers can condition on more precise information, investment inefficiency falls. In summary, the **likelihood of the most inefficient examples of risk-shifting decreases the most** due to endogenous learning.

On the other hand, when the investment is negatively correlated with assets-in-place (case 2), an increase in investment inefficiency *decreases* the information sensitivity of equity. This leads to a decrease in the precision of the information acquired by investors, reducing the quality of the information available in the price. As a result, investment inefficiency rises, i.e., the **likelihood of the most inefficient examples of debt overhang increases the most** due to endogenous learning.

We end this section illustrating our main results using two numerical examples.

## 4.3   Agency Problems: Numerical Examples

Figure 3: **Risk-shifting**



(a)                                    (b)
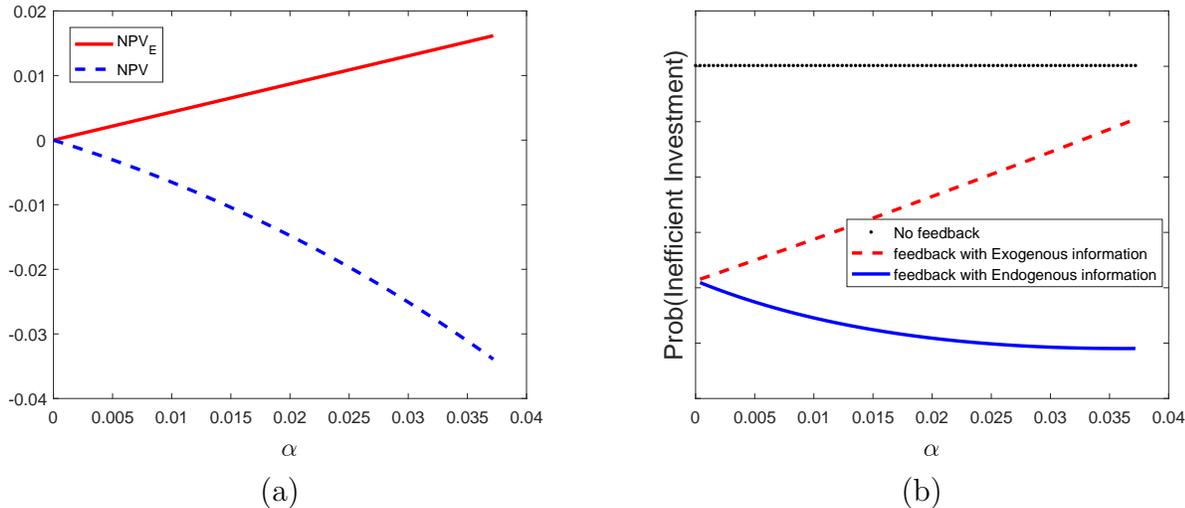
The figure plots the project NPV, $NPV_E$ and the probability of inefficient investment under different learning environments as a function of $\alpha$. Other key parameter values are set to: $\tau_Z = 0.5$, $\mu_z = 0.2$, $\tau_e = 0.4$, the distributions $G_H$ and $G_L$ are exponential with parameters $\lambda_H = 0.5$ and $\lambda_L = 1.5$ respectively, and $D = 1$.

In Figure 3, we consider a set of investment projects which are positively correlated with assets-in-place, thus generating the potential for risk-shifting. From panel (a), it is clear that $\alpha$ is indeed

a proxy for increasingly inefficient investments: as $\alpha$ increases, the $NPV$ of the project decreases. Panel (b) plots $P1$, the ex-ante probability of inefficient investment as defined in (19). Absent any feedback effect (e.g., if investors were unable to acquire private information), such projects are always undertaken (since $NPV_E(\mathcal{F}_0) > 0$); hence, $P1$ is a constant, 100% (dotted line). If the manager learns from prices, but the precision of investors' private information is exogenously specified (dashed line), the probability of inefficient investment is less than one: as noted above, conditioning on prices reduces investment inefficiency. Note that $P1$ increases with $\alpha$: while $K$ remains constant, the threshold for efficient investment, $K_0$, is increasing in $\alpha$ which increases the likelihood that an investment made was inefficient. Despite this countervailing force, however, when investors can choose how much information to acquire (solid line), we observe the opposite trend. The probability of *inefficient risk-shifting decreases as investment inefficiency increases*: increasingly inefficient projects induce investors to learn more information, which decreases the likelihood that the worst examples of risk-shifting are implemented.

Figure 4: **Debt overhang**
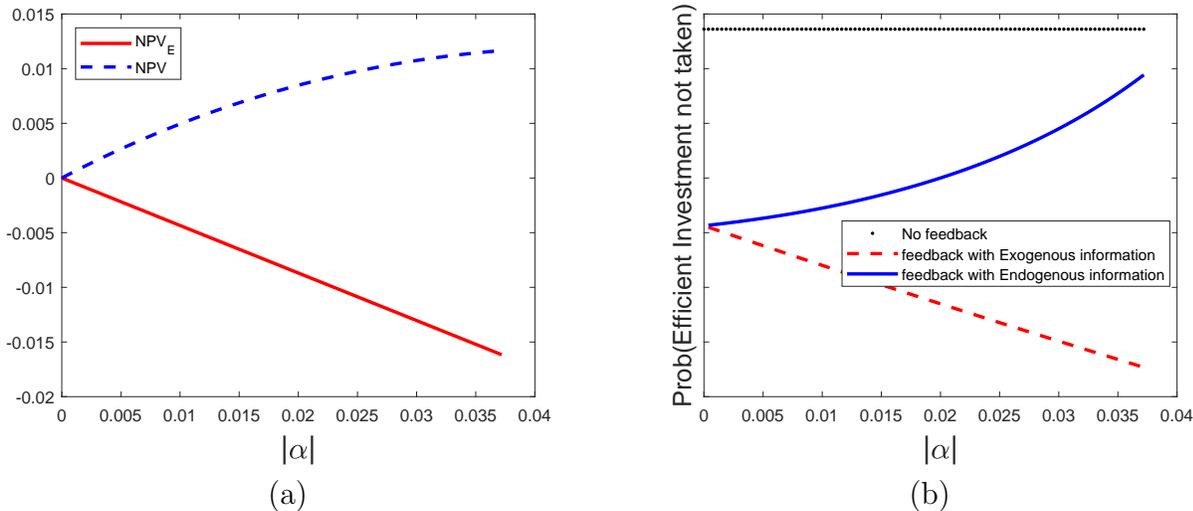


The figure plots the project NPV, $NPV_E$ and the probability of efficient investment not taken under different learning environments as a function of $|\alpha|$. Other parameter values are set to: $\tau_Z = 0.5$, $\mu_z = 0.2$, $t_E = 0.4$, the distributions $G_H$ and $G_L$ are exponential with parameters $\lambda_H = 0.5$ and $\lambda_L = 1.5$ respectively, and $D = 1$.

In Figure 4, we consider a set of investment projects which are negatively correlated with assets-in-place (i.e., $\alpha < 0$), creating the potential for debt overhang. Panel (a) illustrates that $|\alpha|$ is a proxy

for increasingly efficient investment projects: as $|\alpha|$ increases, the $NPV$ of the project increases. As above, panel (b) plots $P1$, but in this case it measures the probability of efficient investment *not taken*. If the manager cannot condition on prices, efficient projects are never undertaken (since $NPV_E(\mathcal{F}_0) < 0$) and $P1$ is a constant, 100% (dotted line). With (i) the feedback effect and (ii) and exogenous investor information (dashed line), efficient investments are less likely to be foregone. Specifically, while $K$ is fixed, decreasing $\alpha$ lowers $K_0$, which increases the likelihood that if an investment is foregone it is also inefficient. However, with endogenous investor information (solid line), the *probability of efficient investment not taken increases with project efficiency*: investors choose to learn less, increasing the likelihood that the worst examples of debt overhang persist.

## 4.4 Managerial Information Acquisition

The analysis in our benchmark model focuses on the impact of investors' information acquisition on the relative likelihood of risk-shifting and debt overhang. One natural question which arises is whether these results arise generally, when agent (e.g., manager) payoffs are correlated with an equity claim, or if there is something unique about learning in financial markets. To answer this question, we now analyze a setting in which the firm manager (who is implicitly compensated as a function of the equity payoff) can acquire information. To isolate the channel of interest, we shut down the financial market in what follows.[34]

Suppose the manager has access to the same information technology possessed by investors: he observes a signal, $s_m = z + \epsilon_m$ with $\epsilon_m \sim N(0, \tau_m^{-1})$, where $\tau_m$ is chosen by the manager at a cost, $C(\tau_m)$. As before, the manager invests if and only if he believes it increases the expected payoff to equity. For example, we show in the proof of Proposition 4.6, that in case one this is equivalent to investing if and only if

$$s_m > f(K, \tau_m), \tag{21}$$

where $f(\cdot, \cdot)$ is defined in equation (13). In case two, of course, the threshold is flipped: the manager

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[34]Alternatively, this is equivalent to assuming that the manager's information is a strictly finer partition of the available information.

invests if $s_m < f(K, \tau_m)$. With this threshold, we can then express the expected net present value of equity before observing $s_m$. This is

$$E[NPV_E(\mathcal{F}_m)] = \begin{cases} |\alpha| \int_{f(K,\tau_m)}^{\infty} \left\{ (2+\gamma) \Phi \left( \frac{\tau_z \mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1+\tau_z+\tau_m)}} \right) - 1 \right\} dF(s_m) & \text{in case 1} \\ |\alpha| \int_{-\infty}^{f(K,\tau_m)} \left\{ 1 - (2+\gamma) \Phi \left( \frac{\tau_z \mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1+\tau_z+\tau_m)}} \right) \right\} dF(s_m) & \text{in case 2.} \end{cases} \tag{22}$$

At date zero, the firm manager faces the following objective function:

$$\max_{\tau_m} \mathbb{E}[NPV_E(\mathcal{F}_m)] - C(\tau_m).$$

With this, we can now analyze how the manager's decision to learn varies with the type of investment.

**Proposition 4.6.** *In both cases, the optimal $\tau_m$ increases with $|\alpha|$, i.e., the manager learns more about more informationally-sensitive investments.*

**Corollary 4.7.** *Managerial learning reduces the likelihood of the most inefficient types of risk-shifting and debt overhang.*

This result stands in direct contrast to our earlier result. When investors can choose how much to learn, the effect on investment inefficiency is asymmetric: the worst examples of risk-shifting are further reduced, while the impact of the feedback channel on debt overhang is attenuated.

For intuition, consider the manager's problem. As is clear from (22), the benefit of aquiring information scales with $|\alpha|$. Thus, the manager's information acquisition scales with the uncertainty regarding the project's cashflows, i.e., the information sensitivity of the *project*. This is in contrast to investors' incentive, which increases with the information sensitivity of *equity*. In particular, the manager learns in order to decide whether or not to invest in the project; investors, however, learn in order to decide whether or not to invest in the firm's equity. Learning is valuable to the extent that it informs these choices. In contrast to investors, the manager's decision to invest depends only

upon the project characteristics.[35]

That this can lead to a difference in incentives is due to the sign of the projects' correlation with assets-in-place. In case 1, as $\alpha$ increases, the information sensitivity of the project and the equity claim increases: both the manager and investors choose to acquire more information. In case 2, as $|\alpha|$ increases, the information sensitivity of the project increases, but the information sensitivity of equity falls: the project is negatively correlated with assets-in-place. Hence, the manager's incentive to learn increases as the project becomes more efficient whereas investors choose to learn less.

In Figure 5, we plot the precision of the private signal chosen by the manager (dashed line) and the investor (solid line) when faced with identical cost functions. In panel (a), we consider a set of investment projects which are positively correlated with assets-in-place ($\alpha > 0$), thus generating the potential for risk-shifting. Both of them increase as *as investment inefficiency increases*: increasingly inefficient projects induce managers and investors to learn more information, which decreases the likelihood that the worst examples of risk-shifting are implemented. In panel (b), we consider a set of investment projects which are negatively correlated with assets-in-place ($\alpha < 0$), thus generating the potential for debt-overhang. Echoing corollary 4.7, the manager chooses to learn more about the most efficient projects; investors, on the other hand, reduce their information acquisition.

# 5    Discussion and Extensions

The specific assumptions we make are for analytic tractability and to highlight the underlying mechanism in the clearest manner. However, our main results are robust to a relaxation of these assumptions along multiple dimensions. We discuss these below while also noting several additional implications that may obtain.

Investors are risk-neutral in our model and so we constrain them to take positions that lie in the interval $[0, 1]$. In section 8.1, found in the appendix, we relax this assumption and allow investors to short sell. Doing so increases investors' incentive to learn: if an investor's private signal is less opti-

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[35]The manager's implicit compensation (as a function of the terminal payoff to equity) implies that he is effectively endowed with a claim to the assets-in-place.

Figure 5: **Managerial vs Investor learning**



(a) Case 1 ($\alpha > 0$)

(b) Case 2 ($\alpha < 0$)
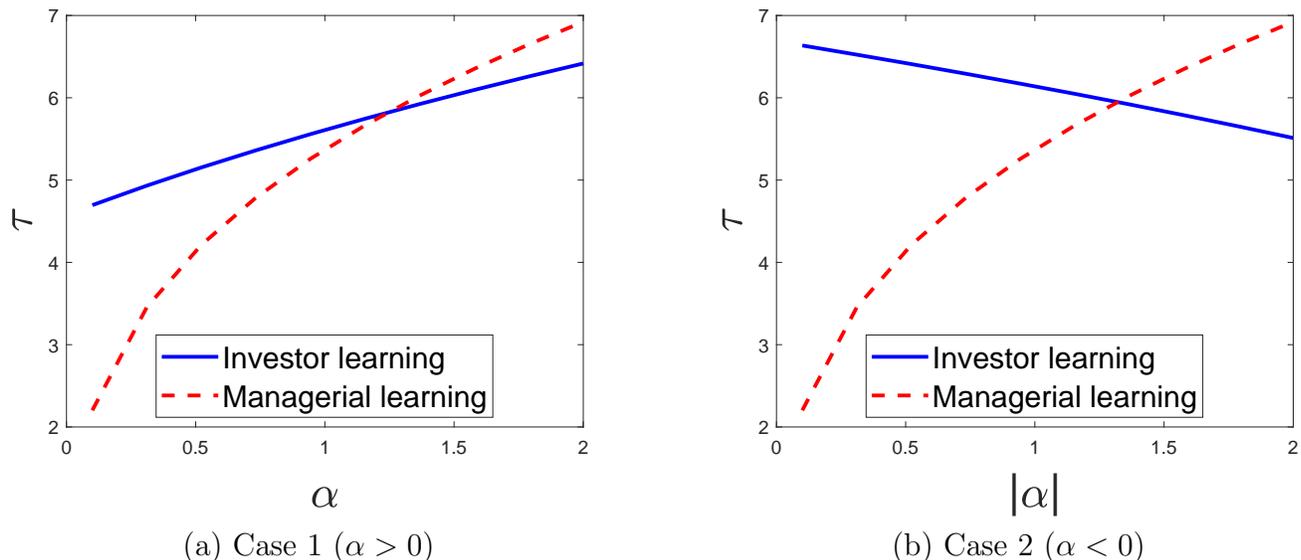
The figure shows the precision of the signal optimally chosen by the manager (dotted line) relative to that chosen by the investor (solid line) when both face identical cost functions. The left panel plots the precision chosen in the face of risk-shifting ($\alpha > 0$). The right panel plots the precision chosen in the presence of debt overhang ($\alpha < 0$). Other parameters are: $\tau_z = 1$, $\gamma = 0.2$, $\Delta E(0) = 2$ and cost function $C(\tau) = 0.0008\tau^2$.

mistic than the marginal investor's, he can now take a short position, increasing the ex-ante potential for trading gains. However, this incentive does not change the relative information acquisition across different types of projects: he still learns more as $\alpha$ increases, consistent with the results presented in the benchmark model.

We also assume that investors trade in a perfectly competitive market. As Edmans et al. (2015) makes clear, when investors are strategic (i.e., they account for the impact their trade has on the price), they respond asymmetrically to "good" and "bad" information. In particular, investors who are pessimistic about the investment's prospects will trade less aggressively on their private information. While this would reduce investors' ex-ante incentive to acquire information, the relative information acquisition across projects remains the same, preserving our main results.

In the benchmark model, we consider an economy in which the price of equity provides information to the firm manager. Given the role that risky debt plays in generating the investment inefficiencies we analyze, it is natural to consider the implications of traded debt in our setting. Above the

threshold ($\underline{q_E}$), however, the price of debt is discontinuous and necessarily non-monotonic: when the manager invests (case 1) or chooses to abstain (case 2), it is at the expense of debt holders. This precludes the existence of a financial equilibrium, preventing us from analyzing the impact of debt formally. However, we note that, like equity, the information sensitivity of debt is increasing in $\alpha$. This suggests that, if investors could also trade a debt claim, it would only serve to amplify the impact of endogenous learning.

In order to focus on the the impact of the feedback effect and investors' endogenous learning, we have chosen to work within a relatively simple framework which shuts down other frictions. For example, our analysis does not solve for the manager's optimal contract.[36] Instead, we take as given that manager chooses to maximize the terminal value of an equity (i.e., long-term compensation). One alternative is that the manager instead chooses to invest to maximize the date one equity price (i.e., short-term compensation). In such a setting, the equilibrium price is always monotonic: the manager's investment threshold matches that of the marginal investor's. It is straightforward to show that ours results are robust to any contract which is a linear combination of short-term and long-term incentives: under both objectives, the expected information sensitivity of equity responds similarly to a change in $\alpha$.[37]

Similarly, we assume that the manager and investors start with common prior beliefs. It is reasonable, however, to believe that while investors are better informed about external conditions, including the state of the macroeconomy or industry trends (all reflected in their beliefs about $q$), the manager may possess more "firm-specific" or internal information (contained in his beliefs about the state-dependent cash flow). In this vein, we endow the manager with private information about the distribution of the firm's existing assets in section 8.2, found in the appendix. We show that the

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[36] In the presence of managerial learning from prices, achieving this is a non-trivial exercise (e.g., see Lin, Liu, and Sun (2015).)

[37] Though of interest, we do not consider whether or not it would be optimal for the manager to commit to an alternative investment threshold. As Strobl (2014) shows, in the presence of endogenous information acquisition, committing to an alternative (non-zero) threshold can be optimal since it can incent investors to acquire more information. In such settings, debt can be used to facilitate such a commitment and so investments which appear inefficient (given the information available) may be ex-ante efficient if they facilitate investor learning. We concur, but emphasize that our metrics (and results) are designed to capture measured ex-post inefficiency. Moreover, if the firm issues (or buys back) debt for any other reason (e.g., tax benefits), then some fraction of our measure will capture decisions which are both ex-ante and ex-post inefficient. We thank our discussant, Günter Strobl, for highlighting this issue.

main mechanism of our baseline model remain the same - while the level of information acquisition can increase or decrease for a given project, information acquisition is still increasing in $\alpha$.

For simplicity, we assume that the investment project is zero-cost. As an earlier version of the paper showed, this is without loss of generality when the investment is financed with cash (as part of the existing assets). One alternative is to assume that existing shareholders (separate from our informed investors) contribute the required funds for investment based on the date one equity price. Increasing the fraction contributed from existing investors is equivalent to assuming that the cash paid out from the assets-in-place decreases. It is straightforward to show that, regardless of the fraction paid out in cash, information acquisition increases in $\alpha$, preserving our main results.

As is clear from the preceding discussion, $\alpha$ plays a crucial role in our analysis. Moreover, in the benchmark model, $\alpha$ also plays a double role: it controls both the information sensitivity of the project (and therefore equity) as well as the $NPV_E$ of the project. As we show in section 8.3, it is the former role which drives our main comparative static. Specifically, we consider an alternative parametrization in which an increase in the investment's riskiness leaves the $NPV_E$ constant. Nevertheless, we show that our main channel is preserved: changes in the the volatility (information sensitivity) of cash flows drive information acquisition. As a result, the most inefficient examples of risk-shifting are attenuated the most while the opposite is true in the presence of debt overhang.

We conclude this section with a few of the empirical predictions of the benchmark model.

**Empirical Implications**

As should be clear from our analysis, the model would predict that an econometrician will find it more difficult to detect risk-shifting relative to debt overhang. This is driven by both the likelihood of such actions occurring as well as the econometrician's ability to label them as such. If we assume, as seems reasonable, that the projects which are easiest to identify are those in which the gap between $NPV$ and $NPV_E$ is the largest, then these are precisely the projects for which the difference in ex-ante likelihood is the largest as well. On the other hand, the model suggests that both price informativeness and investment-to-price sensitivity should be higher (lower) when firm managers

have the opportunity to indulge in risk-shifting (debt overhang).

The importance of our mechanism is likely to differ across assets and over time. The strength depends upon both the quality of investors' private information and the extent to which the manager relies on such information for making investment decisions. This suggests that cross-sectional differences should arise: for example, in a stable, mature industry, this channel may be less relevant. This channel also relies on financial market conditions: the market's ability to aggregate it into an informative signal is essential. This is more likely to be the case in well-developed markets and in settings where the demand of informed investors constitute a large fraction of overall order flow.

Our paper also contributes to the debate on the relative advantages of public versus private ownership. Generically, access to the information contained in prices lowers the incidence of agency problems in our model: this could have many benefits including, for instance, lowering the cost of issuing debt. However, the magnitude of this effect will depend upon the types of projects to which a firm has access. When risk-shifting is a larger concern (perhaps because the firm is closer to financial distress), the impact should be larger than when the main concern is debt overhang. This suggests that the benefits of this feedback channel should vary both in the cross section and over time.

# 6   Information Complementarity

In our framework, settings in which agency problems can arise necessarily exhibit substitutability in information acquisition across investors. While such a result is common in the larger market microstructure literature, it stands in contrast to the results of (Dow et al., 2017), who emphasize the possibility of multiple equilibria and the presence of complementarity in feedback models. In what follows, we show how our model can replicate their results as a special case and extend their analysis to account for the role of a firm's existing assets.

We begin by establishing conditions under which complementarity can arise.

**Proposition 6.1.** *For the marginal value of acquiring information to increase in the precision of others' information, i.e. $\frac{\partial^2 EU}{\partial \tau_i \partial \tau_p} > 0$ (Complementarity), it must be that*

*1. the project is ex-ante suboptimal, i.e. $NPV_E(\mathcal{F}_0) < 0$, in case 1, and*

*2. the project is ex-ante optimal, i.e. $NPV_E(\mathcal{F}_0) > 0$ in case 2.*

To understand these results, it is useful to isolate the two economic forces in our setting that determine how other investors' information acquisition affects the marginal value of learning. First, there is the standard substitutability effect (such as that found in Grossman and Stiglitz (1980)) which decreases the marginal value of acquiring information: the price becomes more informative and so there is less value in private learning. Second, there is a novel effect due to the endogenous investment decision. In particular, the degree to which managers condition on the information contained in the price depends upon its quality. The direction of this second effect depends on two factors: the ex-ante $NPV_E$ and the correlation between the assets-in-place and the investment payoff.

If the risky project is ex-ante optimal, the default decision is to take the project. Conditioning on the price introduces the possibility that the firm will choose not to invest and moreover, the likelihood of investment decreases when more precise information is available. In case 1, when the investment is positively correlated with the assets in place, this reduces the expected information sensitivity of equity, lowering each investor's incentive to learn. As a result, there is strategic substitutability across investors. On the other hand, if the project is ex-ante suboptimal, the firm's default choice is to pass on the investment. As a result, conditioning on a price which is more informative *increases* the possibility of investment, since it lowers the threshold price at which the manager will chooose to invest. In case 1, this increases the expected information sensitivity, which increases the marginal value of information. As a result, when others learn more it can "crowd in" private information. When this latter effect dominates the traditional Grossman-Stiglitz effect, learning exhibits complementarity.

This result, and the possibility of multiple equilibria that it generates, is very similar to what is found in (Dow et al., 2017). In their setting, however, the firm does not have any assets in place; as a result, an investment project of *any* type increases the information sensitivity. Essentially, this corresponds to the first case in our model but sets $\Delta E(F, 0) = 0$, i.e., assets in place are informationally-insensitive.[38]

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[38]In our setting, $\Delta E(F, 0) = 0$ if the debt security operates as a pass-through.

Our analysis generalizes their result but also extends the analysis to allow for investments which would *lower* information sensitivity. In particular, if the investment is negatively correlated with the firm's assets in place, as it is in the second case, the results of (Dow et al., 2017) are reversed. If the project is ex-ante suboptimal, as others learn more, investment becomes more likely, which *lowers* the expected information sensitivity. This discourages private information acquisition, in contrast to what arises in case 1. On the other hand, if the project is ex-ante optimal, more precise information in the price makes investment less likely, which *increases* the expected information sensitivity of equity. That is, in case 2, learning across investors exhibits strategic complementarity when the ex-ante $NPV_E$ is positive.

Figure 6: Incentive to acquire information as a function of price informativeness

The figure plots the marginal value of acquiring information as a function of the precision of the information contained in the price for projects with differing levels of ex-ante profitability, i.e. $NPV_E$. Parameter values are set to $\tau_i = \tau_Z = \tau_n = 1$. In case 1, $\Delta E(\mathbf{y}) = 3$, $\Delta E(\mathbf{0}) = 1$ while in case 2, these are flipped.
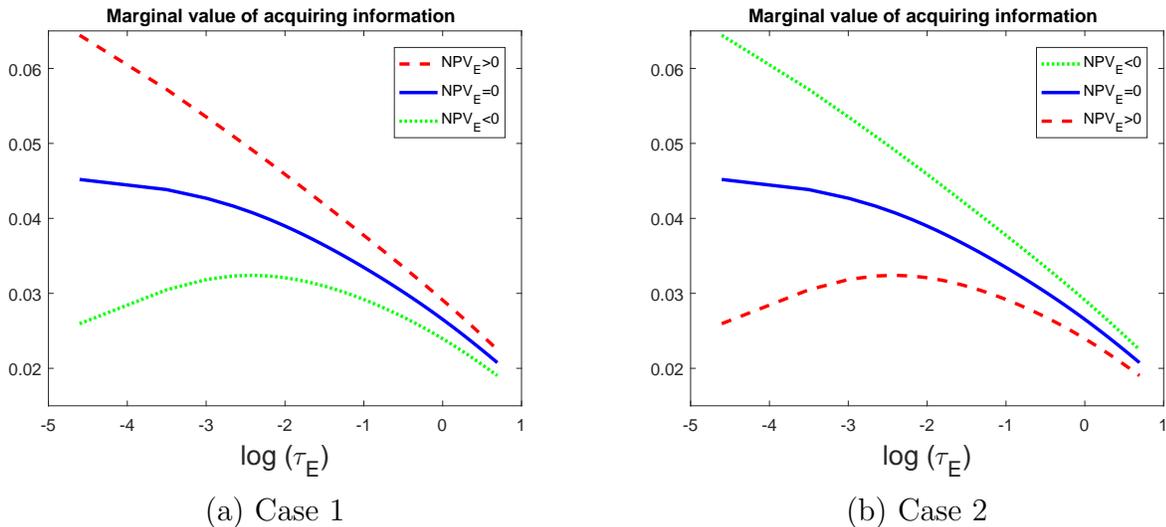


(a) Case 1

(b) Case 2

Figure 6 provides a numerical illustration of these effects. In the first panel, investment is positively correlated with assets in place; as a result, the marginal value of learning increases with others' information acquisition only when the ex-ante $NPV_E$ is sufficiently negative (the dotted line). Note that, eventually, as $\tau_p$ increases, the standard substitutability effect dominates so that the marginal value is non-monotonic in the information acquisition of others. In the second panel, where invest-

ment is negatively correlated with assets in place, this logic is reversed: complementarity only arises when the $NPV_E$ is sufficiently positive (the dashed line).

# 7    Conclusion

In this paper, we analyze the implications for investment efficiency when the riskiness of a firm's cash flows is endogenously driven by a novel "feedback loop": managers' decision to invest (which alters cash flow volatility) depends upon investors' decision to acquire information (which depends upon cash flow volatility). We develop a novel, tractable, non-linear REE which incorporates a feedback loop between security prices and the firm's investment decision. Using this framework, we argue that, in the presence of risky debt, learning from prices generically eliminates some inefficient investment decisions. We then show that investors' endogenous learning creates an asymmetric effect which depends upon the type of investment opportunity. Specifically, the riskiest examples of potential risk-shifting are least likely to be adopted, while the opposite is true when debt overhang is feasible: the most efficient (and risk-reducing) projects are most likely to be abandoned.

Our model also provides a foundation that can facilitate several promising directions for future research. For example, in our model, the manager takes as given the firm's capital structure: specifically, outstanding equity and any previously issued debt. Note, however, that in imperfectly integrated markets, the choice of capital structure influences the quality of the aggregate information available in the economy. This provides a potentially novel motivation for issuing debt: in such settings, the manager will choose the firm's capital structure to maximize total information production to improve the quality of his investment decisions. Another important direction for future work is towards a unified theory of project choice and information acquisition. In our model, the manager has access to a single project; however, ceteris paribus, the manager prefers opportunities which encourage learning. Understanding what projects managers choose to consider (with our without debt in the capital structure) is important in understanding firms' realized investment decisions.

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# Appendix: Supplemental Results & Proofs

**Lemma 7.1.** *If $z$ is normally distributed and $q = \phi[z]$, then*

$$\mathbb{E}[q] = \Phi\left(\frac{\mathbb{E}[z]}{\sqrt{1 + \mathbb{V}[z]}}\right) \tag{23}$$

**Proof of Lemma 2.1.** By definition, $(\alpha, \gamma)$ satisfies

$$E_L(y_L) - E_L(0) \equiv -\alpha$$

$$E_H(y_H) - E_H(0) \equiv \alpha(1 + \gamma).$$

This implies

$$\Delta E(\mathbf{y}) \equiv E_H(y_H) - E_L(y_L) \tag{24}$$

$$= E_H(0) + \alpha(1 + \gamma) - E_L(0) + \alpha \tag{25}$$

$$= \Delta E(\mathbf{0}) + \alpha(2 + \gamma). \tag{26}$$

Note that $\Delta E(\mathbf{y}) = E_H(y_H) - E_L(y_L)$. Taking partial derivative with respect to $y_H$, we get

$$\frac{\partial \Delta E(\mathbf{y})}{\partial y_H} = \frac{\partial E_H(y_H)}{\partial y_H} = \frac{\partial}{\partial y_H} \int_{D-y_H}^{\infty} (x - D + y_H) dG_H = 1 - G_H(D - y_H) > 0.$$

Similarly, taking partial derivative with respect to $y_L$, we get

$$\frac{\partial \Delta E(\mathbf{y})}{\partial y_L} = -\frac{\partial E_L(y_L)}{\partial y_L} = -\frac{\partial}{\partial y_L} \int_{D-y_L}^{\infty} (x - D + y_L) dG_L = -1 + G_L(D - y_L) < 0.$$

From equation, 26, it is clear that information sensitivity of equity increases with $\alpha$. ∎

**Proof of Lemma 3.1.** Note that the value of investment given firm manager's information set is

$$NPV_E|\mathcal{F}_m = V_E(\text{with investment}) - V_E(\text{absent investment})$$

$$= E_L(y_L) - E_L(0) + (\Delta E(\mathbf{y})) - \Delta E(0))E(q|\mathcal{F}_m).$$

Managers who has the interest of equity holders will invest when $NPV_E|\mathcal{F}_m > 0$, which translates to condition 11 (with case 1 investment i.e., $(\Delta E(\mathbf{y})) - \Delta E(0)) > 0$).

With case 2 investment i.e., when $(\Delta E(\mathbf{y})) - \Delta E(0)) < 0$, the condition is flipped and manager invests when he is sufficiently pessimistic:

$$\mathbb{E}[q|\mathcal{F}_m] < \frac{E_L(0) - E_L(y_L)}{\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})} = \frac{1}{2+\gamma} \equiv K. \qquad \textbf{(Case 2)} \qquad (27)$$

∎

**Proof of Proposition 3.2.** In case 1, the equilibrium exists when price increases at $\underline{q_E}$ i.e.,

$$E_L(0) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{0}) < E_L(y_L) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{y}).$$

This can be rewritten as

$$\mathbb{E}[q|s_i = h(z,u), p_E] > \frac{E_L(0) - E_L(y_L)}{\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})} \equiv K$$

$$\iff \frac{\tau_z \mu_z + (\tau_e + \tau_p)f(K, \tau_p)}{\sqrt{\psi(1+\psi)}} > \Phi^{-1}(K)$$

$$\iff \tau_z \mu_z + (\tau_e + \tau_p)\frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z + \tau_p)(1 + \tau_z + \tau_p)}\right] - \tau_z \mu_z}{\tau_p} > \Phi^{-1}(K)\sqrt{\psi(1+\psi)}$$

where $\psi = \tau_z + \tau_e + \tau_p$. Simplifying this condition gives us:

$$\mu_z < \frac{\Phi^{-1}(K)}{\tau_z \tau_e}\left[(\tau_e + \tau_p)\sqrt{(\tau_z + \tau_p)(1 + \tau_z + \tau_p)} - \tau_p\sqrt{\psi(1+\psi)}\right] \qquad (28)$$

In case 2, the equilibrium exists when there is a price drop at $\underline{q_E}$ i.e.,

$$E_L(0) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{0}) > E_L(y_L) + \mathbb{E}[q|s_i = h(z,u), p_E]\Delta E(\mathbf{y}).$$

This can be rewritten as

$$\mathbb{E}[q|s_i = h(z,u), p_E] > \mathbb{E}[q|p_E] = K$$

Note that this is the same condition as in case 1 and simplifying this condition gives us 28. ∎

**Proof of Proposition 3.3.** Let the information set (filtration) $\mathcal{F}$ be more informative than $\mathcal{G}$ (i.e., $\mathcal{G}$ is a coarser filtration: $\mathcal{G} \subset \mathcal{F}$). Let $a^F$ (and $U^F$) and $a^G$ (and $U^G$) denote the optimal demands (and corresponding expected utilities) under filtrations $\mathcal{F}$ and $\mathcal{G}$. The fact that $\mathcal{G} \subset \mathcal{F}$ implies that $U^F \geq U^G$. Hence expected utility weakly increases with more information.

Expected utility in case 1 is given by

$$EU = \mathbb{E}[\Delta E(\mathbf{0})(q_i - q_E)\mathbb{1}_{q_i > q_E}\mathbb{1}_{q_E < \underline{q_E}} + \Delta E(\mathbf{y})(q_i - q_E)\mathbb{1}_{q_i > q_E}\mathbb{1}_{q_E > \underline{q_E}}]$$

$$= \Delta E(\mathbf{0}) \int_{-\infty}^{q_E} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E)dF_{q_i|q_E}(q_i) + \Delta E(\mathbf{y}) \int_{\underline{q_E}}^{\infty} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E)dF_{q_i|q_E}(q_i)$$

where $F_x(y)$ is the cdf of random variable x evaluated at point y. Note that

$$s_i|s_p \sim \mathcal{N}\left(\frac{\tau_z \mu_z + \tau_p s_p}{\tau_z + \tau_p}, \frac{1}{\tau_z + \tau_p} + \frac{1}{\tau_i}\right)$$

Let $w_i = \frac{\tau_z \mu_z + \tau_i s_i + \tau_p s_p}{\sqrt{\psi_i(1+\psi_i)}}$, $w_E = \frac{\tau_z \mu_z + (\tau_e + \tau_p)s_p}{\sqrt{\psi(1+\psi)}}$ where $\psi_i = \tau_z + \tau_i + \tau_p$ and $\psi = \tau_z + \tau_e + \tau_p$. Then $q_i = \Phi(w_i)$, $q_E = \Phi(w_E)$ and

$$w_i|s_p \sim \mathcal{N}\left(\sqrt{\frac{\psi_i}{1+\psi_i}}\frac{\tau_z \mu_z + \tau_p s_p}{\tau_z + \tau_p}, \frac{\tau_i}{(1+\psi_i)(\tau_z + \tau_p)}\right) \tag{29}$$

Using change of variables, expected utility can be rewritten as

$$EU(\tau_i, \tau_p) = \Delta E(\mathbf{0}) \int_{-\infty}^{f(\tau_p, K)} dF_{s_p}(s_p) \int_{w_E}^{\infty} \{\Phi(w_i) - \Phi(w_E)\} dF_{w_i|s_p}(w_i) + \Delta E(\mathbf{y}) \int_{f(\tau_p, K)}^{\infty} dF_{s_p}(s_p) \int_{w_E}^{\infty} \{\Phi(w_i) - \Phi(w_E)\} dF_{w_i|s_p}(w_i)$$

Define $H(s_p, \tau_p, \tau_i) = \int_{w_E}^{\infty} \{\Phi(w_i) - \Phi(w_E)\} dF_{w_i|s_p}(w_i)$. Note that $H$ is always positive. Using this definition, we can rewrite expected utility as

$$EU(\tau_i, \tau_p) = \Delta E(\mathbf{0}) \int_{-\infty}^{f(\tau_p, K)} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p) + \Delta E(\mathbf{y}) \int_{f(\tau_p, K)}^{\infty} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p) \tag{30}$$

In case 2, the expected utility can be written as

$$EU = \Delta E(\mathbf{y}) \int_{-\infty}^{q_E} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E) dF_{q_i|q_E}(q_i) + \Delta E(\mathbf{0}) \int_{\underline{q_E}}^{\infty} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E) dF_{q_i|q_E}(q_i)$$

$$= \Delta E(\mathbf{y}) \int_{-\infty}^{f(\tau_p, K)} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p) + \Delta E(\mathbf{0}) \int_{f(\tau_p, K)}^{\infty} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p)$$

**Result:** $H(s_p, \tau_p, \tau_i)$ increases with $\tau_i$.

*Proof:* Recall that

$$H(s_p, \tau_p, \tau_i) = \int_{w_E}^{\infty} \{\Phi(w_i) - \Phi(w_E)\} dF_{w_i|s_p}(w_i) \tag{31}$$

$$\approx \phi(w_E) \int_{w_E}^{\infty} (w_i - w_E) dF_{w_i|s_p}(w_i) \tag{32}$$

$$= \phi(w_E) \left[ \mu_i \Phi\left(\frac{\mu_i}{\sigma_i}\right) + \sigma_i \phi\left(\frac{\mu_i}{\sigma_i}\right) \right] \tag{33}$$

where $\mu_i = \sqrt{\frac{\psi_i}{1+\psi_i}} \frac{\tau_z \mu_z + \tau_p s_p}{\tau_z + \tau_p} - w_E$, $\sigma_i = \frac{\tau_i}{(1+\psi_i)(\tau_z + \tau_p)}$. Differentiating H wrt $\tau_i$,

$$\frac{\partial H}{\partial \tau_i} = \phi(w_E) \left[ \Phi\left(\frac{\mu_i}{\sigma_i}\right) \frac{\partial \mu_i}{\partial \tau_i} + \phi\left(\frac{\mu_i}{\sigma_i}\right) \frac{\partial \sigma_i}{\partial \tau_i} \right]$$

It is obvious that when both the mean and variance of distribution of $w_i|s_p$ increases with $\tau_i$, H increases with $\tau_i$ as well. Next, we will show that this result holds more generally. Taking derivative of $\mu_i$ and $\sigma_i$ with respect to $\tau_i$, we get $\frac{\partial \mu_i}{\partial \tau_i} = \frac{\mu_i}{2\psi_i(1+\psi_i)}$ and $\frac{\partial \sigma_i}{\partial \tau_i} = \frac{\sigma_i(1+\tau_Z+\tau_p)}{2\tau_i(1+\psi_i)}$. So, H increases with $\tau_i$ if

$$\lambda\Phi(\lambda) + \phi(\lambda)(1 + \frac{\tau_Z + \tau_p}{\tau_i})(1 + \tau_Z + \tau_p) > 0$$

where $\lambda = \frac{\mu_i}{\sigma_i}$. It is clear that

$$\lambda\Phi(\lambda) + \phi(\lambda) > 0 \qquad \forall \lambda > 0 \implies \frac{\partial H}{\partial \tau_i} > 0 \quad \forall \lambda > 0$$

The challenge is to show it for negative values of $\lambda$. Using Chebychev's inequality for standard normal random variable X, we know that

$$E[X|X > \lambda] > \lambda.$$

Note that lhs of the above expression can be simplified as $E[X|X > \lambda] = \frac{\phi(\lambda)}{\Phi(-\lambda)}$. Substituting this, we get

$$-\lambda\Phi(-\lambda) + \phi(-\lambda) > 0 \qquad \forall \lambda \implies \frac{\partial H}{\partial \tau_i} > 0 \quad \forall \lambda$$

Taking partial derivative of expected utility with respect to $\alpha$ in case 1 gives us

$$\frac{\partial EU}{\partial \alpha} = (\Delta E(\mathbf{0}) - \Delta E(\mathbf{y}))H(f(\tau_p,K),\tau_p,\tau_i)f_{s_p}(f(\tau_p,K))\frac{\partial f(\tau_p,K)}{\partial \alpha} + \frac{\partial \Delta E(\mathbf{y})}{\partial \alpha} \int\limits_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i)dF_{s_p}(s_p)$$

$$= (\Delta E(\mathbf{0}) - \Delta E(\mathbf{y}))H(f(\tau_p,K),\tau_p,\tau_i)f_{s_p}(f(\tau_p,K))\frac{\sqrt{(\tau_z+\tau_p)(1+\tau_z+\tau_p)}}{\tau_p\phi(\Phi^{-1}(K))}\frac{\partial K}{\partial \alpha} + \frac{1}{K_0}\int\limits_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i)dF_{s_p}(s_p)$$

$$= \frac{1}{K_0}\int\limits_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i)dF_{s_p}(s_p)$$

$$> 0$$

The fact that H is monotonic in $\tau_i$ implies that

$$\frac{\partial^2 EU}{\partial \tau_i \partial \alpha} = \frac{1}{K_0}\int\limits_{f(\tau_p,K)}^{\infty} \frac{\partial H(s_p,\tau_p,\tau_i)dF_{s_p}(s_p)}{\partial \tau_i} > 0.$$

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This implies that the marginal value of acquiring information increases with $\alpha$. ∎

**Proof of Proposition 3.4.** Investors' maximization problem has unique symmetric solution when $\frac{\partial EU(\tau_i, \tau_p)}{\partial \tau_i}|_{\tau_i=\tau_j=\tau \forall j} = \frac{\partial C(\tau_i)}{\partial \tau_i}|_{\tau_i=\tau}$. has one solution. Since the cost function is convex, the rhs of above equation is increasing in $\tau$. Investors' FOC has unique solution when lhs is decreasing in $\tau$. This is true when

$$\frac{\partial^2 EU(\tau_i, \tau_p)}{\partial \tau_i^2}|_{\tau_i=\tau_j=\tau} + \frac{\partial^2 EU}{\partial \tau_i \partial \tau_p}|_{\tau_i=\tau_j=\tau} < 0.$$

This is true given the concavity of EU and when there is substitutability across investors. ∎

**Proof of Lemma 4.1.** (1) In case 1, we can rewrite the condition for $NPV(\mathcal{F}) < 0$ as

$$\frac{\mathbb{E}[q|\mathcal{F}]}{1 - \mathbb{E}[q|\mathcal{F}]} < \frac{-y_L}{y_H}$$

Similarly, $NPV_E(\mathcal{F}) < 0$ if

$$
\begin{aligned}
\frac{\mathbb{E}[q|\mathcal{F}]}{1 - \mathbb{E}[q|\mathcal{F}]} &< \frac{E_L(0) - E_L(y_L)}{E_H(y_H) - E_H(0)} \\
&= \frac{\int_D^\infty (-y_L) dG_L + \int_D^{D-y_L} [x - (D - y_L)] dG_L}{\int_D^\infty y_H dG_H + \int_{D-(y_H)}^D [x - (D - (y_H))] dG_H} \\
&= \left[\frac{-y_L}{y_H}\right] \left[\frac{1 - G_L(D) + \frac{\int_D^{D-y_L}[x-(D-y_L)]dG_L}{-y_L}}{1 - G_H(D) + \frac{\int_{D-y_H}^D[x-(D-y_H)]dG_H}{y_H}}\right] \\
&< \frac{-y_L}{y_H}
\end{aligned}
$$

The last inequality holds because it is always the case that (1) $\int_D^{D-y_L}[x - (D - y_L)]dG_L < 0$ and $\int_{D-y_H}^D [x - (D - y_H)]dG_H > 0$, while (2) $G_H(D) < G_L(D)$ holds by assumption of FOSD without investment. This argument shows that, in case 1, $NPV_E(\mathcal{F}) < 0 \implies NPV(\mathcal{F}) < 0$ i.e., debt overhang is not possible in case 1. By the same logic, it is straightforward to see that conditions exist under which $NVP_E(\mathcal{F}) > 0$, while $NPV(\mathcal{F}) < 0$ i.e., risk-shifting is possible in case 1.

(2) In case 2, we can rewrite the condition for $NPV(\mathcal{F}) > 0$ as

$$\frac{1 - \mathbb{E}[q|(\mathcal{F})]}{\mathbb{E}[q|(\mathcal{F})]} > \frac{-y_H}{y_L}$$

Similarly, $NPV_E(\mathcal{F}) > 0$ if

$$\frac{1 - \mathbb{E}[q|(\mathcal{F})]}{\mathbb{E}[q|(\mathcal{F})]} > \frac{E_H(0) - E_H(y_H)}{E_L(y_L) - E_L(0)}$$

$$= \frac{\int_{D-y_H}^{\infty}(-y_H)dG_H + \int_{D}^{D-y_H}(x-D)dG_H}{\int_{D-y_L}^{\infty}y_L dG_L + \int_{D-y_L}^{D}(x-D)dG_L}$$

$$= \left[\frac{-y_H}{y_L}\right]\left[\frac{1 - G_H(D-y_H) + \frac{\int_{D}^{D-y_H}(x-D)dG_H}{-y_H}}{1 - G_L(D-y_L) + \frac{\int_{D-y_L}^{D}(x-D)dG_L}{y_L}}\right]$$

$$> \frac{-y_H}{y_L}$$

The last inequality holds because it is always the case that (1) $\int_{D}^{D-y_H}(x-D)dG_H > 0$ and $\int_{D-y_L}^{D}(x-D)dG_L < 0$, while (2) $G_H(D-y_H) < G_L(D-y_L)$ holds by assumption of FOSD with investment. This argument shows that, in case 2, $NPV_E(\mathcal{F}) > 0 \implies NPV(\mathcal{F}) > 0$ i.e., risk shifting is not possible in case 2. By the same logic, it is straightforward to see that conditions exist under which $NVP_E(\mathcal{F}) < 0$, while $NPV(\mathcal{F}) > 0$ i.e., debt overhang is possible in case 2. $\blacksquare$

**Proof of Lemma 4.2.** (1) Taking partial derivative of NPV wrt $\alpha$, we get

$$\frac{\partial NPV}{\partial \alpha} = q_0\frac{\partial y_H}{\partial \alpha} + (1 - q_0)\frac{\partial y_L}{\partial \alpha} \tag{34}$$

$$= \left((1+\gamma)\frac{q_0}{1 - G_H(F - y_H)} - \frac{1 - q_0}{1 - G_L(F - y_L)}\right) \tag{35}$$

$$= \left(\frac{q_0(1+\gamma)(1 - G_L(F - y_L)) - (1 - q_0)(1 - G_H(F - y_H))}{(1 - G_H(F - y_H))(1 - G_L(F - y_L))}\right) \tag{36}$$

This is less than zero when

$$\gamma < \frac{(1 - q_0)(1 - G_H(F))}{q_0(1 - G_L(F))} - 1 \equiv \bar{\gamma}$$

Moreover, for $NPV_E$ to be positive, we need $\gamma > \frac{1}{q_0} - 2 \equiv \underline{\gamma}$ $\blacksquare$

**Proof of Corollary 4.3.** The first part follows from the proof of Lemma 4.1. It shows that

(1) In case 1, any project for which $\mathbb{E}[q|s_p] < \underline{q_E}$ is inefficient (i.e., $NPV(s_p) < 0$).

(2) In case 2, any project for which $\mathbb{E}[q|s_p] < \underline{q_E}$ is efficient (i.e., $NPV(s_p) > 0$).

Moreover, as long as $\tau_p > 0$, given the unbounded support for both $z$ and $u$, there is always a non-zero probability that the manager observes some $s_p$ such that $\mathbb{E}[q|s_p] < \underline{q_E}$. ∎

**Proof of Proposition 4.4.** Using a change of variables, we can rewrite the probability of inefficient investment as

$$\int_{f(K,\tau_p)}^{f(K_0,\tau_p)} dF_{s_p} = \sqrt{\tau_z^{-1} + \tau_p^{-1}} \left[ \Phi\left( \frac{f(K_0,\tau_p) - \mu_z}{\sqrt{\tau_z^{-1} + \tau_p^{-1}}} \right) - \Phi\left( \frac{f(K,\tau_p) - \mu_z}{\sqrt{\tau_z^{-1} + \tau_p^{-1}}} \right) \right] \tag{37}$$

where $dF_{s_p}$ is the cdf of distribution of $s_p$.

(i) Probability of inefficient investment is given by

$$\sqrt{\tau_z^{-1} + \tau_p^{-1}} \Phi\left( \underbrace{\frac{\Phi^{-1}(K_0)\sqrt{(1 + \tau_z + \tau_p)\tau_z} - \mu_z\sqrt{(\tau_z + \tau_p)\tau_z}}{\sqrt{\tau_p}}}_{\equiv \varpi_1} \right) - \Phi\left( \underbrace{\frac{\Phi^{-1}(K)\sqrt{(1 + \tau_z + \tau_p)\tau_z} - \mu_z\sqrt{(\tau_z + \tau_p)\tau_z}}{\sqrt{\tau_p}}}_{\equiv \varpi_2} \right)$$

Differentiating this probability wrt $\tau_p$ gives us

$$\propto \phi(\varpi_1) \left( -\frac{\Phi^{-1}(K_0)(1 + \tau_z)}{\sqrt{1 + \tau_z + \tau_p}} + \frac{\mu_z\tau_z}{\sqrt{\tau_z + \tau_p}} \right) - \phi(\varpi_2) \left( -\frac{\Phi^{-1}(K)(1 + \tau_z)}{\sqrt{1 + \tau_z + \tau_p}} + \frac{\mu_z\tau_z}{\sqrt{\tau_z + \tau_p}} \right) \tag{38}$$

$$= \frac{1 + \tau_z}{\sqrt{1 + \tau_z + \tau_p}} \left( \phi(\varpi_2)\Phi^{-1}(K) - \phi(\varpi_1)\Phi^{-1}(K_0) \right) + \frac{\mu_z\tau_z}{\sqrt{\tau_z + \tau_p}} (\phi(\varpi_1) - \phi(\varpi_2)) \tag{39}$$

Note that $K_0 > K$ implies that $\varpi_1 > \varpi_2$. We want the above expression (39) to be negative. First note that, condition $\gamma > \bar{\gamma}$ implies that $\varpi_2 < 0$.

If $\frac{\phi(\varpi_1)}{\phi(\varpi_2)} < 1$, the second term in equation 39 is negative. Moreover, if $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)}$, the first term in equation 39 is also negative.

(ii) Probability of inefficient investment conditional of investment taking place is

$$\frac{\Phi(\varpi_1) - \Phi(\varpi_2)}{1 - \Phi(\varpi_2)} = \frac{\Phi(-\varpi_2) - \Phi(-\varpi_1)}{\Phi(-\varpi_2)} = 1 - \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$$

Differentiating the above with respect to $\tau_p$ gives us

$$\propto \phi(\varpi_1) \left( -\frac{\Phi^{-1}(K_0)(1 + \tau_z)}{\sqrt{1 + \tau_z + \tau_p}} + \frac{\mu_z \tau_z}{\sqrt{\tau_z + \tau_p}} \right) - \phi(\varpi_2) \left( -\frac{\Phi^{-1}(K)(1 + \tau_z)}{\sqrt{1 + \tau_z + \tau_p}} + \frac{\mu_z \tau_z}{\sqrt{\tau_z + \tau_p}} \right) \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$$

(40)

$$= \frac{1 + \tau_z}{\sqrt{1 + \tau_z + \tau_p}} \left( \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)} \phi(\varpi_2) \Phi^{-1}(K) - \phi(\varpi_1) \Phi^{-1}(K_0) \right) + \frac{\mu_z \tau_z}{\sqrt{\tau_z + \tau_p}} \left( \phi(\varpi_1) - \phi(\varpi_2) \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)} \right)$$

(41)

If $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)} < \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$, the conditional probability of inefficient investment decreases with more learning. So, the necessary condition for both to be true is $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)} < \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$ ∎

**Proof of Proposition 4.5.** This follows directly from Proposition 3.3 and 4.4. ∎

**Proof of Proposition 4.6.** Note that after observing the signal $s_m$, the posterior is

$$E[q|s_m] = \Phi \left( \frac{\tau_z \mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1 + \tau_z + \tau_m)}} \right).$$

Note that the value of investment given firm manager's information set is

$$NPV_E(\mathcal{F}_m) = E_L(y_L) - E_L(0) + (\Delta E(\mathbf{y})) - \Delta E(0))E(q|s_m).$$

Managers who has the interest of equity holders will invest when $NPV_E(\mathcal{F}_m) > 0$, which translates to condition

$$E[q|s_m] > K \equiv \frac{-(E_L(y_L) - E_L(0))}{(\Delta E(\mathbf{y})) - \Delta E(0)} \iff s_m > f(K, \tau_m)$$

With case 2 investment i.e., when $(\Delta E(\mathbf{y})) - \Delta E(0)) < 0$, the condition is flipped and manager

invests when he is sufficiently pessimistic:

$$E\left[q|s_m\right] < K \equiv \frac{-(E_L(y_L) - E_L(0))}{(\Delta E(\mathbf{y})) - \Delta E(0))} \iff s_m < f\left(K, \tau_m\right)$$

Before observing signal, the expected net present value of equity is given by,

$$E[NPV_E\left(\mathcal{F}_m\right)] = \begin{cases} |\alpha| \int_{f(K,\tau_m)}^{\infty} \left\{ (2+\gamma) \, \Phi\left(\frac{\tau_z\mu_z + \tau_m s_m}{\sqrt{(\tau_z+\tau_m)(1+\tau_z+\tau_m)}}\right) - 1 \right\} dF\left(s_m\right) & \text{in case 1} \\ |\alpha| \int_{-\infty}^{f(K,\tau_m)} \left\{ 1 - (2+\gamma) \, \Phi\left(\frac{\tau_z\mu_z + \tau_m s_m}{\sqrt{(\tau_z+\tau_m)(1+\tau_z+\tau_m)}}\right) \right\} dF\left(s_m\right) & \text{in case 2.} \end{cases}$$

In case 1 (Risk-shifting), marginal benefit of acquiring information is

$$\frac{\partial E[NPV_E\left(\mathcal{F}_m\right)]}{\partial \tau_m} = |\alpha| \frac{\partial}{\partial \tau_m} \int_{f(K,\tau_m)}^{\infty} \left\{ -1 + (2+\gamma) \, \Phi\left(\frac{\tau_z\mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1 + \tau_z + \tau_m)}}\right) \right\} dF\left(s_m\right)$$

and the marginal benefit increases with $|\alpha|$ if

$$\frac{\partial}{\partial \tau_m} \int_{f(K,\tau_m)}^{\infty} \left\{ -1 + (2+\gamma) \, \Phi\left(\frac{\tau_z\mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1 + \tau_z + \tau_m)}}\right) \right\} dF\left(s_m\right) > 0.$$

This implies that when ever marginal benefit is positive, it increases with $|\alpha|$. Next, we argue that marginal benefit of learning is postive for the manager.

Let the information set $\mathcal{F}$ be more informative than $\mathcal{G}$ (i.e., $\mathcal{G}$ is a coarser filtration: $\mathcal{G} \subset \mathcal{F}$). Assume that filtration $\mathcal{G}$ is of precision $\tau_m$ and filtration $\mathcal{F}$ includes filtration $\mathcal{G}$ and another signal of precision $\Delta \tau_m > 0$. By blackwell's theorem, the fact that $\mathcal{G} \subset \mathcal{F}$ impies that $NPV_E(\mathcal{F}) > NPV_E^($\mathcal{G})$. Hence marginal benefit of learning is positive for the mananger.

In case 2 (Debt-overhang), marginal benefit of acquiring information is

$$\frac{\partial NPV_E\left(\tau_m\right)}{\partial \tau_m} = |\alpha| \frac{\partial}{\partial \tau_m} \int_{-\infty}^{f(K,\tau_m)} \left\{ 1 - (2+\gamma) \, \Phi\left(\frac{\tau_z\mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1 + \tau_z + \tau_m)}}\right) \right\} dF\left(s_m\right).$$

and the marginal benefit increases with $|\alpha|$ if

$$\frac{\partial}{\partial \tau_m} \int_{-\infty}^{f(K,\tau_m)} \left\{ 1 - (2+\gamma) \Phi \left( \frac{\tau_z \mu_z + \tau_m s_m}{\sqrt{(\tau_z + \tau_m)(1 + \tau_z + \tau_m)}} \right) \right\} dF(s_m) > 0.$$

This implies that when ever marginal benefit is positive, it increases with $|\alpha|$. Using the same reasoning as above (for case 1), we can argue that marginal benefit is always positive. This implies that it increases with $|\alpha|$. ∎

**Proof of Corollary 4.7.** This follows directly from Proposition 4.6. ∎

**Proof of Proposition 6.1.** Recall that expected utility of acquiring information of precision $\tau_i$ when prices reveal information of precision $\tau_p$ is given by

$$EU(\tau_i, \tau_p) = \Delta E(\mathbf{0}) \int_{-\infty}^{f(\tau_p, K)} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p) + \Delta E(\mathbf{y}) \int_{f(\tau_p, K)}^{\infty} H(s_p, \tau_p, \tau_i) dF_{s_p}(s_p)$$

Taking partial derivative with respect to $\tau_p$ gives us

$$\frac{\partial EU}{\partial \tau_p} = \Delta E(\mathbf{0}) \int_{-\infty}^{f(\tau_p, K)} \frac{\partial \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_p} ds_p + \Delta E(\mathbf{y}) \int_{f(\tau_p, K)}^{\infty} \frac{\partial \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_p} ds_p - $$

(42)

$$(\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})) \frac{\partial f(\tau_p, K)}{\partial \tau_p} H(f(\tau_p, K), \tau_p, \tau_i) f_{s_p}(f(\tau_p, K), \tau_p)$$

(43)

Taking partial derivative of the above equation with respect to $\tau_i$ gives us

$$\frac{\partial^2 EU}{\partial \tau_i \partial \tau_p} = \Delta E(\mathbf{0}) \int_{-\infty}^{f(\tau_p, K)} \frac{\partial^2 \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_i \partial \tau_p} ds_p + \Delta E(\mathbf{y}) \int_{f(\tau_p, K)}^{\infty} \frac{\partial^2 \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_i \partial \tau_p} ds_p - $$

(44)

$$(\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})) \frac{\partial f(\tau_p, K)}{\partial \tau_p} \frac{\partial H(f(\tau_p, K), \tau_p, \tau_i)}{\partial \tau_i} f_{s_p}(f(\tau_p, K), \tau_p)$$

(45)

The first two terms capture standard Grossman Stiglitz effect: if others acquire more information,

investor i has less incentive to acquire information and the terms are obviously negative. Lets focus on the third term. Using equation 13, we can write

$$\frac{\partial f(\tau_p, K)}{\partial \tau_p} = \frac{\partial}{\partial \tau_p} \left( \frac{\Phi^{-1}(K) \left[ \sqrt{(\tau_z + \tau_p)(1 + \tau_z + \tau_p)} \right] - \mu_z \tau_z}{\tau_p} \right) < 0 \tag{46}$$

$$\Longleftrightarrow \text{ The project if -ve } NPV_E \tag{47}$$

This implies that if the project is negative NPV equity, the third term is positive, there could be complementarity. Otherwise, if the project is positive NPV equity, the third term is negative, there is always substitutability.

(2) Lets focus on case 2 now. In this case,

$$\frac{\partial EU}{\partial \tau_p} = \Delta E(\mathbf{y}) \int_{-\infty}^{f(\tau_p, K)} \frac{\partial \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_p} ds_p + \Delta E(\mathbf{0}) \int_{f(\tau_p, K)}^{\infty} \frac{\partial \left( H(s_p, \tau_p, \tau_i) f_{s_p}(s_p, \tau_p) \right)}{\partial \tau_p} ds_p + $$

$$\tag{48}$$

$$(\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})) \frac{\partial f(\tau_p, K)}{\partial \tau_p} H(f(\tau_p, K), \tau_p, \tau_i) f_{s_p}(f(\tau_p, K), \tau_p) \tag{49}$$

Here again, lets focus on the third term. We can have complementarity only if the third term is positive. This is true if $\frac{\partial f(\tau_p, K)}{\partial \tau_p} < 0$ i.e., the project is positive NPV equity. ■

# 8  Appendix: Extensions

In this section, we demonstrate that our main results are robust to several natural extensions of the benchmark model analyzed in the main text.

## 8.1 Allowing for Short Sales

In this section, we relax the benchmark model's short sale constraint. As before, we impose finite position limits (to bound the demand of the risk-neutral investors); however, we assume that equity investors can now buy *or short* no more than one share. In order to ensure that markets always clear, noise trader demand is modified so that they now purchase $2\Phi(u)$ units of the outstanding equity.[39]

With no other modifications to the benchmark model, we posit that investors will still follow a threshold strategy: an investor buys one unit of equity if $s_i > h(z, u)$; otherwise, they sell short one unit of equity. This strategy implies that the market clearing condition is now

$$1 = \underbrace{[1 - \Phi\left(\sqrt{\tau_e}\left(h(z, u) - z\right)\right)]}_{\text{informed investor demand}} - \underbrace{\Phi\left(\sqrt{\tau_e}\left(h(z, u) - z\right)\right)}_{\text{short sales}} + \underbrace{2\Phi(u)}_{\text{liquidity demand}}.$$

Rewriting this expression shows that markets clear if and only if $h(z, u) = z + \frac{u}{\sqrt{\tau_e}}$. This is just as in the benchmark case considered in the main text, and so the analysis of the financial market equilibrium remains the same.

Given this, we turn to investors' information acquisition incentives in the absence of short sale constraints. For instance, in case 1, the investor's expected utility (expected trading gains) is

$$EU = \mathbb{E}\left(\underbrace{|q_i - q_E|}_{\text{buy if } q_i > q_E \text{and sell otherwise.}} \left[\Delta E(\mathbf{0}) \overbrace{\mathbb{1}_{q_E < \underline{q_E}}}^{\substack{\text{Do not} \\ \text{Invest}}} + \Delta E(\mathbf{y}) \overbrace{\mathbb{1}_{q_E > \underline{q_E}}}^{\text{Invest}}\right]\right). \tag{50}$$

The key difference between this expression and (16), from the benchmark model, is the first term. This corresponds to the expected difference in beliefs between the investor and the marginal investor, given that the investor can take either a long or short position. It is straightforward to show that proposition 3.3 still holds in this economy. Furthermore, and unsurprisingly, the ability to short increases investors' incentive to learn.

**Proposition 8.1.** *The marginal value of acquiring information is higher than in the case with short*

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[39]Were this not the case, then for some extreme realizations of $z$, $u$, the market would not clear, providing additional information to investors about the security's value. This assumption avoids this unnecessary complication.

*sale constraints. Moreover, the marginal value of acquiring information increases with $\alpha$.*

Given the proposition above, the rest of our results regarding the impact of feedback and the relative prevalence of risk-shifting and debt overhang follow.

## 8.2 Managerial Private Information

In our benchmark analysis, the manager and investors start with common prior beliefs. In what follows, we relax this assumption by endowing the manager with private information about the payoff distribution of the assets-in-place. While structuring his private knowledge in this fashion yields large benefits from a tractability perspective (investors and manager private information are orthogonal), we believe it is also well-motivated in practice. In particular, while managers may possess more "firm-specific" or internal information, investors are likely to be better informed about external conditions, including the state of the macroeconomy, industry trends, and fluctuations in consumer demand. Moreover, in such a setting, the firm manager is still incented to learn from the price of traded equity before making investment decisions.

To capture the difference in their beliefs about the existing asset's payoff, we denote the distribution of cash flows given the manager's information set as $G_L^m$ (in the low state) and $G_H^m$ (in the high state). Using this, we define

$$E_s^m(\delta) \equiv \int_{D-\delta}^{\infty} (x + \delta - D)dG_s^m$$

as the equity value (conditional on the state of the world), given the manager's information set. Similarly, we define the manager's perception of information sensitivity as $\Delta E^m(\mathbf{0})$ without investment and $\Delta E^m(\mathbf{y})$ with investment.

As in the benchmark model, we conjecture that the price of equity reveals a signal $s_p$ to all agents, including the manager. Using our new notation and the logic presented in our main analysis, the firm manager invests in case 1 when

$$\mathbb{E}[q|s_p] > \frac{E_L^m(0) - E_L^m(y_L)}{\Delta E^m(\mathbf{y}) - \Delta E^m(\mathbf{0})} \equiv K^m.$$

We note that the cutoff, $K_m$, will differ depending upon the information received by the manager. For simplicity, we assume the signal structure of the firm manager is such that $K^m \in \{K_1, K_2\}$, $K_1 < K_2$ and $\mathbb{P}[K_m = K_1] = q^m$.

In contrast to our main analysis, in this setting, investors are not always certain whether the project will be taken or not, given their uncertainty regarding $K_m$. Given this additional complication, we conjecture the following functional form for the price of equity:

1. Suppose the information contained in the price $(s_p)$ is sufficiently negative, so that $E[q|s_p] \leq K_1$. Then the manager will not invest for sure and the price of equity is

$$P_E = P_E^{NI} \equiv E_L(0) + E[q|s_i = h(z, u), p_E]\Delta E(\mathbf{0})$$

2. Suppose the information contained in the price $(s_p)$ is sufficiently positive, so that $E[q|s_p] \geq K_2$. Then the manager will invest for sure and the price of equity is

$$P_E = P_E^{I} \equiv E_L(y_L) + E[q|s_i = h(z, u), p_E]\Delta E(\mathbf{y})$$

3. In all other cases, investors are not sure whether project will be undertaken, since $K_1 < E[q|s_p] < K_2$. Since investors' private information is orthogonal to firm manager's information, investors cannot use their private information to forecast the likelihood that the project will be taken. As a result, in this region, we write the price of equity as

$$P_E = P_E^{NI} q_m + P_E^{I}(1 - q_m)$$

As in the baseline model, we can restrict our primitives in such a way that the price is monotonic (as in Proposition 3.2), confirming the existence of the conjectured financial market equilibrium.

Given this, we can rewrite the investor's incentive to acquire information in the presence of

managerial private information. For example, in case 1, the investor's expected utility is now

$$EU_{new} = \mathbb{E} \left( (q_i - q_E) \underbrace{\mathbb{1}_{q_i > q_E}}_{\substack{\text{buy if} \\ q_i > q_E}} \left[ \Delta E(\mathbf{0}) \overbrace{\mathbb{1}_{E[q|F_m] < K^m}}^{\substack{\text{Do not} \\ \text{Invest}}} + \Delta E(\mathbf{y}) \overbrace{\mathbb{1}_{E[q|F_m] > K^m}}^{\text{Invest}} \right] \right)$$

$$EU_{new} = EU_{old}(K_1) q_m + EU_{old}(K_2)(1 - q_m),$$

where $EU_{old}$ is defined as in equation 30. Thus, expected utility in this setting is simply a weighted average of the expected utility previously analyzed in the benchmark model. This implies that the marginal value of information for investors is given by

$$\frac{\partial EU_{new}}{\partial \tau_i} = q_m \frac{\partial EU_{old}(K_1)}{\partial \tau_i} + (1 - q_m) \frac{\partial EU_{old}(K_2)}{\partial \tau_i},$$

which implies that the implications of proposition 3.3 remain the same even if the firm manager has private information. Taken together, this implies that our main results remain qualitatively robust to such a modification.

## 8.3 Alternative Parameterization: constant $NPV_E$ constant

In our model, the ex-ante efficiency of the project, $NPV(\mathcal{F}_0)$, is a function of many variables. Hence, in the benchmark setting, we parameterize our model in such a way that a single variable ($\alpha$) will serve as a proxy for the investment's riskiness (and as we show in Lemma 4.2, inefficiency). More specifically, as we change $\alpha$, the investment threshold $K$ remains fixed but both the $NPV(\mathcal{F}_0)$ and $NPV_E(\mathcal{F}_0)$ of the project change with $\alpha$. In this section, we show that our main results are robust to an alternative parameterization in which we vary project characteristics (specifically, increasing the riskiness) while keeping $NPV_E(\mathcal{F}_0)$ constant.

First, define $q_0[q|\mathcal{F}_0] = \Phi\left(\mu_z \sqrt{\frac{\tau_z}{1+\tau_z}}\right)$. Let the project characteristics be such that

$$E_L(y_L) = E_L(0) + \theta_L$$

$$E_H(y_H) = E_H(0) + \theta_H.$$

Here, $\theta_L$ and $\theta_H$ denote the change in equity value due to investment in the low and high state, respectively. The project has constant $NPV_E(\mathcal{F}_0)$ if

$$(1 - q_0)\theta_L + q_0\theta_H = constant \equiv NE.$$

For any project, there are only 2 degrees of freedom: $\theta_L, \theta_H$. By keeping $NPV_E(\mathcal{F}_0)$ constant, we are constraining the degrees of freedom to just one. In this case,

$$NPV_E(\mathcal{F}_0) = q_0\theta_H + (1 - q_0)\theta_L = NE \tag{51}$$

$$NPV(\mathcal{F}_0) = q_0 y_H + (1 - q_0)y_L \tag{52}$$

$$K = \frac{E_L(0) - E_L(y_L)}{\Delta E(\mathbf{y}) - \Delta E(\mathbf{0})} = \frac{-\theta_L}{\theta_H - \theta_L} \tag{53}$$

**Risk shifting with constant $NPV_E(\mathcal{F}_0)$**

For risk shifting, we need to be in case 1, i.e., we need $\theta_L < 0 < \theta_H$ (as well as $NPV_E > 0$ and $NPV < 0$). Given $\beta > 0$, let $\theta_L = -\beta$. Since $NPV_E(\mathcal{F}_0) = NE$ is a constant, equation (51) and (53) implies $\theta_H = \frac{NE + (1 - q_0)\beta}{q_0}$ and $K = \frac{\beta}{NE + \beta}q_0$. Since NE is a constant, K increases from 0 to $q_0$ as we increase $\beta$. First, we want to show that $\beta$ is a proxy for inefficiency, i.e., $\frac{\partial NPV(\mathcal{F}_0)}{\partial \beta} < 0$. Taking the partial derivative of $NPV(\mathcal{F}_0)$ with respect to $\beta$, we get

$$\frac{\partial NPV(\mathcal{F}_0)}{\partial \beta} = q_0 \frac{\partial y_H}{\partial \beta} + (1 - q_0)\frac{\partial y_L}{\partial \beta} \tag{54}$$

$$= \frac{1 - q_0}{q_0}\frac{q_0}{1 - G_H(F - y_H)} - \frac{1 - q_0}{1 - G_L(F - y_L)} \tag{55}$$

$$= (1 - q_0)\left(\frac{G_H(F - y_H) - G_L(F - y_L)}{(1 - G_H(F - y_H))(1 - G_L(F - y_L))}\right) \tag{56}$$

$$< 0 \tag{57}$$

Intuitively, while equity holders benefit from the successful outcomes of high-risk (high $\beta$) projects, the losses from unsuccessful outcomes are borne by debt holders. Furthermore, not only is there a transfer of wealth from debt holders to equity holders but there is a reduction in enterprise value - as $\beta$ increases, these projects becomes increasingly more socially inefficient.

Investors also account for the change in $\beta$ when they decide how much information to acquire. Taking the partial derivative of an investor's expected utility with respect to $\beta$ in case 1 gives us

$$
\begin{aligned}
\frac{\partial EU}{\partial \beta} &= (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_p,K),\tau_p,\tau_i) f_{s_p}(f(\tau_p,K)) \frac{\partial f(\tau_p,K)}{\partial \beta} + \frac{\partial \Delta E(F,c)}{\partial \beta} \int_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i) dF_{s_p}(s_p) \\
&= (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_p,K),\tau_p,\tau_i) f_{s_p}(f(\tau_p,K)) \frac{\sqrt{(\tau_z+\tau_p)(1+\tau_z+\tau_p)}}{\tau_p \phi(\Phi^{-1}(K))} \frac{\partial K}{\partial \beta} + 2 \int_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i) dF_{s_p}(s_p) \\
&= -H(f(\tau_p,K),\tau_p,\tau_i) f_{s_p}(f(\tau_p,K)) \frac{\sqrt{(\tau_z+\tau_p)(1+\tau_z+\tau_p)}}{\tau_p \phi(\Phi^{-1}(K))} \frac{NE}{\beta+NE} + 2 \int_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i) dF_{s_p}(s_p) \\
&= -H(f(\tau_p,K),\tau_p,\tau_i) exp \left[ \frac{\left(\Phi^{-1}(K)\right)^2}{2} \left(1 - \frac{\tau_z(1+\tau_z+\tau_p)}{\tau_p}\right) \right] \sqrt{\frac{\tau_z(1+\tau_z+\tau_p)}{\tau_p}} \frac{NE}{\beta+NE} + 2 \int_{f(\tau_p,K)}^{\infty} H(s_p,\tau_p,\tau_i) dF_{s_p}(s_p)
\end{aligned}
$$

If NE is small, as $\beta$ increases, the first term tends to zero and the second term dominates. This implies that the marginal value of acquiring information increases with $\beta$.

## Debt Overhang with constant $NPV_E$

For debt overhang, we need to be in case 2, i.e., we need $\theta_L > 0 > \theta_H$, $NE < 0$ and $NPV > 0$. Let $\theta_L = \beta$ where $\beta > 0$. Since the $NPV_E$ is a constant, $\theta_H = \frac{NE - (1-q_0)\beta}{q_0}$ and $K = \frac{\beta}{\beta - NE} q_0$. As we increase $\beta$, $K$ increases from 0 to $q_0$ First, we want to show that $\beta$ is a proxy for efficiency, i.e., $\frac{\partial NPV}{\partial \beta} > 0$. Taking the partial derivative of the $NPV$ with respect to $\beta$, we get

$$
\begin{align}
\frac{\partial NPV(\mathcal{F}_0)}{\partial \beta} &= \frac{1}{2} \left( \frac{\partial y_H}{\partial \beta} + \frac{\partial c_L}{\partial \beta} \right) \tag{58} \\
&= \frac{1}{2} \left( \frac{-1}{1 - G_H(F - y_H)} + \frac{1}{1 - G_L(F - c_L)} \right) \tag{59} \\
&= \frac{1}{2} \left( \frac{G_L(F - c_L) - G_H(F - y_H)}{(1 - G_H(F - y_H))(1 - G_L(F - c_L))} \right) \tag{60} \\
&> 0 \tag{61}
\end{align}
$$

$\beta$ is, indeed, a proxy for efficiency in this setting.

We now turn to the information acquisition incentives of investors. Taking the partial derivative of an investor's expected utility with respect to $\beta$ in case 2 gives us

$$
\begin{aligned}
\frac{\partial EU}{\partial \beta} &= (\Delta E(F,c) - \Delta E(F,0)) H(f(\tau_p,K),\tau_p,\tau_i) f_{sp}(f(\tau_p,K)) \frac{\partial f(\tau_p,K)}{\partial \beta} + \frac{\partial \Delta E(F,c)}{\partial \beta} \int\limits_{-\infty}^{f(\tau_p,K)} H(s_p,\tau_p,\tau_i) dF_{sp}(s_p) \\
&= (\Delta E(F,c) - \Delta E(F,0)) H(f(\tau_p,K),\tau_p,\tau_i) f_{sp}(f(\tau_p,K)) \frac{\sqrt{(\tau_z+\tau_p)(1+\tau_z+\tau_p)}}{\tau_p \phi(\Phi^{-1}(K))} \frac{\partial K}{\partial \beta} - 2 \int\limits_{-\infty}^{f(\tau_p,K)} H(s_p,\tau_p,\tau_i) dF_{sp}(s_p) \\
&= H(f(\tau_p,K),\tau_p,\tau_i) f_{sp}(f(\tau_p,K)) \frac{\sqrt{(\tau_z+\tau_p)(1+\tau_z+\tau_p)}}{\tau_p \phi(\Phi^{-1}(K))} \frac{NE}{2\beta-NE} - 2 \int\limits_{-\infty}^{f(\tau_p,K)} H(s_p,\tau_p,\tau_i) dF_{sp}(s_p)
\end{aligned}
$$

The second term is always negative, whereas the first term is also negative since $NE < 0$. In this case, as the project become more efficient, an investor's marginal benefit of acquiring information decreases.