# A Dynamic Theory of Lending Standards*


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**Abstract**

We develop a dynamic model of credit markets in which both lending standards and the quality of potential borrowers are endogenous. Competitive lenders face an evolving pool of potential borrowers with the same publicly-observable characteristics. Each lender privately decides on its lending standards: whether to pay a cost to screen out some unprofitable borrowers or instead to lend (or not) to all potential borrowers. Lending standards – defined as the degree of screening – have externalities and are dynamic strategic complements: tighter screening worsens the future pool of borrowers, increasing the incentive to screen in the future. We show that lending standards can amplify and propagate fluctuations, and that even temporary adverse changes in fundamentals can have amplified and long-lasting effects on lending volume, credit spreads, and default rates. Further, lending standards may be inefficiently tight and we discuss several policies such as government support for lending that can help ameliorate this inefficiency, along with several pitfalls to avoid.



**JEL codes:** D82, G21, G01, G10

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# 1 Introduction

Through the allocation of external financing, lending standards play an important role in the economy, determining which entrepreneurs get funding, which consumers buy houses, and which firms grow. Figure 1 plots a measure of lending standards in the market for commercial and industrial (C&I) loans from banks. Lending standards, by this metric and many others, are highly cyclical, tightening in recessions and loosening in booms. For example, the recent credit boom-bust cycle was associated with relatively lax lending standards in the lending boom of the mid-2000's, when credit spreads and default rates were low, and relatively tight lending standards during credit crunch and recession that followed, when spreads and rates were high. Notably, while the volume of lending has increased since the crisis, lending standards have not loosened much.

In this paper we develop a dynamic model of lending standards and analyze whether lending standards amplify and propagate fluctuations, whether they are efficiently set by private markets, and, if not, whether government policy can improve the allocation of credit and real economic outcomes. In our model, a credit market consists of a mass of competitive banks and a pool of potential borrowers who are initially identical conditional on public or readily-available information (e.g. identical within credit score brackets). Each instant, borrowers are drawn from the pool and approach banks in search of a loan to fund an investment project. Projects differ by the type of borrower, and can either be of high or low quality, implying a positive or negative net present value for the investment project and thus for the loan. Lenders choose lending standards.

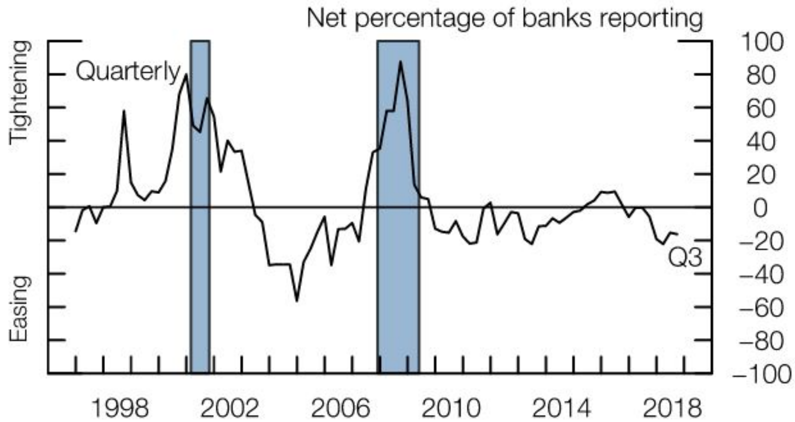
Lending standards have two dimensions. First, lenders can lend less or condition the terms of a loan on public information. For example, a lender might deny loans to all borrowers with credit scores below some threshold. In our model, provided that the typical loan in a given credit market is profitable, a bank can lend to all applicants in that market. Alternatively, the bank may lend to only a subset of borrowers or, if the typical loan has a negative net present value, the bank can freeze credit to this market and make no loans.

Our main focus however is on the second dimension of lending standards: lenders can acquire additional information about the future payoff of lending to a potential borrower before deciding whether or not to originate the loan.[1] For example, lenders might interview a potential borrower, conduct a detailed valuation of their business plan, or verify reported information such as employment or income. In our model, lenders can tighten lending standards by expending costly effort to create private information about the future payoff of lending to that borrower and condition lending on this information. Importantly, the information is private and non-verifiable, and borrowers whose loan applications are declined at one bank may apply at another bank later.

The dynamics of our model are determined by the the interplay between lending standards and borrower quality. By rejecting borrowers found to be low-quality, a bank with tighter lending standards worsens the pool of potential borrowers for all banks in the future. Thus, the current

[1]These two types of lending standards are not completely distinct. Loan terms can be used to screen borrowers. Section 6 discusses how our main results would apply under this alternative screeing technology.

**Figure 1:** Change in Bank Lending Standards for C&I Loans




*Note:* Figure shows the shows the share of banks reporting tightening less the share reporting loosening. Banks can also report no change in standards. *Source:* Senior Loan Officer Opinion Survey on Bank Lending Practices, Board of Governors of the Federal Reserve System (2018).

quality of potential borrowers reflects past lending standards. At the same time, lending standards depend on the current quality of potential borrowers. The key to understanding the dynamics of lending standards is the implication that lending standards are *dynamic strategic complements*. When one bank tightens lending standards, other banks are later confronted with more adversely selected potential borrowers, which increases their incentive to follow suit and tighten standards as well.

The key to understanding our normative results is that lending standards have *negative externalities.* Lending standards that lead to the rejection of some potential borrowers raise the share of low-quality applicants that apply for loans in the future which wastes banks' resources on screening more low-quality projects and/or increases the share of low-quality loans made. Since this cost that is not internalized by any individual bank, government intervention to prevent excessively tight lending standards can be beneficial. We characterize constrained-efficient policy and its implementation. Among other results, we show conditions under which policies such as government-subsidized loan guarantees can increase welfare, and under which the same policy can be detrimental to welfare if instead implemented after a delay.

Our first main result is that credit markets exhibit hysteresis. Markets with the same set of fundamentals may see persistently different lending volume, credit spreads, default rates, and lending standards depending on their specific history (i.e. their initial conditions). While at any point in time, the equilibrium of our model is unique, there are multiple steady states in the single state variable in our model—the share of high-quality potential borrowers in the pool, the *pool quality*. When the pool quality is high, banks do not find it worth the effort to check the quality of all applicants just to avoid the occasional low-quality borrower, and normal lending standards are optimal, which do not reduce the quality of the pool. When the pool quality is low, however, banks find it worthwhile to screen borrowers and so reject many low-quality borrowers which contributes to the low average quality of the pool. Accordingly, in the steady state with normal lending standards ("pooling steady state"), the volume of lending is high and, both because lending involves no screening costs and because the average quality of borrowers is high, loan spreads are low. In the steady state with tight lending standards ("screening steady state"), the volume of lending is low and, both because lending involves screening costs and because these costs are in equilibrium born only by the borrowers who are funded, loan spreads are high.

As a result of the multiplicity of steady states, temporary changes in market fundamentals—e.g. shifts in the payoff structure of borrowers' projects or the share of good borrowers entering the pool—can set in motion a self-reinforcing dynamic culminating in a *permanent* shift in the credit market equilibrium. Thus, absent interventions or changes in market fundamentals of the opposite sign, the endogenous response of lending standards can amplify and propagate fluctuations in fundamentals. Section 3.2 contains an example that illustrates these properties. This feature of our model is consistent with the limited relaxation of lending standards following the Great Recession documented in Figure 1.

Our second main result is that government intervention can improve private market outcomes, and in particular, that policies that relax lending standards can increase efficiency. These results follow in part from the fact that the two steady states in our model are Pareto-ranked: the planner always prefers the pooling equilibrium to the screening equilibrium. This ranking is a result of the negative externality associated with tight lending standards.[2] To characterize optimal policy in general, we solve a dynamic planning problem for the socially optimal path of lending standards. Optimal lending standards are concisely characterized by a *social* threshold of average borrower quality that always lies strictly below the private threshold. Thus, for intermediate levels of average borrower quality, loose lending standards are socially optimal but banks find it privately optimal to impose tight lending standards. It is important to note that because the pool of potential borrowers is a common resource, there is no way for individual banks to recover the costs of loose lending standards from later profits absent collective, i.e. government, actions.

The implication of this difference between private and socially optimal lending standards is that for small temporary declines in the pool quality, no intervention is needed, as the private threshold is not crossed. However, for large enough temporary declines in pool quality, the optimal policy response is an intervention that ensures that banks do not screen and that credit standards remain normal. Such temporary support for lending markets involves short-run costs and long-run benefits through improvements in the pool of potential borrowers. Further, we show that the short-run costs are larger the later an intervention occurs, so that *early timing* of interventions becomes crucial. In fact, if the government prefers not to intervene in the steady state with tight lending standards, then a long enough delay by the government allows the pool quality to fall below the social threshold,

[2] It is worth noting that in our model, lending standards do not influence the quality of new potential borrowers entering the pool of potential borrowers (as in Hu (2018), discussed below). However, because interest rates are lower, the incentive for borrowers to be higher quality is greater when lending standards are loose than when they are tight, as we discuss in Section 6.

so that intervention becomes detrimental to welfare. Section 5 contains an example that illustrates these properties. This normative feature of our model is consistent with government support for lending markets during or following temporary downturns.

It is important to note that optimal support for lending improves outcomes, but does not reach the first-best. While inconsistent with our modelling assumptions, the best policy would be to would eliminate the externality associated with screening, for example by making the outcome of any valuation public. We discuss the existence of credit registries in practice and in theory and why they may not correct the externality of evaluation in Section 4.2.

While we focus on the second dimension of lending standards – the decision to aquire costly information and screen borrowers – we also show that there are situations in which banks impose tight lending standards of the first type – restricting lending without screening. For example, if the quality of the average borrower in the pool is low enough, banks do not lend at all because they can neither cover the costs of screening nor lend profitably without screening. Because this type of lending standard does not have the externality associated with screening, the private threshhold for these tight lending standards can be inefficiently high or low, depending on incidental modelling choices.

More interestingly, for some parameter values, there is an intermediate range of pool quality in which banks impose tighter lending standards by rationing credit instead of screening, a situation we refer to as *slow thawing*. The logic behind credit rationing in our model is quite different from the typical credit rationing due to adverse selection (Stiglitz and Weiss (1981), Mankiw (1986)). At these levels of pool quality, banks lend without screening, but if banks lent to all borrowers, the pool quality would improve so rapidly over time that borrowers would choose to delay borrowing and wait for lower interest rates in the future. Previously-rejected borrowers would not delay however, making it unprofitable for banks to lend. In equilibrium, banks constrain lending to the point where not-previously-rejected borrowers are indifferent between delaying and not. Section 3.3 contains an example of this phenomenon, in which growth of lending volume and decline in interest rates is non-monotonic, slow, then rapid, then slow again as the market converges to the pooling steady state.

We discuss extensions to our model, as well as the relative importance of different modelling assumptions, in Section 6.

**Related literature** In our analysis lenders may face adverse selection. This is because the pool of borrowers may have been "cream skimmed," with good borrowers being systematically removed from the pool by lenders who employ tight lending standards. This source of adverse selection is also the central feature in Fishman and Parker (2015) and Bolton et al. (2016) and there gives rise to a strategic complementarity with regard to information acquisition that is conceptually similar to ours. That is, lenders find it more profitable to acquire information about potential borrowers if other lenders are acquiring such information. Theirs are static models where the complementarity leads to multiple equilibria, e.g., one with information acquisition and one without. And since these models are static, they cannot address the evolution, over time, of the quality of the borrower pool which is the focus of the dynamic model developed here. The intertemporal aspect of our model is key, shaping equilibrium behavior, e.g., the multiplicity of steady states but uniqueness of equilibrium, as well as the policy prescriptions, e.g., the importance of the timing of interventions.

Ruckes (2004) and Dell'Ariccia and Marquez (2006) also analyze static models of lending standards but neither features the strategic complementarity we emphasize. In Ruckes (2004) lending standards are strategic substitutes. This is because lenders simultaneously acquire information about borrowers and if a borrower is rejected or quoted a high interest rate, the borrower cannot then seek out other potential lenders. In Dell'Ariccia and Marquez (2006) there is cream skimming by informed lenders but these lenders are endowed with their information.

Hu (2018) analyzes a dynamic model and, like in our analysis, banks choose lending standards and the quality of the borrower pool evolves over time. A key difference is that in Hu (2018), lending standards are strategic substitutes, due to the positive response of the average quality of newly-entering borrowers to tighter lending standards. With this specification, Hu (2018) finds a number of interesting results regarding economic recoveries, e.g., his model may exhibit double dip recoveries.

Here, the incentive to collect information regarding an asset is higher if agents in the past have collected information, and hence there has been cream skimming. Asyrian, Fuchs, and Green (2019) and Zou (2019) analyze models in which the incentive to collect information is higher if agents in the future are expected to collect information. This is because a buyer of the asset today may want to sell the asset in the future. There is no motive to resell a loan in our analysis and so this effect is not present in our analysis.

In our model, when lenders determine whether to lend they do not observe borrowers' prior activity, e.g., how long the borrowers have been in the market for a loan or whether the borrowers have been previously denied credit. Since bank borrowing rates are publicly observed, such information would be informative regarding whether has been denied credit and is thus a type-$L$ and not worth financing. Similarly, in Daley and Green (2012, 2016) the history of offers received by the informed party is not observable. By contrast, in Chari et al. (2014) the history of the privately-informed party is observed by potential trading partners; their equilibrium features a partial separation of the high- and low-quality types.

In Axelson and Makarov (2017), borrowing rates are not publicly observed (borrowers make private offers to banks). So knowing that a borrower's offer was rejected does not necessarily indicate a bad borrower; rather the borrower may have made a low offer. Another key difference is that in their analysis, acquiring information on a borrower is costless and hence the information acquisition choice is trivial. These features of their model lead to the following interesting result: introducing a credit registry that tracks borrowers' loan application histories, but not the borrowing rates offered, can lead to more adverse selection and quicker market break down.

A number of papers study dynamic adverse selection models without information acquisition. Daley and Green (2012, 2016) and Malherbe (2014) analyze models where current markets can break down when high-quality sellers remain absent, waiting for market prices to improve over time. This behavior is related to, but distinct from, our slow thawing dynamics. In these models, the path of market prices over time separates good sellers from bad. In our slow thawing dynamics, the equilibrium *composition* of borrowers does not change, only the *speed* of lending is reduced. Zryumov (2015) and Caramp (2017) study models where bad sellers strategically enter when market prices are good. This, in and of itself, does not lead to a market shutdown (lower prices positively select entrants), but as Caramp (2017) emphasizes, the bigger presence of bad sellers can raise the likelihood of adverse selection induced market failures in the future.

In Gorton and He (2008) lending standards vary over time because of a different sort of strategic interaction among lenders. Theirs is a repeated games model of tacit collusion among banks. In equilibrium no bank defects from the collusive arrangement but a "punishment phase" can be triggered. The punishment phase entails tight lending standards which in turn implies less lending and lower bank profits and is interpreted as a credit crunch. So even though the quality of borrowers does not vary over time, lending standards do vary over time.

Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), and Dang, Gorton, and Holmstrom (2009), among others, analyze how the trading of debt securities minimizes adverse selection problems. Their results follow because the payoff on debt is less sensitive to the condition of the underlying assets as compared to other securities. Our analysis also features debt securities, though ours is a trivial security design setting. Debt would remain optimal, however, even with a more general setting.

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## 2 A Model of Lending Standards

Time is continuous and runs from $t$ to infinity, $t \in [0, \infty)$. There are two sets of agents: a unit mass of potential borrowers who have no capital and are looking to fund projects and a large mass $\mathcal{J}$ of competitive banks. All agents are risk neutral and have discount rate $\rho > 0$. The main state variable in the model is the quality composition of the pool of borrowers, defined below and denoted by $x_t$, which both determines and is influenced by the main control variables, banks' lending standards, denoted by $z_{jt}$, which will turn out to be identical across banks in equilibrium.

### Borrowers and banks

**Borrowers.** At Poisson rate $\kappa > 0$, a potential borrower receives an investment opportunity. This opportunity is a project that requires an up-front investment of 1. Borrowers have no capital and must fund the investment externally. If the borrower raises the funds and makes the up-front investment at time $t$, then the project returns both a pledgeable cash flow at time $t + T$ and a non-pledgeable private benefit $u > 0$ (in present value) to the borrower. With this private benefit all borrowers will have the incentive to finance their project, even if they know they will receive no monetary benefit.

To capture differences in borrower quality, we assume that there are two types of borrowers: type $H$ ("high quality") and type $L$ ("low quality"). Type-$H$ borrowers always have positive NPV investment opportunities. The pledgeable cash flow of a type-$H$ borrower's project is $D_H$, with net excess return $r_H \equiv e^{-\rho T} D_H - 1 > 0$.[3] Type-$L$ borrowers always have negative NPV projects, with pledgeable cash flow $D_L$ and net excess return $r_L \equiv e^{-\rho T} D_L - 1 < 0$. A borrower's type is permanent, always type $H$ or always type $L$. We refer to $r^{\Delta} \equiv \frac{r_H - r_L}{-r_L} > 0$ as the (normalized) *return difference* between the investments of the two types.

When an investment opportunity arises, borrowers choose whether or not to apply to the competitive banking sector for a unit of funding to implement their project. A borrower that applies for funding is either approved or denied depending on whether she satisfies the bank's lending standards. If a borrower is funded, she invests in her project, exits the pool to run the project. Alternatively, if the borrower does not apply for funding or is denied funding, she returns to the pool where at rate $\kappa > 0$ a new investment opportunity arises.

Potential borrowers in the pool "die" and leave the pool at Poisson rate $\delta > 0$. One can interpret 'dying' borrowers as ones who will no longer receive investment opportunities.

New borrowers arrive as type $H$ with exogenous probability $\lambda$ and as type $L$ with probability $1 - \lambda$. For much of the paper we make the following assumptions about the inflow if new borrowers so that the size of the pool is constant at 1:

both potential borrowers that die and borrowers that are funded are immediately replaced in the pool by new potential borrowers. As a result, it will suffice to keep track of the fraction of type-$H$ borrowers in the pool of potential borrowers at time $t$, $x_t \in [0, 1]$. We relax these assumptions and allow for fluctuations in the size of the pool of borrowers in Section 5.

Finally, we assume throughout that the average project of a borrower entering the pool has positive NPV based on the pledgeable cash flow.

**Assumption 1.** *The average investment project has a positive net present value* $\lambda r_H + (1 - \lambda) r_L > 0$.

**Banks and lending standards.** Banks make two decisions. First, they decide whether to be *active* or *inactive*. Second, conditional on being active, they choose their *lending standard*, that is, how aggressively to screen potential borrowers.

At any instant $t$, a bank may choose to be *active*, in which case it enters a competitive lending market, where it may receive a loan application by a borrower. Alternatively a bank may choose to be inactive in which case it makes no loans and consequently receives no loan applications. Let $\theta_{jt}$ denote the probability that bank $j$ is active at time $t$. While generally all banks are active in our model, e.g. at all steady-states where $x = const$, there may be a region in the state space with equilibrium *credit rationing*, $\theta_{jt} < 1$, where banks offer fewer loans than borrowers demand.

[3] It is a net excess return because the per-period log return $\frac{1}{T} \ln(1 + r_H) = \frac{1}{T} \ln(D_H) - \rho$.

An active bank $j$ also chooses a *lending standard* $z_{jt} \in [0, \overline{z}]$, where $\overline{z} \in [0, 1]$. With lending standard $z_{jt}$, a type-$L$ borrower is identified as such with probability $z_{jt}$, in which case her loan is denied.[4] Otherwise, the borrower's loan is approved. As along as screening is imperfect, $z_{jt} < 1$, some type-$L$ borrowers get financed. A bank's cost of utilizing the lending standard $z_{jt}$ is $\tilde{c} z_{jt}$, where $c \equiv \frac{\tilde{c}}{-r_L} > 0$ is the (normalized) marginal cost. The most lax lending standard corresponds to $z_{jt} = 0$, in which case all loan applications are deemed to meet the lending standards of bank $j$. Banks choose lending standards to maximize expected profit. Given a lending standard $z_{jt}$, banks offer to lend 1 in exchange for a promised loan payment at time $t + T$ equal to $D_{jt}$.

Due to symmetry and competition, it is without loss of generality to assume that all banks choose the same probability of being active, $\theta_t$, the same lending standard $z_t$, and the same loan payment $D_t$. With a loan face value of $D_t$, repayment is $\min\{D_t, D\}$, where $D$ is the payoff on the investment, $D_L$ or $D_H$ depending on borrower type. Since type-$L$ borrowers have negative NPV investments, $D_t > D_L$ for a bank to break even in expectation. Thus, type-$L$ borrowers always default. The repayment $D_t$ is without loss of generality bounded above by $D_H$ since any higher $D_t$ will not generate additional repayment. So if a loan is made (meaning the bank can break even) then type-$H$ borrowers will not default. We define $r_t \equiv e^{-\rho T} D_t - 1$ as the *credit spread* charged by the bank since $\rho + \frac{1}{T} \ln(1 + r_t)$ is per-period (log) return on a loan that does not default. We note that $r_t$ always lies in $(r_L, r_H)$.

**Information structure.** Before screening, banks cannot distinguish between type-$H$ and type-$L$ borrowers.[5] Borrowers have no private information about their type when they enter the pool. And for as long as a borrower has no such private information, we call her an *average borrower*. Some type-$L$ borrowers learn that they are type-$L$ after being denied funding by a bank because of a failure to meet the lending standard. We call these borrowers *rejected borrowers*.[6] The shares of average and rejected borrowers are endogenously determined. For instance, the lower the lending standard $z_t$, the fewer rejected borrowers will be in the pool. All agents have common knowledge of the structural parameters of the lending market and the initial fraction of type-$H$ borrowers in the pool, $x_0 \in [0, \lambda]$.[7] Also, all agents can infer past, current, and future $x_t$.

## A borrower's problem

Taking the path of credit spreads $\{r_t\}$ as given, borrowers with investment opportunities choose whether to apply for a loan at each time $t$. Let $\varphi_t^a$ denote the probability that an average borrower

[4] A type-$H$ borrower is never misidentified as a type $L$, a modeling assumption important for tractability, as discussed in Section 6.

[5] Observe that for each individual bank, previously screened loan applicants will represent a zero mass in the pool of borrowers and can therefore be ignored.

[6] Thus there are three types of borrowers in the pool at any time: average borrowers who are actually type $H$, average borrowers who are actually type $L$, and rejected borrowers (always type $L$). We show however that optimal behavior depends only on the share of type-$H$ borrowers, $x_t$, because the behavior of all type-$L$ borrowers is the same.

[7] Here, $x_0 = \lambda$ corresponds to a pool consisting entirely of average borrowers.

with an investment opportunity applies for a loan—as opposed to waiting in hope of an improvement in borrowing opportunities. Let $\varphi_t^r$ denote the probability that a rejected borrower with an investment opportunity applies for a loan. Letting $J_t^a$ and $J_t^r$ denote the value functions of an average borrower and a rejected borrower, respectively, the optimal strategies for the two satisfy the following Hamilton-Jacobi-Bellman equations:

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa\theta_t\varphi_t^a \left\{\lambda\left(r_H - r_t + u\right) + (1-\lambda)(1-z_t)u + (1-\lambda)z_t J_t^r - J_t^a\right\} + \dot{J}_t^a - \delta J_t^a \tag{1a}$$

$$\rho J_t^r = \max_{\varphi_t^r \in [0,1]} \kappa\theta_t\varphi_t^r \{(1-z_t)(u - J_t^r)\} + \dot{J}_t^r - \delta J_t^r, \tag{1b}$$

where $J_t^a$ and $J_t^r$ satisfy the transversality conditions $\lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^a = \lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^r = 0$. For an average borrower, (1a) reflects three possible outcomes that may occur when she has an investment opportunity, is matched with an active bank, and chooses to apply for financing: with probability $\lambda$ she is type $H$ and is funded, receiving a monetary payoff of $r_H - r_t$ and $u$ in private benefits; with probability $(1-\lambda)(1-z_t)$, she is type $L$ but satisfies the lending standard, receiving a payoff of $u$ in private benefits; and with probability $(1-\lambda)z_t$, she is type $L$ and does not satisfy the lending standard (is rejected), receiving a payoff of $J_t^r$. For a rejected borrower who has an investment opportunity and is matched with an active bank, (1b) reflects the fact that with probability $1 - z_t$, she satisfies the lending standard and receives a payoff of $u$ in private benefits; otherwise, she continues as a rejected borrower.

With strategies $\{\varphi_t^a, \varphi_t^r\}$, there is a flow of

$$\kappa_{Ht} \equiv \kappa\varphi_t^a x_t \tag{2}$$

type-$H$ borrowers applying for loans. Note that all of the type-$H$ borrowers belong to the sub-pool of average borrowers. There is a flow of

$$\kappa_{Lt} \equiv \kappa\varphi_t^a \frac{1-\lambda}{\lambda} x_t + \kappa\varphi_t^r \frac{\lambda - x_t}{\lambda} \tag{3}$$

type-$L$ borrowers applying for loans. For the derivation of (3), let $A_t$ denote the share of average borrowers at time $t$, with $1 - A_t$ being the share of rejected borrowers at time $t$. The fraction of type-$H$ borrowers in the whole pool is $x_t = A_t\lambda$. The flow of type-$L$ borrowers equals $\kappa\varphi_t^a A(1 - \lambda) + \kappa\varphi_t^r(1 - A)$. Substituting in $A_t = x_t/\lambda$ yields (3). In equilibrium, it will be the case that $\varphi_t^a = \varphi_t^r = 1$, so that $\kappa_{Ht} = \kappa x_t$ and $\kappa_{Lt} = \kappa(1 - x_t)$.

## A bank's problem

Since there is a flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications by borrowers at time $t$, it is without loss to assume that there are at most a flow of $\kappa_{Ht} + \kappa_{Lt}$ active banks at time $t$. As will be seen below, there are cases where some banks remain inactive in equilibrium, leaving only $\theta_t\left(\kappa_{Ht} + \kappa_{Lt}\right)$ active banks, with

$\theta_t \in [0,1]$. A fraction $\theta_t$ of the flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications is then received by the $\theta_t(\kappa_{Ht} + \kappa_{Lt})$ active banks.

Conditional on flows $\kappa_{Ht}, \kappa_{Lt}$ and credit spread $r_t$, an active bank's lending standard $z$ solves

$$\Pi_t(r_t) \equiv \max_{z \in [0,\bar{z}]} \kappa_{Ht} r_t + \kappa_{Lt}(1-z) r_L - (\kappa_{Ht} + \kappa_{Lt})\tilde{c} z. \tag{4}$$

Taking $z$ as given, Bertrand competition among banks then determines $r_t$ by

$$\Pi_t(r_t) = 0. \tag{5}$$

Whenever this cannot be satisfied by any finite $r_t$, no bank will find it profitable to lend. In this case, we set $r_t = \infty$ and $\theta_t = 0$.

### Evolution of the borrower pool

The evolution of the quality of the borrower pool is given by

$$\dot{x}_t = \theta_t \kappa_{Lt}(1-z_t)\lambda - \theta_t \kappa_{Ht}(1-\lambda) + \delta(\lambda - x_t), \tag{6}$$

which is the combination of three distinct forces: the first term accounts for the $\theta_t \kappa_{Lt}(1-z_t)$ type-$L$ borrowers who are funded and replaced with a fraction $\lambda$ of type-$H$ borrowers; the second term accounts for the $\theta_t \kappa_{Ht}$ type-$H$ borrowers who are funded and replaced with a fraction $1-\lambda$ of type-$L$ borrowers; and the third term accounts for the $\delta\lambda$ type-$H$ borrowers being born and $\delta x_t$ borrowers dying each instant.

### Equilibrium

We define an equilibrium as follows:

**Definition 1.** Given an initial share of type-$H$ borrowers $x_0 \in [0,1]$ in the pool, an *equilibrium* consists of a path of the fraction of type-$H$ borrowers $\{x_t\}$, credit spreads $\{r_t\}$, shares of active banks $\{\theta_t\}$, borrowers' application decisions $\{\varphi_t^a, \varphi_t^r\}$, implied application flows of type-$H$ and type-$L$ borrowers $\{\kappa_{Ht}, \kappa_{Lt}\}$, and screening choices $\{z_t\}$ such that

- $\{\varphi_t^a, \varphi_t^r\}$ solve each type's maximization problem (1) given $\{r_t, z_t, \theta_t\}$,
- $\{\kappa_{Ht}, \kappa_{Lt}\}$ are determined by (2) and (3),
- $z_t$ solves the bank's maximization problem (4) given $\{r_t, \kappa_{Ht}, \kappa_{Lt}\}$,
- $r_t$ is determined by the zero profit condition for banks (5) given $\kappa_{Ht}, \kappa_{Lt}$ whenever possible; if not, $r_t = \infty$ if $\kappa_{Ht} = 0$,
- $\{x_t\}$ follows the law of motion (6),

- at no time $t$ can a bank raise its profit $\Pi_t$ by being active, charging a rate $\tilde{r} < r_t$ that average borrowers would weakly prefer to waiting, and lending to the entire set of borrower applicants (a flow of $\kappa x_t$ type-$H$ and a flow of $\kappa(1 - x_t)$ type-$L$ borrowers).

A *steady state (equilibrium)* is an equilibrium in which all equilibrium objects $\{x_t, r_t, \theta_t, \varphi_t^a, \varphi_t^r, z_t\}$ are constant over time.

To study variation in lending standards, we make the following assumption on parameters throughout the remainder of this paper.

**Assumption 2.** *The cost of bank screening $c$ is not too low or too high:*

$$1 - \lambda < c < 1 - x^s + \bar{z}^{-1} \min\left\{x^s r^\Delta - 1, 0\right\},$$

*where* $x^s = \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z})+\delta\kappa^{-1}}$.

The first inequality in Assumption 2 ensures that the bank screening cost $\tilde{c}$ is high enough that tight lending standards do not strictly dominate normal lending standards. The second inequality ensures that there can exist a steady state in which tight lending standards are optimal.

# 3 Equilibrium characterization

The model's tractability allows for a tight analytical characterization of the set of equilibria, starting with steady-state equilibria.

## 3.1 Steady-state equilibria

Borrowers' and banks' behavior simplifies significantly in a steady-state equilibrium.[8] Borrowers, facing the same interest rate $r_t = r$ at all times, have no incentive to wait and therefore choose $\varphi_t^a = \varphi_t^r = 1$. In fact, they strictly prefer borrowing to waiting,

$$\lambda\left(r_H - r_t + u\right) + (1-\lambda)(1-z_t)u + (1-\lambda)z_t J^r - J^a > 0, \tag{7}$$

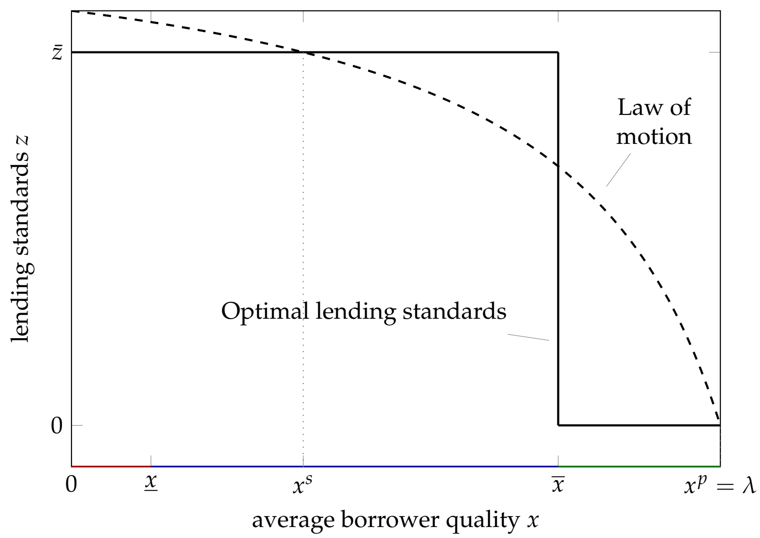
because, for any bank to be active, the loan rate needs to be weakly below the highest pledgeable payoff, $r \leq r_H$, but then $J^a > 0$, which is equivalent to (7) in a steady state. Under these conditions, all banks are active in a steady state, $\theta = 1$.

The steady-state quality of the pool $x$ and the steady-state lending standard $z$ are jointly determined, by the interaction of two forces. On the one hand, the law of motion of $x$, (6), implies that when $\dot{x} = 0$,

$$x = \lambda - \lambda \frac{(1-\lambda)\, z}{(1-\lambda z) + \delta\kappa^{-1}}. \tag{8}$$

[8]Since prices and quantities are constant, we drop the time subscripts for this subsection.

**Figure 2:** The two forces shaping steady-state equilibria.




*Note:* This figure shows two curves whose intersections yield the steady-state pool quality $x$ and the steady-state lending standard $z$. The solid line represents the optimal choice of the lending standard, (9). The dashed line represents the pool quality $x$ that is caused by any given lending standard $z$ through the law of motion.

This equation highlights that tighter lending standards—higher $z$—are associated with a lower steady-state quality of the pool of borrowers $x$, as more low-quality borrowers are rejected by banks. This effect is greater when the effects of lending standards on the pool are more persistent (low death rate $\delta$) or when opportunities to invest arise more frequently (high $\kappa$) and so potential investors are evaluated more frequently.

On the other hand, banks solve (4) and choose tighter lending standards $z$ precisely when the pool is more adversely selected,

$$z = \begin{cases} 0 & \text{if } x > \overline{x} \\ [0, \overline{z}] & \text{if } x = \overline{x} \\ \overline{z} & \text{if } x < \overline{x} \end{cases}, \quad \text{where } \overline{x} \equiv 1 - c. \tag{9}$$

The combination of the two equations (8) and (9) is illustrated in Figure 2. Both represent downward-sloping relationships between $x$ and $z$, and given Assumption 2 admit three intersections, each of which represents a steady-state equilibrium. This logic is summarized in the following proposition.

**Proposition 1** (Steady state equilibria)**.** *There exist three steady-state equilibria:*

*(i) A* pooling *steady state with $z = 0$ and $x^p = \lambda$.*

*(ii) A* screening *steady state with $z = \overline{z}$ and $x = x^s \equiv \lambda - \lambda \frac{(1-\lambda)\overline{z}}{(1-\lambda\overline{z}) + \delta\kappa^{-1}}$.*

*(iii) A* mixed *steady state with $z = \frac{\lambda - \overline{x}}{\lambda - \lambda\overline{x}}\left(1 + \delta\kappa^{-1}\right) \in (0, \overline{z})$ and $x = \overline{x}$.*

The root of the multiplicity is a *dynamic strategic complementarity* among banks. According to (9), banks naturally respond to a lower quality pool by tightening their lending standards; however, according to (8), tighter lending standards worsen the pool itself, creating an even bigger incentive for banks to tighten their standards in the future. This reasoning rationalizes the existence of the pooling and screening equilibria, see Figure 2. The mixed steady state formally exists but will turn out to be unstable and therefore play no role in the remainder of the analysis.

The pooling and screening steady states have the following important characteristics.

**Corollary 1** (Quality of funded borrowers)**.** *In the pooling steady state,*

1. *the credit spread $R$ is lower.*
2. *more projects are funded: $\kappa$ relative to $\kappa x + \kappa(1-x)(1-z)$ in the screening steady state.*
3. *the default rate is higher: the share of funded borrowers who are of type $L$ is $\lambda$ relative to $\frac{(1-x^s)(1-z)}{x^s+(1-x^s)(1-z)} < \lambda$ in the screening steady state.*

The first point follows from the observation that a lower pool quality, ceteris paribus, hurts banks' profits, and therefore requires larger credit spreads for banks to break even. This is true, even if banks choose to screen more, in which case credit spreads rise partly due to greater screening costs. The second point follows from the fact that screening reduces the flow of borrowers that receive funding. The third point has the following subtlety. In fact, when $\delta = 0$ (no birth and death from the pool of potential borrowers), the default rate, which in our model equals the share of type $L$ borrowers among all funded borrowers, is equal to $1-\lambda$ in *any* steady state. Indeed, in the screening steady state, the imposition of tight lending standards exactly balances the low average project quality in the pool. This leads to the same share of bad projects being funded as in the pooling equilibrium.[9]

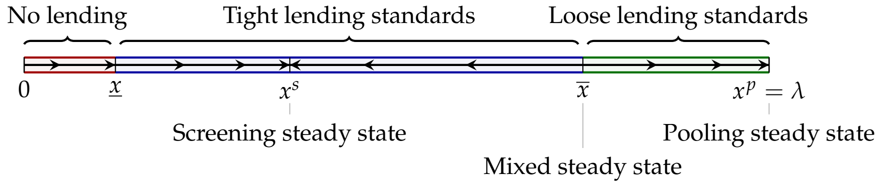
This may give the impression that tight lending standards cannot be social optimal. They are costly and may not improve the quality of funded borrowers while lowering the volume of funding. Yet, as the following section emphasizes, this is a *dynamic* model where the transition to one of the steady states plays a crucial role.

### 3.2 Transitional dynamics

An important factor that simplifies the steady state analysis is the fact that banks are always active in a steady state, $\theta = 1$. This is no longer true in equilibria with dynamics. In particular, there are now up to two regions in which banks may choose to remain inactive. Naturally, this is the case when the quality of the pool $x$ is very low, so that even a breakeven loan rate at its maximum of $r = r_H$ is not enough to recoup the losses incurred from lending to the many type-$L$ borrowers in

[9] The results in Corollary 1 are robust to alternative assumptions on the dynamics of the borrower pool, e.g. assuming a constant inflow, rather than a constant pool size.

**Figure 3:** State space and banks' optimal strategies.




the pool. Formally, this requires $x$ to be so low that $\Pi(r_H) < 0$, or equivalently,

$$\theta(x) = \begin{cases} 0 & \text{if } x < \underline{x} \\ [0,1] & \text{if } x = \underline{x} \end{cases}, \quad \text{where } \underline{x} \equiv \frac{1 - \overline{z} + c\overline{z}}{r^{\Delta} - \overline{z}} \tag{10}$$

Somewhat more surprisingly, however, banks may also remain inactive in a subset of the pooling region, where inactivity can endogenously reduce the volume of lending and slow down the speed of convergence to the pooling steady state. We refer to this region as "slow-thawing" region and it is described in detail in Section 3.3. Until then, we assume parameters are such that there is no such region:

**Assumption 3** (No slow thawing)**.** *Assume that there is no slow-thawing region, that is,* $\theta(x) = 1$ *for all* $x \geq \underline{x}$.

For the sake of exposition, this assumption is stated in terms of endogenous objects. The analytical condition is stated in the next section.

Under Assumption 3, the equilibrium transitional dynamics are as follows.

**Proposition 2** (Transitional dynamics without slow thawing)**.** *Suppose Assumption 3 holds and* $x_0 \in [0, \lambda]$ *is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium, in which banks' activity policy satisfies (10) for* $x \leq \underline{x}$*, their lending standards are given by (9), and borrowers never wait,* $\varphi_t^a = \varphi_t^r = 1$*. As* $t \to \infty$*, the credit market converges to*

*(i) the screening steady state,* $x_t \to x^s$*, if* $x_0 < \overline{x}$*.*

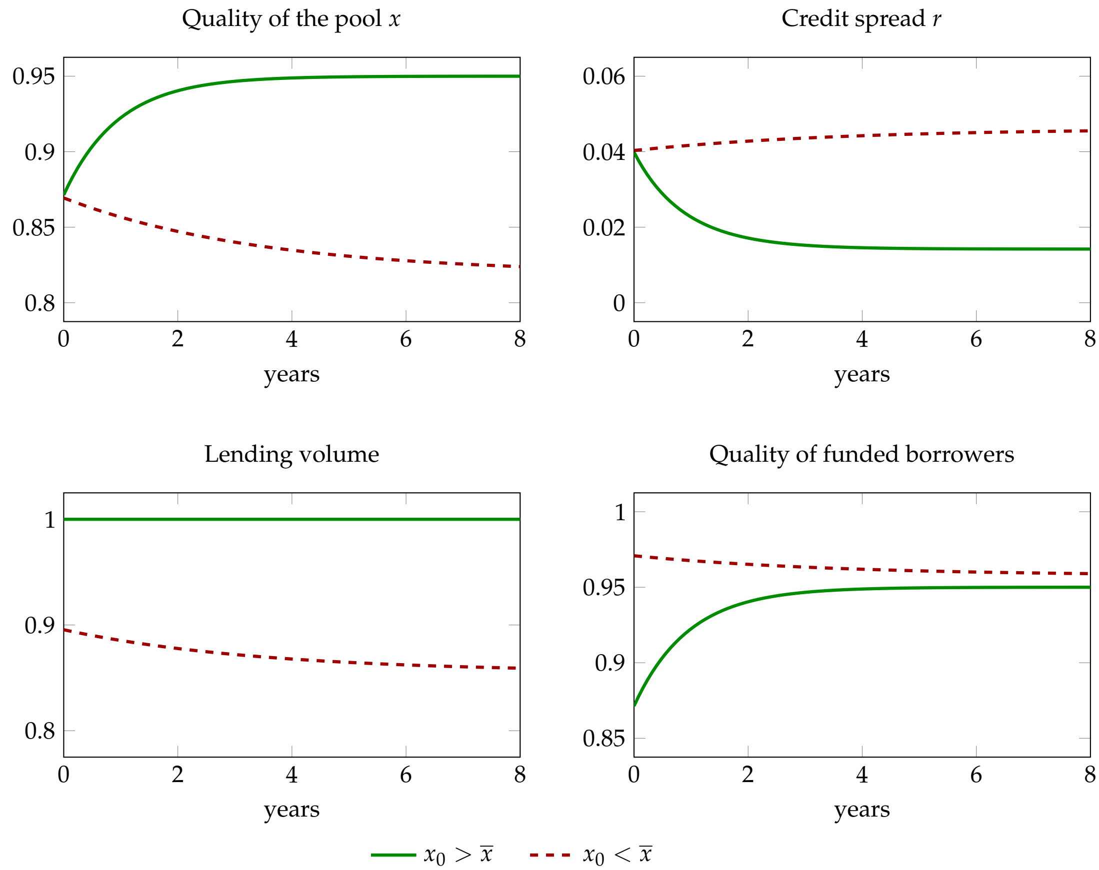
*(ii) the mixed steady state,* $x_t \to \overline{x}$*, if* $x_0 = \overline{x}$

*(iii) the pooling steady state,* $x_t \to x^p$*, if* $x_0 > \overline{x}$

Proposition 2 provides a complete characterization of the equilibrium transitional dynamics. Despite the multiplicity of steady states (Proposition 1), there is a unique equilibrium for any $x \neq \bar{x}$, giving unambiguous model predictions..

The model predictions can be seen in Figure 3, which illustrates the state space of the credit market and highlights the transitional dynamics in the three different regions of bank behavior: the "no lending" region for low pool qualities, where banks are inactive ($\theta_t = 0$) and the pool quality

**Figure 4:** The self-reinforcing property of lending standards.



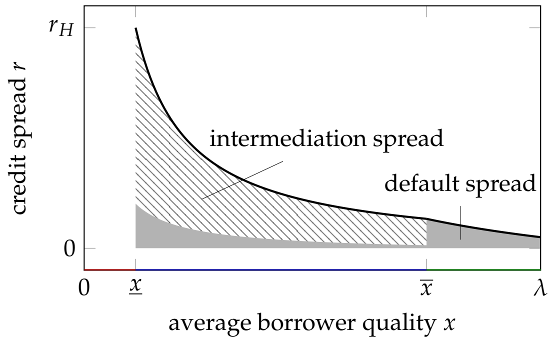

*Note.* This figure shows two sets of transitional dynamics in a credit market without slow thawing. Green and solid is a market starting at $x_0 = \overline{x} + \epsilon$ and therefore banks have normal lending standards; red and dashed is a market starting at $x_0 = \overline{x} - \epsilon$ and therefore banks impose tight lending standards. The parameters used for this simulation are as follows: TBD.

improves only due to death and birth; the "tight lending standards" region, where banks screen borrowers $z_t = \overline{z}$ and the market approaches the screening steady state; and the "normal lending standards" region where banks choose $z_t = 0$ and the market returns to the pooling steady state.

A crucial part of the diagram is at $x = \overline{x}$. This point represents a sharp boundary between the tight and normal lending standards regions and gives rise to an important model prediction, a "bifurcation" property: when $x_0$ lies above $\overline{x}$, the credit market converges to the pooling steady state; when $x_0$ lies below $\overline{x}$, however, the self-reinforcing nature of tight lending standards pushes the market to the screening steady state.

The bifurcation property also comes out in Figure 4 where we simulate the credit market with two different initial values for $x_0$, one just above $\overline{x}$ (green, solid) and one just below $\overline{x}$ (red, dashed). As can be seen, this small difference in initial conditions leads to quite different evolutions of pool qualities $x$, credit spreads $r$, and lending volumes $\kappa_{Ht} + \kappa_{Lt}(1 - z)$. The final panel of Figure 4 shows

**Figure 5:** Breakeven credit spread as function of pool quality $x$.




*Note:* Grey is the component of the credit spread that is due to default risk (the *default spread*). Hatched is the component of the credit spread that is due to intermediation costs (the *intermediation spread*).

the evolution of the quality of funded borrowers, $\kappa_{Ht}/\left(\kappa_{Ht}+\kappa_{Lt}(1-z)\right)$, which is one minus the default rate. The market with the relatively *lower* initial pool quality initially has a much lower lending volume and default rate, as banks are imposing tight lending standards. Interesting, the two markets initially have quite similar credit spreads. Over time, and foreshadowing our results on efficiency and optimal policy, the slightly lower initial pool quality causes convergence to a steady-state with much higher credit spreads and lower lending volume but quite similar default rates (asymptotically identical according to Corollary 1).

Interestingly, we can characterize both equilibrium credit spreads $r_t$ across credit markets and the relative roles of both expected default and intermediation costs. First, how do credit spreads vary with pool quality $x_t$? Markets with higher $x_t$ have lower default rates for any given lending standard, but also are more likely to have normal lending standards, and so higher default rates and higher bank funding costs. The second effect suggests that a higher $x$ could be associated with a higher default rate and so a higher credit spread, but this turns out not to be the case. As the following proposition formally proves, $r_t$ is still inversely related to $x_t$.

**Proposition 3** (Equlibrium credit spread)**.** *The equilibrium credit spread $r_t = r(x)$ is decreasing in the fraction of type-H borrowers $x$ and is given by*

$$r_t = r(x) = \begin{cases} \infty & \text{if } x < \underline{x} \\ 1 + (r_L)x^{-1}\left\{c\overline{z} + (1-\overline{z})(1-x)\right\} & \text{if } \underline{x} \le x < \overline{x} \\ 1 + (r_L)x^{-1}\left\{1-x\right\} & \text{if } x \ge \overline{x} \end{cases} . \tag{11}$$

Using (11), we can decompose $r(x)$ into a *default spread*, $-r_L x^{-1}(1-z(x))(1-x) > 0$ where $z(x)$ is the optimal screening choice given $x$; and into an *intermediation spread* $-r_L x^{-1} c z(x) > 0$. Figure 5 plots the credit spread $r(x)$ and these two components over the state space, illustrating the inverse relationship of $r(x)$ with pool quality $x$. The shaded areas in Figure 5 highlight that the default spread changes discretely at $x = \overline{x}$ as banks switch between screening and not screening, but this change is offset by an equally large change in the spread due to the costs of intermediation. The spread rises significantly due to intermediation costs at lower pool qualities $x$. The decoupling of credit spreads and credit risk in this region of the state space provides a new rationale for why, at times, credit spreads may appear to be high given the credit risk. He and Milbradt (2014) attribute such high credit spreads to low liquidity. Alternatively one might rely on risk aversion as an explanation. Here the high credit spread derives from the costs of intermediation.

The monotonicity of $r(x)$ is also reflected in Figure 4, with a rising loan rate for the credit market with the lower quality of potential borrowers and a falling loan rate for the market with higher quality. The falling loan rate raises an obvious question: wouldn't average borrowers have an incentive to wait for lower loan rates? The answer is yes in certain cases. Credit can be restricted even with normal lending standards, so that both lending volume and the improvement of the pool of borrowers are slowed, and credit markets recover ("thaw") much more slowly than otherwise.

### 3.3 Slow thawing

When $x_0$ is just above $\overline{x}$, it is possible that if $\theta_t = 1$ and $r(x)$ were as defined in Proposition 3, then the increase in $x_t$ over time would lead to so rapid a decline in $r_t$ that average borrowers would strictly prefer not to accept loans but would instead prefer to wait for lower credit spreads ($\varphi^a = 0$). This, however, would cause a market shutdown because no bank is willing to lend to rejected borrowers only, and therefore cannot be an equilibrium. Instead, the equilibrium must exhibit a slower speed of transition so that the improvement in the pool of potential borrowers and the decline in rates both occur more slowly and average borrowers are willing to accept loans in equilibrium. For this transition to be slower, it must be that not all banks are active ($\theta_t < 1$), which can only be the case if there are no profits to be made from making a new loan (see Definition 1). This is precisely the case when borrowers are also indifferent between waiting and applying for loans. The following proposition proves that these strategies are indeed an equilibrium.

**Proposition 4** (Slow thawing)**.** *There exists a threshold $\hat{x} \in (0, x^p)$, such that: (i) if $\hat{x} \leq \overline{x}$, there is no slow thawing; if (ii) $\hat{x} > \overline{x}$, then for any $x \in [\overline{x}, \hat{x})$, a positive fraction of banks are inactive $\theta(x) < 1$, where*[10]

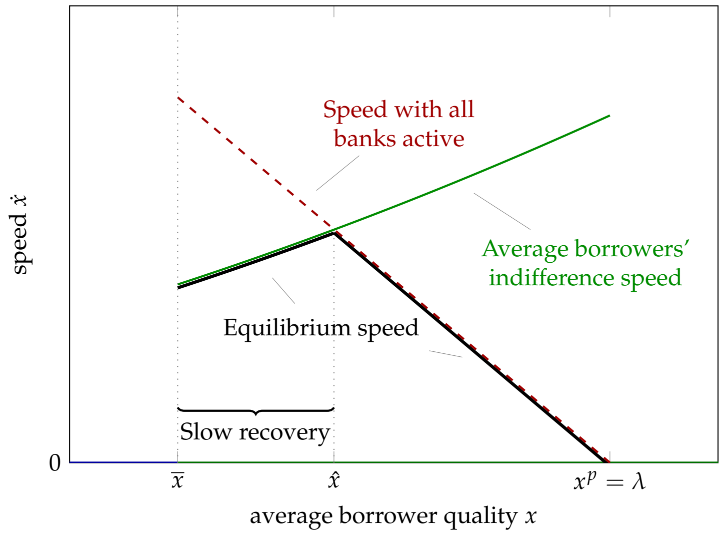
$$\theta(x) = \frac{(\rho + \delta)\,(r_H - r(x))}{-\kappa r'(x)(\lambda - x)} - \delta\kappa^{-1}. \tag{12}$$

*where $r(x) = -r_L x^{-1}\{1 - x\} > 0$. Borrowers are indifferent and apply for loans, $\varphi_t^a = \varphi_t^r = 1$. $\hat{x}$ is determined as the unique solution to $\theta(\hat{x}) = 1$ in $(0, x^p)$.*

The intuition for the expression in (12) comes directly from the indifference condition of average borrowers. Again, focus on the instructive special case where $u \to 0$. Then, the HJB of an average

[10] For the sake of readability, we assume here that the utility benefit from running a project is comparatively small, $u \to 0$.

**Figure 6:** Slowly thawing credit markets.




*Note.* This figure illustrates when there exists a region with "slow thawing" where credit markets recover only very slowly from a crisis. The green solid line represents the speed at which the pool quality needs to improve for average borrowers to be exactly indifferent between applying for loans (strictly preferred below the curve) and waiting (strictly preferred above). The red dashed line represents the speed of improvement when all banks are active. The equilibrium speed (black solid line) is the minimum of both curves.

borrower is given by

$$\rho J_t^a = \max_{\varphi_t^a \in [0,1]} \kappa \theta_t \varphi_t^a \left\{ \lambda \left( r_H - r_t \right) - J^a \right\} + \dot{J}^a - \delta J^a$$

with indifference between applying for a loan or not requiring that $J^a(x) = \lambda \left( r_H - r(x) \right)$. Substituting this back into the HJB yields an equation for the speed $\dot{x}$ at which the pool needs to improve for average borrowers to be indifferent,

$$\underbrace{-\lambda r'(x)\dot{x}}_{\text{benefit of waiting}} = \underbrace{(\rho + \delta)\lambda \left( r_H - r(x) \right)}_{\text{opportunity cost of waiting}}. \tag{13}$$

When is $\dot{x}$ the equilibrium speed? Precisely when $\theta_t$ is such that $\dot{x}$ satisfies the law of motion of $x$, (6). Together, (13) and (6) give (12).

Figure 6 schematically illustrates this logic. The green solid line represents the speed $\dot{x}$ at which average borrowers are indifferent between borrowing now and waiting for the pool to improve. This is an increasing line as the benefit of waiting declines the closer $x$ is to the pooling steady state. The red dashed line represents the speed at which the pool quality improves when all banks choose to be active. Clearly, where this line falls in the "borrow" region, it is also equal to the equilibrium speed, shown in black solid. Where it is in the "wait" region, however, for $x < \hat{x}$, it cannot be an equilibrium. There, a fraction $1 - \theta(x)$ of banks choose to be inactive, bringing down the equilibrium speed to match the one along the indifference curve. This leads to a hump-shaped thawing speed: initially little lending due to the threat of average borrowers waiting, a period of slow thawing as lending volume and the pool quality accelerate, followed by a period of normal convergence to the steady state.[11]

What determines how likely or how strong this period of slow-down is? The following corollary reveals the roles of interest rates, project payoffs, and meeting frequencies.

**Corollary 2.** *Fix a quality of the borrower pool $x \in (\overline{x}, x^p)$ and let $\dot{x}$ denote the speed of improvement in the pool's quality. Then:*

1. *Worse projects slow down the recovery: $\dot{x}$ falls with lower $r_L, r_H$.*
2. *Low aggregate interest rates, $\rho$, can backfire: if $x$ is in the slow-thawing region, $x < \hat{x}$, $\dot{x}$ falls with lower $\rho$.*
3. *Easy access to banks does not speed up the recovery: if $x$ is in the slow-thawing region, $x < \hat{x}$, greater meeting frequencies $\kappa$ do not raise $\dot{x}$; for $x > \hat{x}$, $\dot{x}$ rises with $\kappa$.*

Most noteworthy are the comparative statics in 2 and 3. When $\rho$ (or $\delta$) is low, holding $r_L, r_H$ fixed, this effectively makes average borrowers more willing to wait, shifting down the indifference curve in Figure 6 and slowing down the recovery. This channel suggests that typical expansionarymonetary policy – reducing bank funding rates to raise lending volume and stimulate credit markets – can backfire, or at least be less effective, in aiding the recovery from a financial crisis.

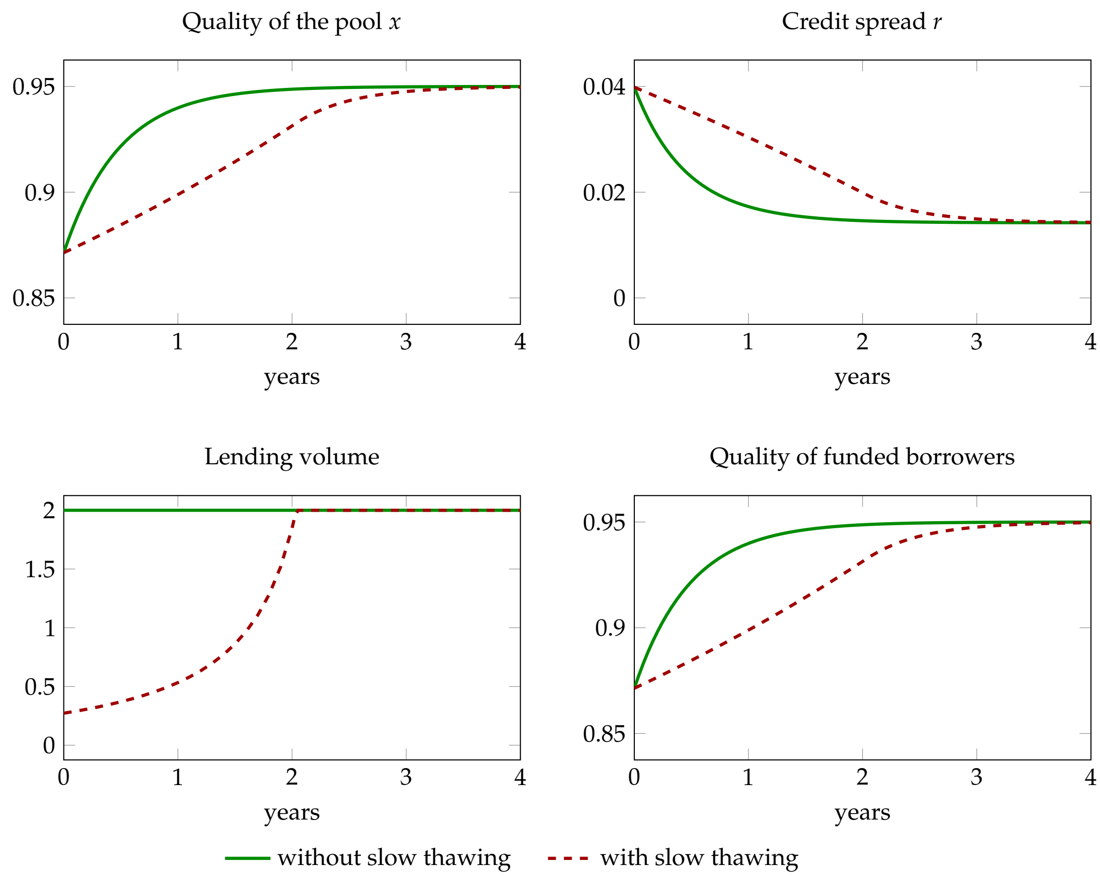
When the meeting frequency $\kappa$ of borrowers and banks increases, the red line in Figure 6 increases. This naturally increases the speed of the recovery towards the steady state outside the slow-thawing region. Inside that region, however, it has no effect. In fact, even when $\kappa \to \infty$, the transition towards the pooling steady state is slow and entirely determined by the indifference condition (13).

Figure 7 juxtaposes the transitional dynamics with slow thawing (solid green line) and the transitional dynamics without slow thawing (dashed red line). The latter was computed by ruling out slow thawing by assumption, imposing $\varphi_t^a = 1$, $\theta_t = 1$, and dropping equilibrium equation 7 in place of . .assuming that potential borrowers are myopic in the sense that (and only in the sense that) when they have the opportunity to invest, they approach the competitive banking sector and accept the loan and invest rather than optimally choosing whether instead to wait for their next opportunity to borrow ,. As is visible in the first panel, slow thawing can greatly slow the transition back to the pooling steady state and lead to a relatively low lending volume.

In closing, it is important to note that a similar region with slow thawing can also appear in the region between $\underline{x}$ and $x^s$ and slow down the convergence to the screening steady state from the left. Since our focus lies on the transitions between the pooling and screening steady states, we have relegated the characterization of that region to Appendix B.

[11] Note that Figure 6 does not show $\dot{x}$ just to the left of $\bar{x}$ because it is negative. By Proposition 3), $\dot{x} < 0$ implies $\dot{r} < 0$. With spreads decreasing over time, there is no incentive to delay and so no region of slow thawing.

**Figure 7:** Slowly thawing credit markets.




*Note.* The plots compare two transitions back to the pooling steady state. Green solid is a transition without "slow thawing", where average borrowers always accept current loan offers and banks do not ration credit; red dashed is a transition with slow thawing, where banks ration credit in equilibrium.

# 4 Efficiency

At the heart of the positive model predictions is a dynamic strategic complementarity: when current banks operate tight lending standards and screen out low-quality borrowers, future banks prefer tight lending standards as well. We next characterize the (constrained) efficient outcomes.[12]

## 4.1 Constrained efficient policy

In our concept of constrained efficiency, we allow the planner to control banks' activity and screening decisions, subject to borrowers' application decisions, so as to maximize the sum of agents' utilities.[13] To keep the derivations and exposition clear, we focus on the case where the private benefit from running the project $u$ is vanishingly small, $u \to 0$. In this section, it is further assumed that the planner can set the path of market interest rates $\{r_t\}$, and therefore prevent average borrowers from waiting (i.e. there is no slow thawing). We discuss relaxing this assumption below.

The constrained efficient planning problem is then given by

$$\max_{z_t \in [0,\overline{z}], \theta_t \in [0,1]} \int_0^\infty e^{-\rho t} \kappa \theta_t \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt \tag{14}$$

subject to the law of motion of $x_t$, (6). The solution to this problem is given in the next proposition.

**Proposition 5** (Second-best policy)**.** *There exists a threshold $\overline{x}^* \in [0, \overline{x})$ such that the second-best planner sets:*

$$z_t = \begin{cases} \overline{z} & \text{if } x_t < \overline{x}^* \\ 0 & \text{if } x_t > \overline{x}^* \end{cases} \tag{15}$$

*The threshold $\overline{x}^*$ is the largest $x$ that satisfies*

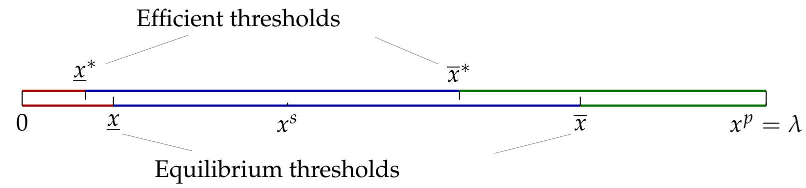
$$\underbrace{\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} r_H + (1 - \overline{z}) \left(1 - \frac{\rho x + \alpha^s x^s}{\rho + \alpha^s}\right) r_L - \tilde{c}\overline{z}}_{\text{Average social benefit of screening}} \geq \underbrace{\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L}_{\text{Average social benefit of pooling}} \tag{16}$$

*where $\alpha^p = \kappa + \delta$, $\alpha^s = \kappa + \delta - \overline{z}\lambda\kappa$. In particular, for any $x_t \in (\overline{x}^*, \overline{x})$, equilibrium lending standards are (second-best) inefficiently tight.*

[12] The unconstrained efficient allocation (first-best) in our model would allow the planner to only fund average borrowers, which, given Assumption 2, he would do without screening.

[13] Since borrowers and banks are risk-neutral, this is without loss when the planner has access to transfers between agents.

**Figure 8:** Constrained efficient vs. equilibrium lending standards.




*For any $\overline{x}^* > x^s$, the optimal policy for bank activity is given by*

$$\theta_t = \begin{cases} 0 & \text{if } x_t < \underline{x}^* \\ 1 & \text{if } x_t > \underline{x}^* \end{cases}$$

*for some $\underline{x}^* \in [0, \overline{x}^*)$.*

Proposition 5 reveals that the optimal policy is similar in spirit to the equilibrium: when the quality of the pool is relatively high, $x > \overline{x}^*$, normal lending standards, $z = 0$, are optimal; when it is not, tight lending standards are optimal. But, importantly, the cutoffs for the optimal policy and for the market equilibrium differ: There exists a region in the state space, $(\overline{x}^*, \overline{x})$, where equilibrium lending standards are too tight relative to the constrained-efficient outcome.

To develop an intuition for this finding, imagine the current pool quality is $x$ and banks operate normal lending standards, $z = 0$, in all periods from now on so that the credit market ultimately converges to the pooling steady state $x^p$. In (16), one can think of $\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$ as the time-averaged fraction of type-$H$ borrowers funded. The weight on current $x$ is $\rho$, as with greater discounting the present becomes relatively more important; the weight on the (long-run) steady state $x^p$ is $\alpha^p = \kappa + \delta$, which is the speed at which $x$ converges to $x^p$. The average social benefit of screening is therefore the weighted average surplus from lending to each type of borrower,

$$\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L.$$

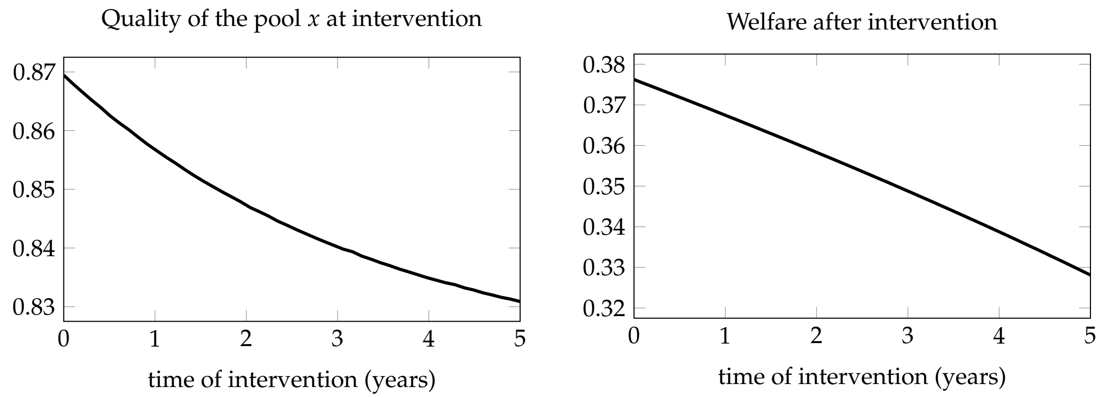
An analogous expression describes the social benefit of screening, where we additionally account for both the costs of screening and the fact banks successfully screen a fraction $\bar{z}$ of of low-quality borrowers, giving rise to (16).

In contrast, the private cut-off, $\overline{x}$, is the largest value satisfying

$$\underbrace{xr + (1 - \overline{z})(1 - x) r_L - \tilde{c}\overline{z}}_{\text{Average private benefit of screening}} \quad \geq \quad \underbrace{xr + (1 - x) r_L}_{\text{Average private benefit of pooling}} \tag{17}$$

which is calculated using the *current* fraction $x$ of type-$H$ borrowers and entirely ignores the dynamic

**Figure 9:** Early interventions dominate late ones.




*Note.* This figure shows how intervention policies affect a credit market that is transitioning towards the screening steady state. The horizontal axis shows the time at which an intervention starts (where 0 corresponds to the immediate, constrained efficient intervention).

consequences from screening and pooling. In particular, since in the relevant region it holds that

$$\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} < x < \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$$

agents privately ignore the dynamic costs from screening relative to pooling. Therefore, $\overline{x}^* < \overline{x}$. The private and social thresholds are shown in Figure 8.

Another way to highlight the differential dynamic consequences of pooling and screening is to compare steady states.

**Corollary 3.** *When both steady states exist (Assumption 2), the screening steady state has strictly lower welfare than the pooling steady state.*

If $\delta = 0$, this result would be a simple consequence of the fact that screening potential borrowers is costly and the quality of funded borrowers is independent of the steady state (Corollary 1). But with $\delta > 0$, screening borrowers has a social benefit because a share of them are never funded. Still, the corollary shows that, as long as the cost $c$ of screening is not too low (in which case the pooling steady state ceases to exist), welfare of the pooling steady state always dominates welfare of the screening steady state.

There are two important practical implications from the existence of a non-empty interval $(\overline{x}^*, \overline{x})$ where the equilibrium diverges from the optimal allocation.

*1. Intervention timing matters.* Figure 9 simulates the characteristics of a credit market that starts at a given $x_0 \in (\overline{x}^*, \overline{x})$ for various times when an intervention starts (on the horizontal axis). The later the time of intervention is, the lower is the quality of the borrow pool when the policy switches from screening to pooling (left panel). Later intervention times thus increase the short-run losses incurred at the start of the intervention and are therefore welfare-inferior to early interventions. In fact, after a sufficiently long time, if $x_t$ has fallen below $\overline{x}^*$, intervening may even be welfaredominated by not intervening at all and allowing the market to converge to the screening steady state, despite its having lower welfare than the pooling steady state. That is, a late intervention may be worse than a policy of not intervening at all, a result that underscores the importance of the timing of interventions in our model. However, it may instead be the case that even at the screening steady state, it is still optimal to intervene and relax lending standards. In this case, intervention is always optimal (when the quality of the pool is weakly above that in the screening steady state).

*2. Better screening technology may be detrimental to welfare.* Suppose the cost $\tilde{c}$ of operating tight lending standards falls. While it is clear that such a reduction in costs necessarily raises efficiency in any steady-state equilibrium, it can decrease welfare because it raises thresholds both for the market convergence to a screening equilibrium and for the efficient intervention, $\overline{x}$ and $\overline{x}^*$. Therefore, if a market is just recovering from a crisis, with $x_0$ just above $\overline{x}$, such a technological improvement may cause $\overline{x}$ to rise above $x_0$ and thereby prevent a recovery and lead to a reduction in welfare. If $\overline{x}^*$ also rises above $x_0$ then it is too costly for policy to mitigate this decline in welfare.

A decrease in costs $\tilde{c}$ represents an improvement in private information technology. What happens if instead public information technology (e.g. credit reporting) improves? A crude way to capture such a change is as an increase in $\delta$, an increase in the probability that rejected borrowers die. While the death of average borrowers has no effect on equilibrium as they are replaced in the pool by an equal measure of new average borrowers, a greater death rate of rejected borrowers does matter for equilibrium. A larger $\delta$ increases (decreases) the speed of convergence when $x$ is increasing (decreasing), and raises the pool quality in the screening steady state, so therefore unambiguously increasing welfare. Thus, the welfare effects of improving public information are unambiguously positive.

### 4.2 First-best policy

While inconsistent with our modelling assumptions, the first-best policy would be to would eliminate the externality associated with screening, for example by making the outcome of any valuation public. While one might think that some credit registries serve this purpose, in fact most registries track the existing credit and delinquencies rather than applications and certainly do not document evaluations. The model speaks to why. Making public bad information destroys private value.

### 4.3 Implementation of the constrained optimum

There are several ways in which a government or a regulating authority could implement the constrained efficient outcome, that is, normal lending standards when $x \in (\overline{x}^*, \overline{x})$, where we continue to assume that there is no slow thawing. Since such an intervention entails short-run losses (see Figure 9) and the model's banking sector is competitive, either the government or type-$H$ borrowers have to carry that burden.

An example for a policy in the first category is a government-funded loan insurance program. In this case, the government would provide an insurance benefit $b > 0$ to be paid whenever a borrower defaults. Assuming $b$ is in present value terms, this incentivizes banks to pool precisely if

$$\frac{b}{-r_L} > 1 - \frac{c}{1-x}.$$

This condition is satisfied for $b = 0$ in the region $x > \overline{x}$ where pooling is privately optimal. It requires nonzero insurance benefits $b = b(x) > 0$ when $x < \overline{x}$. As function of the pool quality, $b(x)$ is decreasing in $x$. This means, a typical intervention starting from some $x_0 < \overline{x}$ requires large insurance benefits early on, which are then phased out over time, until they disappear entirely.

Our model is thus consistent with the ability of government loan guarantees to increase the efficiency of credit markets by decreasing lending standards and interest rates. Examples of such loan guarantees in the US include mortgage markets, student loans, and credit for international trade. All these loan guarantees require that the borrower meet eligibility requirement and/or condition rates on readily-available public information. Also notably, the retraction of a loan guarantee arguably characterized the start of the US financial crisis of 2008-2009. By allowing Lehman Brothers to fail, the US government retracted an implicit guarantee of the short-term debt of large financial institutions. In response, lenders (buyers of short-term commercial paper of financial institutions) tightened lending standards and interest rates rose and lending volumes declined.[14]

Because this implementation entails a subsidy from the government to the banking sector, this policies reduces credit spreads. But a policymaker could implement the optimum instead by taxing future interest payments and refunding to all lenders (the equivalent of a loan guarantee funded from future bank interest earnings from loans of the same cohort). Such a policy would not entail a subsidy from the taxpayer so the burden clearly falls upon type-H borrowers. As a result, the policy increases credit spreads.

We assume that the private sector cannot observe which borrowers are rejected. But if the government could monitor banks and measure either lending standards or rejections (but, like the private sector, not observe the identity of the rejected agent), then another policy that implements the constrained optimum is to tax lending standards or rejections at a high enough rate to ensure normal lending standards (when $x \in (\overline{x}^*, \overline{x})$). This policy is the most direct: taxing the activity – tight lending standards – that has the negative externality. Interestingly, this policy increases equilibrium spreads even though no tax is collected. But because the profits from type-*H* borrowers have to cover the losses from lending to more type-L projects, this policy raises credit spreads.

Note finally, that all such policies are not privately optimal without market-wide collusion. Thus, there is a role for government in a competitive market.

[14]Of particular import was the withdrawal of lending by market mutual funds, which as structured did not have the ability to evaluate borrowers. Further, by regulation they were not allowed to lend more than 0.5% of their assets to any institution limited which limited the profits that might otherwise cover the cost of higher lending standards. In our model, this would be a restriction that any one bank could only provide a fraction of the funds needed by any given borrower, so lending would require several banks.

### 4.4 Limits to constrained efficiency

In practice, regulations or policies like government-funded loan subsidy or insurance programs are rarely undertaken for the entire financial sector, but instead usually apply only to certain types of institutions, such as traditional banks but not money market mutual funds or shadow banks for example.[15] We consider a situation where the government can only affect the lending decisions of some fraction $\eta \in [0, 1)$ of banks. For lack of a better term we refer to these banks as *government-owned banks*, even though they do not need to be literally owned by the government. We now ask what the optimal policy is under such limiting circumstances. For this section, we focus on the case without slow thawing or inactivity, i.e. $\theta = 1$, and further assume that government-owned banks always charge the same market interest rates as their private competitors.

To state the new planning problem for a given $x_0$, denote by $z^p(x)$ the optimal screening action of a private bank, that is, $z^p(x) = 0$ . Then, the planner solves the same objective as before,

$$\max_{z_t} \int_0^\infty e^{-\rho t} \kappa \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt$$

subject to the same law of motion of $x_t$,

$$\dot{x}_t = (\kappa + \delta)\,(\lambda - x_t) - \kappa z_t \lambda (1 - x_t),$$

with the exception that $z_t$ is now subject to an additional constrained,

$$z_t \in [(1 - \eta) z^p(x_t), (1 - \eta) z^p(x_t) + \eta \bar{z}], \tag{18}$$

rather than the entire interval $[0, \bar{z}]$. This is owed to the fact that only a fraction $\eta$ of overall lending standards can be controlled by the government.

Constraint (18) is no mild alteration to the planning problem. As we show in Appendix A.11, the optimal policy still takes a threshold form, where now there is a threshold $\bar{x}^*(\eta)$ that depends on the share of government-owned banks $\eta$. For low levels of $\eta$, this threshold is exactly equal to $\bar{x}$, that is, the planner does not want to implement any other allocation than the competitive equilibrium. Only when $\eta$ is sufficiently large,

$$\eta > 1 - \frac{(\kappa + \delta)\,(\lambda - \bar{x})}{\kappa \bar{z} \lambda (1 - \bar{x})}, \tag{19}$$

does the planner find it optimal to intervene in some region of the state space, $\bar{x}^*(\eta) < \bar{x}$.

Why is not optimal for the planner to intervene for low $\eta$? The planner's motivation for intervention is to shift away from convergence to the screening steady state to convergence to the

[15] For example, in the US financial crisis of 2008, the government extended deposit guarantees from traditional banks to money market mutual funds, but also did not extend the guarantees to other short-term debt markets. Similarly, the government bailed out the government-sponsored mortgage lending agencies and traditional banks but not private label securitizers nor mortgage brokers.

pooling steady state. If, however, $\eta$ is below the threshold in (19), the planner is not able to relax overall lending standards enough to induce the state $x_t$ to move towards pooling for values of $x_t$ close to the private threshold $\overline{x}$. Thus, it lacks the "firepower" to get to pooling and instead of getting stuck somewhere between the screening and pooling steady states, it chooses not to intervene at all.

## 5 A credit boom-bust cycle

One of the most salient features of the credit market is that it appears to come in "boom-bust cycles." In our model, booms and lending standards interact; neither is solely exogenously driving the other. We now demonstrate how our model can give rise to boom-bust dynamics. While such dynamics can be driven by exogenous movements in either lending standards or inflows to the pool of potential borrowers, in this section we study a credit boom shock driven by changes in the pool of borrowers.

### The credit boom

We feed into the model a "market size shock" which allows a flow rate $\mu$ of new borrowers to enter the pool until time $T$. The new borrowers are assumed to have a lower fraction of type-$H$ borrowers, $\underline{\lambda} < \lambda$, capturing the idea that these are not borrowers that usually believe they are able to get a loan.[16] Thus, the total size of the pool now evolves according to

$$\dot{N}_t = \mu 1_{\{t \leq T\}} - \delta(N_t - 1). \tag{20}$$

Our model in Section 2 involves a fixed pool size $N = 1$ and therefore needs to be amended to allow for dynamics in $N$. However, as we show in Appendix C, since the law of motion of $N$ is entirely exogenous in the model, the model still applies to a "normalized" version of the credit market, where all absolute quantities (total volume of loans, total welfare, profit, etc) are to be thought of as normalized by $N$. The only adjustment that then needs to be made is that the fraction of type-$H$ borrowers, $x$, now evolves according to

$$\dot{x}_t = \theta_t \kappa_{Lt}(1 - z_t)\lambda_t - \theta_t \kappa_{Ht}(1 - \lambda_t) + N_t^{-1}\left(\delta + \mu 1_{\{t \leq T\}}\right)(\lambda_t - x_t)$$

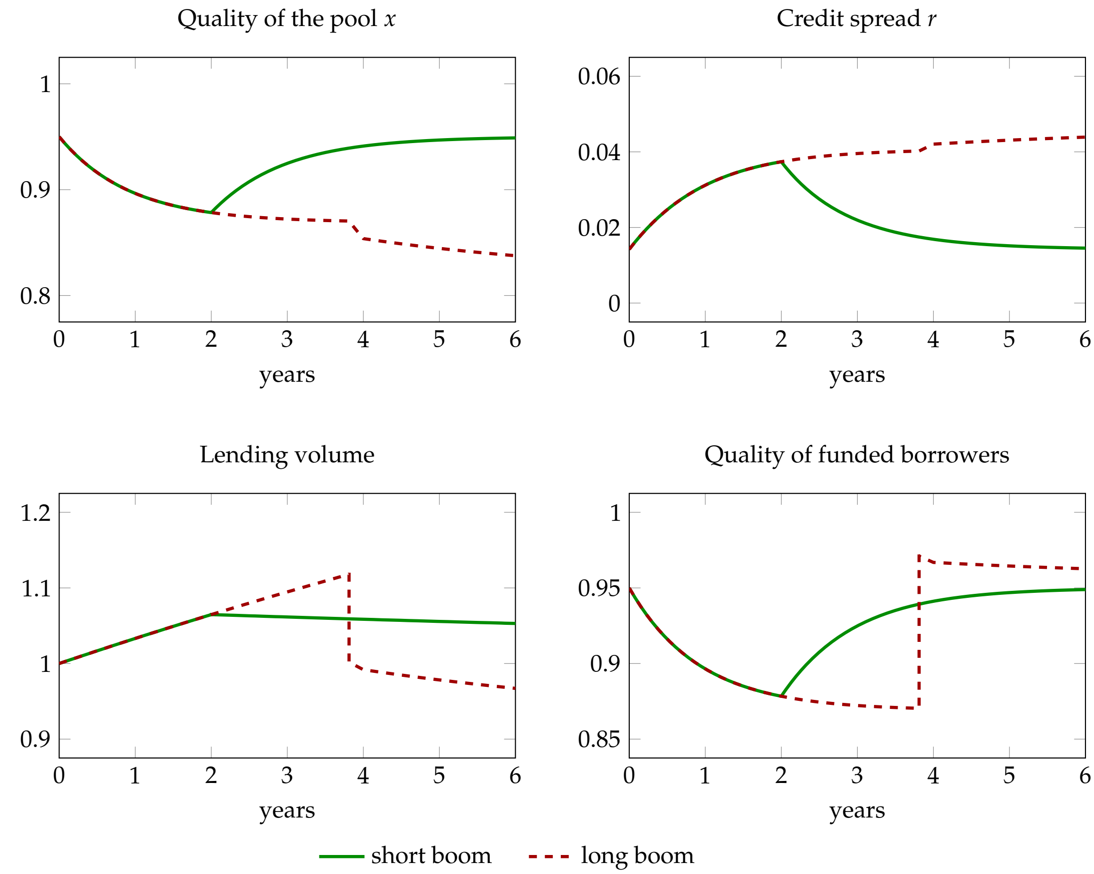
where $\lambda_t$ is the average quality of new borrowers entering into the pool,

$$\lambda_t \equiv \frac{\mu}{\mu + \delta} 1_{\{t \leq T\}} \underline{\lambda} + \frac{\delta}{\mu + \delta} \lambda.$$

For the simulations in this section, we calibrate the model parameters as follows: TBD. Under this parameterization, the market turns out not to have a region with slow thawing.

[16] We allow entering borrowers to know their average quality $\underline{\lambda}$.

**Figure 10:** The boom-bust cycle.




*Note.* This figure shows a credit market in response to two shocks: the green solid line represents a 2-year credit boom, the red dashed line a 4-year credit boom. A "credit boom" is modeled as an inflow of relatively less creditworthy borrowers. The transitions were computed using these parameters: TBD.

## The boom-bust cycle

We simulate the response to two different boom lengths, with $T_0 = 2$ years and $T_1 = 4$ years. The results are shown in Figure 10. As the solid green line shows, the short boom goes hand in hand with an increasing lending volume and a decline in the quality of borrowers, and ends in a soft decline ultimately converging back to the original steady state. Contrast this with the long boom (dashed red line). This boom ends in an abrupt decline in volume, an increase in lending standards, and a permanent transition away from the original steady state.

## 6 Discussion and Extensions

In this section, we discuss the importance of a few of our assumptions and several extension of our model.

The first point to emphasize is that there are potential benefits of lending standards omitted from our model that may be pertinent in some lending markets. In particular, tight lending standards provide an incentive to improve quality along the dimension that is being evaluates. Thus tight standards could increase the average quality of new projects, as potential borrowers exert effort to avoid having type-$L$ projects.[17] Under this assumption, tighter lending standards would have a positive externality: tighter standards today would increase the quality of the pool of potential borrowers for all banks in the future. This would provide a countervailing force to the effect of rejected borrowers in our model that would raise the relative efficiency of a steady state with screening.

Further, we note that tractability motivates some of our specific assumptions about the inflow of new potential borrowers. Our assumptions imply that the pool of potential borrowers has a constant size, which implies that $x$ is our sole state variable. One could instead for example assume that the inflow of new potential borrowers is constant, which would imply that the size of the pool would vary. While the choice of screening would still depend only on $x$, the dynamics of the market would in this alternative depend on the size of the pool as well as on $x$. While this would affect the dynamics of the market, this would not change many of the main lessons of the model, and in particular, the market would still exhibit two steady states (Corollary 1 would hold for a different range of parameter values) and the results on their relative efficiency, Corollary 3 derived in the Section 4.1, would still hold.

Turning to the banking sector, we have assumed that it is competitive so that banks make no profits in equilibrium. We conjecture that the qualitative features of the steady-states, dynamics and welfare results would all remain if banks shared the surplus of a match with a given potential borrower. The analysis is simplified by assuming that potential borrowers have all the bargaining power (or equivalently that banks compete by posting lending terms) as we have done.

Do our main results rely on our specific screening technology? For example, our screening technology never mistakes a type $H$ borrower for a type $L$ borrower. Such mistakes would imply that potential borrowers who are screened do not learn their type with certainty, and so there would be a distribution of beliefs among potential borrowers, with beliefs depending on the number of times a borrower had been screened and rejected for a loan. Such complexity would change the exact formula for $\dot{x}$. And it would complicate the analysis of the slow thawing region by potentially making possible regions with different speeds of slow thawing. Apart from this region however, since potential borrowers with different beliefs behave identically in the model (outside of any slow

[17]As noted earlier, Hu (2018) develops a model with this feature and studies its dynamics.

thawing region), our main results would remain intact.

Other changes to the screening technology are even less consequential. Our model can easily incorporate a screening technology is non-linear in cost. Concavity replicates our current results. Convexity would imply that rather than necessarily screening at a level of $\bar{z}$, banks might choose a lower level instead that equated marginal benefits of screening and marginal costs which would then necessarily depend on, and be increasing in, $x$. We have assumed that screening produces a binary signal, and it would be inconsequential to instead assume a continuous signal (banks would simply choose a cutoff value for their binary decision). Finally, if screening were correlated across banks, this would increase the strategic complementarity at the heart of our model since when one bank screens and rejects a potential borrower, it makes it easier for the next bank to detect that borrower as bad and so raises the private value to screening.

We have discussed the model as model of lending with a debt contract. But in fact, in our model there is an equity contract that delivers exactly the same payoffs to banks and borrowers of each type. This equivalence arises because of the model has only two types of investors so that all good. With more types, our model could become significantly more complex in general. While the exact degree of complexity would depend on how well the screening technology detected different types, the extensions we have considered have all involved more state variables, which raises the possibility of non-linear dynamics that can occur in such systems.

# 7 Concluding Remarks

The implications of our analysis all follow from the dynamic strategic complementarity associated with lending standards. If yesterday's lenders employed tight lending standards then it will be optimal for today's lenders to do the same. This is because yesterday's tight standards adversely affect the quality of today's pool of borrowers; that is, the pool will be cream skimmed. The adverse selection problem created by this cream skimming leads today's lenders to employ the same tight lending standards as yesterday's lenders. And so on for tomorrow's lenders. Alternatively, if yesterday's lenders employed normal lending standards then it will again be optimal for today's lenders to do the same. With normal lending standards, the pool of borrowers will evolve toward a pool comprised of average borrowers; and this is a pool (given our assumptions) that calls for normal lending standards.

Even if tight lending standards are not socially optimal, they may be the unique equilibrium outcome. For that case, we considered the possibility of government intervention that replaces an inefficient unique equilibrium with an alternative more efficient unique equilibrium; one with normal lending standards. Capitalizing on our dynamic model, we showed that while there are interventions that can put the market on the path to the socially efficient steady state outcome, the timing of the intervention is crucial for determining whether the intervention is value enhancing. If the intervention is delayed too long, it will not be value enhancing. An intervention considered involves a temporary subsidized (partial) loan guarantee program. Temporarily offering loan guarantees can eliminate lenders' incentives to employ tough lending standards. This in turn leads to an improvement of the borrower pool and then subsequent lenders will have the incentive to employ normal lending standards even without loan guarantees.

We considered the possibility that the borrower pool temporarily expands through the addition of below-average borrowers. By assumption, even though they are below-average borrowers, they are still good enough to be profitably funded with normal lending standards. The expansion of the borrower pool represents a boom that converts previously unfundable borrowers into now fundable borrowers. This boom may result in an immediate increase in lending volume followed by a bust, where the volume of lending collapses. Underlying the boom is a reliance on normal lending standards and underlying the bust is an endogenous switch to tight lending standards.

Our paper opens up several avenues for future research. For example, the government intervention that is analyzed effectively assumes that the negative shock that put the market on the path to the inefficient steady state is expected to never recur. But suppose it might recur. Is government intervention still optimal? Addressing this question requires a specification of the social cost of a government giveaway, e.g., a subsidized loan guarantee. With that we expect that it can be shown that if the likelihood that the shock recurs is low enough, then government intervention will be value enhancing. This is because with a low probability of a negative shock, the cost of an intervention can be amortized over a longer period of time.

Another extension could allow borrowers to exert effort to improve the quality of their projects. For instance, suppose when an average borrower is matched with a lender but before he is screened, the average borrower privately chooses effort, where more effort increases the likelihood of being a type-$H$ borrower and entails greater personal cost. Will the borrower have a bigger incentive to exert effort to become a type-$H$ borrower when he will be screened or when he will not be screened? For $x_t > \bar{x}$ let $r_t^p(x_t)$ denote the borrowing rate when lenders are pooling. With lenders pooling, the difference between the type-$H$ and type-$L$ borrower payoffs is $r_H - r_t^p(x_t)$ (a type-$H$ receives a cash payoff $r_H - r_t^p(x_t)$ and a private benefit $u$ while a type-$L$ receives only $u$). For $x_t < \bar{x}$ let $r_t^s(x_t)$ denote the borrowing rate when lenders are screening. With lenders screening, the difference between the type-$H$ and type-$L$ borrower payoffs is $r_H - r_t^s(x_t) + u - v(u)$, where $v(u)$ is the expected present value of a type-$L$ borrower's private benefit (accounting for the fact that this type-$L$ borrower might get financed now or in the future or may never be financed). From the preceding analysis - before introducing borrower effort - we have $r_t^p(x') < r_t^s(x'')$, for $x' > \bar{x} > x''$. And since $u$ is small, $u - v(u)$ is small. Therefore, before introducing borrower effort, $r_H - r_t^p(x') > r_H - r_t^s(x'') + u - v(u)$ for $x' > \bar{x} > x''$ and a borrower's incentive to exert effort is greater when banks are not screening. This suggests that tight lending standards do not boost borrowers' incentives to create high-quality projects and there is no counterveiling force to offset the negative effect that screening has on the borrower pool. If anything, borrower effort may amplify the effect of screening. That is, screening reduces the quality of the borrower pool, which in turn leads to lower borrower effort further reducing the quality of the borrower pool, which in turn raises lenders' incentive to screen, and so on. This discussion of borrower effort does not account for the effect of borrower effort on borrowing rates $r_t^p(x_t)$ and $r_t^s(x_t)$. But we conjecture that with effort increasing in $x_t$, the borrowing rate will drop even faster as $x_t$ increases, and therefore accounting for the effect of effort on borrowing rates will reinforce the amplification discussed here.

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# A Proofs and derivations

## A.1 Within-period lending game

## A.2 Steady state equilibria: Proof of Proposition 1

The three pairs $(x, z)$ mentioned in Proposition 1 are solutions to (8) and (9) if $\lambda > \overline{x}$, $x^s < \overline{x}$, and $\frac{\lambda-\overline{x}}{\lambda-\lambda\overline{x}}\left(1+\delta\kappa^{-1}\right) < \overline{z}$. The first two of these hold by Assumption 2 and the third is a straight consequence of the second.

We claim that the three pairs indeed constitute equilibria, with $\theta = 1$, $\varphi^a = \varphi^r = 1$ and with $R$ pinned down by Proposition 3. To prove this, first note that the law of motion (6) as well as the bank's maximization problem (4) are satisfied due to (8) and (9). The zero profit condition (5) pins down the interest rate (see our proof to Proposition 3). Finally, in any steady state the average borrower strictly prefers a loan today, that is,

$$\lambda\left(R_H - R + u\right) + (1-\lambda)(1-z)u + (1-\lambda)zJ^r - J^a > 0,$$

and since $R \leq R_H$ (which holds since $x^s \geq \underline{x}$ with $\underline{x}$ as in (10) due to Assumption 2) we have that $\theta = 1$ and $\varphi^a = \varphi^r = 1$.

## A.3 Proof of Corollary 1

The flow of projects being funded in the pooling steady state is $\kappa$, compared to $\kappa x^s + \kappa(1-x^s)(1-\overline{z})$ in the screening steady state. The credit spread result follows directly from Proposition 3 and the fact that $R(x)$ is strictly decreasing in $x$. The equilibrium default rate is given by

$$\frac{\kappa(1-x)(1-z)}{\kappa(1-x)(1-z)+\kappa x} = \left(1+\frac{x}{(1-x)(1-z)}\right)^{-1}$$

which can further be simplified to

$$(1-\lambda)\left(1+\frac{\lambda z\delta\kappa^{-1}}{(1+\delta\kappa^{-1})(1-z)}\right)^{-1}.$$

Thus, when $\delta = 0$, the equilibrium default rate is always equal to $1-\lambda$, irrespective of the steady state.

## A.4 Proof of Proposition 2

Let $x_0 \in [\underline{x}, \overline{x})$ ($x_0 \in (\overline{x}, \lambda]$). In that case, $z = \overline{z}$ ($z = 0$) is the optimal bank strategy (see (4)), and therefore the law of motion of $x$, (6), necessarily describes the unique equilibrium dynamics of $x$. By Assumption 6, $\theta_t = 1$ and therefore also $\varphi_t^a = 1 = \varphi_t^r$ (due to (7)).

The case $x_0 = \overline{x}$ is straightforward as $\overline{x}$ is a steady state.

Finally, if $x < \underline{x}$, $R(x) = \infty$, which is why the only possible equilibrium involves $\theta_t = 0$. In that region, therefore, the pool improves according to $\dot{x}_t = \delta(\lambda - x_t)$, until $x$ hits $\underline{x}$, in which case the law of motion switches to be the same as the one for an initial quality $x_0 \in [\underline{x}, \overline{x})$.

### A.5 Proof of Proposition 3

The zero profit condition (5) implies that

$$\Pi(R) = \kappa_H(R-1) + \kappa_L(1-z)(R_L-1) - (\kappa_H + \kappa_L)\tilde{c}z = 0.$$

Reformulating this we obtain

$$\kappa x(R-1)/(R_L-1) + \kappa(1-x)(1-z) + \kappa cz = 0$$

$$R = 1 + (1-R_L)\frac{cz + (1-x)(1-z)}{x}$$

which proves Proposition 3.

### A.6 Proof of Proposition 4

Define $\theta(x)$ as in (12) and define $\hat{x}$ implicitly as the unique value of $x < \lambda$ with $\theta(x) = 1$. Such a value exists since $\theta(x)$ is strictly increasing and continuous in $x$ with $\theta(0) = -\delta\kappa^{-1} < 0$ and $\lim_{x\to\lambda} \theta(x) = \infty$.

Assume $\hat{x} > \overline{x}$. Conjecture for any $x_0 \in [\overline{x}, \hat{x})$ that the equilibrium is one with $\theta_t = \theta(x_t)$. To verify the conjecture, we need to show that average borrowers are indifferent between taking a loan and waiting. Assuming $u \to 0$ in (1a), this is equivalent to

$$J_t^a = \lambda(R_H - R(x_t))$$

with

$$rJ_t^a = \dot{J}_t^a - \delta J_t^a.$$

Putting the two together, we obtain (13),

$$-\lambda R'(x)\dot{x} = (r+\delta)\lambda\left(R_H - R(x)\right).$$

The law of motion for $x$ with $\theta < 1$ is $\dot{x}_t = (\kappa\theta + \delta)(\lambda - x)$, which, together with (13) yields (12) and therefore confirms that average borrowers are, by construction, precisely indifferent.

### A.7 Proof of Corollary 3

By Assumption 2, $c \geq 1 - \lambda$. Therefore, welfare in the screening steady state is bounded above,[18]

$$W^s = x^s r^\Delta - (1-\bar{z})(1-x^s) - c\bar{z} \leq x^s \left(r^\Delta + 1 - \bar{z}\right) - (1-\bar{z}) - (1-\lambda)\bar{z} = x^s \left(r^\Delta + 1 - \bar{z}\right) - (1 - \lambda\bar{z})$$

Welfare in the pooling steady state is $W^p = \lambda(r^\Delta + 1) - 1$. Observe that $W^s$ increases in $x^s$, so $W^s$ can only ever be above $W^p$ if $x^s$ is as large as possible. Clearly, given the formula for $x^s$, $x^s$ is largest as a function of $\delta$ if $\delta = \infty$ where $x^s = \lambda$. In that case, we find

$$W^s \leq x^s \left(r^\Delta + 1 - \bar{z}\right) - (1 - \lambda\bar{z}) < \lambda \left(r^\Delta + 1 - \bar{z}\right) - (1 - \lambda\bar{z}) = \lambda \left(r^\Delta + 1\right) - 1 = W^p$$

Therefore, welfare of the pooling steady state always dominates that of the screening steady state.

### A.8 Proof of Proposition 5

We prove Proposition 5 in two steps. First, we determine the efficient screening policy $z^*(x)$ conditional on banks operating. Then we determine the optimal behavior for banks to operate $\theta^*(x)$.

#### A.8.1 Optimal screening policy $z^*(x)$

To do so, let $V(x,z)$ denote the present value of welfare if the current state of the market is $x$ and the screening policy is $z$ from hereafter, that is,

$$V(x,z) \equiv \frac{rx + \alpha^z x^z}{r + \alpha^z}(R_H - 1) + (1-z)\left(1 - \frac{rx + \alpha^z x^z}{r + \alpha^z}\right)(R_L - 1) - \tilde{c}z. \tag{21}$$

where $\alpha^z \equiv \kappa + \delta - \lambda\kappa z$ and $x^z \equiv \lambda - \lambda\frac{(1-\lambda)z}{(1-\lambda z)+\delta\kappa^{-1}}$. Also, denote by

$$v(x,z) \equiv r\left\{x(R_H - 1) + (1-z)(1-x)(R_L - 1) - \tilde{c}z\right\} \tag{22}$$

the flow value of policy $z$ at state $x$. Finally, we call

$$d(x,z) \equiv \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{23}$$

the derivative of $x$ at state $x$ under policy $z$ (see the law of motion in (6)). Observe that

$$rV(x,z) = v(x,z) + V_x(x,z)d(x,z) \tag{24}$$

as well as

$$d(x^s, \bar{z}) = 0 \;\; d(x^p, 0) = 0. \tag{25}$$

[18]We define all welfare expressions here as multiples of $\kappa$, for expositional clarity. $\kappa$ multiplies both $W^s$ and $W^p$ equally.

We first prove the following helpful lemma.

**Lemma 1.** *We have:*

1. If $\lambda\kappa\rho \geq r+\kappa+\delta$, pooling is strictly optimal for any state $x$, i.e. $z^*(x) = 0$.
2. If $\lambda\kappa\rho < r+\kappa+\delta$, $V(x,z)$ has negative cross-partials, $V_{xz} < 0$.
3. If $\lambda\kappa\rho < r+\kappa+\delta$ and $V(x,0) > V(x,z_1)$ for some $z_1 > 0$, then also $V(x,0) > V(x,z_2)$ for any $z_2 \in (0,z_1)$.

*Proof.* Assume $\lambda\kappa\rho \geq r+\kappa+\delta$. Suppose pooling were not strictly optimal for every state $x$. First, if $d(x,z^*(x))$ is ever non-negative for some $x < \lambda$, there must be a steady state at some $x^0 \in [0,\lambda)$ with some $z^*(x^0) > 0$. This cannot be optimal since

$$V(x^0,z^0) < V(x^0,0)$$

is equivalent to (after a few lines of algebra)

$$-\left(r+(1-\lambda)\alpha^p - rx^0\right)\left(\rho\kappa\lambda - (r+\kappa+\delta)\right) < (r+\alpha^s)(r+\alpha^p)c$$

which is true since the left hand side is negative. Second, assume $d(x,z^*(x))$ is positive everywhere. Then, $x^p = \lambda$ is still the unique steady state. Let $\mathbf{V}(x)$ the optimal value function. It has to hold that

$$r\mathbf{V}(x) = v(x,z^*(x)) + \mathbf{V}'(x)d(x,z^*(x)). \tag{26}$$

Rearranging,

$$\mathbf{V}'(x) = \frac{r\mathbf{V}(x) - v(x,z^*(x))}{d(x,z^*(x))} \equiv \mathbf{F}(\mathbf{V}(x),x).$$

Compare this to the ODE describing the value of pooling,

$$V_x(x,0) = \frac{rV(x,0) - v(x,0)}{d(x,0)} = \mathbf{F}^0(V(x,0),x)$$

Observe that $\mathbf{F}(V,x) > \mathbf{F}^0(V,x)$ for any $x$ for which $z^*(x) > 0$.[19] Since $\mathbf{V}(x^p) = V(x^p,0)$, it must be that $\mathbf{V}(x) < V(x,0)$ for $x$ sufficiently small. This contradicts our assumption that $\mathbf{V}(x)$ is the optimal value function. Thus, pooling is optimal for every state.

Assume $\lambda\kappa\rho < r+\kappa+\delta$. Simple algebra based on (21) implies that

$$V_x = \frac{r}{r+\alpha^z}(R_H - 1) + (1-z)\frac{r}{r+\alpha^z}(1-R_L) > 0$$

and

$$V_{xz} = r\frac{\lambda\kappa\rho - (r+\kappa+\delta)}{(r+\alpha^z)^2(1-R_L)} < 0.$$

[19]Note that $\mathbf{V}'(x) > 0$ by a simple envelope argument.

□

With this result in mind, we assume in the following that $\lambda\kappa\rho < r + \kappa + \delta$ and characterize $z^*(x)$.

**Lemma 2.** *Assume $\lambda\kappa\rho < r + \kappa + \delta$. The efficient screening policy $z^*(x)$ is to screen if $x < \overline{x}^*$ and to pool if $x > \overline{x}^*$, where*

$$V(\overline{x}^*, 0) = V(\overline{x}^*, \overline{z}) \tag{27}$$

*as long as the solution to that equation is greater or equal to $x^s$. Otherwise, $\overline{x}^*$ is determined by*

$$v_z(\overline{x}^*, 0) + V_x(\overline{x}^*, 0) d_z(\overline{x}^*, 0) = 0. \tag{28}$$

*Proof.* First, notice that $\overline{x}^*$ is indeed well-defined, in that if the solution to (27) is $x^s$, then (28) is also solved by $x^s$. Assume

$$V(x^s, 0) = V(x^s, \overline{z}).$$

Combining (24) and (25), we can rewrite $V(x^s, 0)$ and $V(x^s, \overline{z})$ and obtain

$$v(x^s, 0) + V_x(x^s, 0) d(x^s, 0) = v(x^s, \overline{z}) + V_x(x^s, \overline{z}) d(x^s, \overline{z}).$$

Since $d(x^s, \overline{z}) = 0$, this can be combined into

$$v(x^s, \overline{z}) - v(x^s, 0) + V_x(x^s, 0) \left( d(x^s, \overline{z}) - d(x^s, 0) \right) = 0 \tag{29}$$

which is equivalent to (28) as $v$ and $d$ are linear in $z$. Moreover, going these steps backwards, if $\overline{x}^* < x^s$, then (29) holds with inequality and therefore

$$V(x^s, 0) > V(x^s, \overline{z}). \tag{30}$$

Now we proceed to our main argument, a proof by contradiction. We distinguish four possible cases.

**Case 1: There exists $x > \overline{x}^*$ with $x \geq x^s$ where screening is optimal.** If true, this would require there to be at least one point $x^0 \in [\overline{x}^*, \lambda)$ where the planner strictly prefers to remain at $x^0$ forever (by choosing strategy $z^0 \in (0, \overline{z}]$ such that $d(x^0, z^0) = 0$) over pooling. In math,

$$V(x^0, z^0) > V(x^0, 0).$$

Since $V$ has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, 0)$ and $V(x^s, z^0) > V(x^s, 0)$, which, by point 3 in Lemma 1, is contradicting either (27) or (30).

**Case 2: There exists $x < \overline{x}^*$ with $x \geq x^s$ where pooling is optimal.** If true, this would require there to be at least one point $x^0 \in (x^s, \overline{x}^*]$ where the planner strictly prefers to remain at $x^0$ forever (by choosing strategy $z^0 \in [0, \bar{z})$ such that $d(x^0, z^0) = 0$) over screening. In math,

$$V(x^0, z^0) > V(x^0, \bar{z}).$$

Since $V$ has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\bar{x}^*, z^0) > V(\bar{x}^*, \bar{z})$, which by point 3 in Lemma 1, contradicts (27).

**Case 3: There exists $x > \bar{x}^*$ with $x \leq x^s$ where screening is optimal.** If true, this would require there to be at least one point $x^0 \in [\bar{x}^*, x^s]$ where the planner strictly prefers to screen with some intensity $z^0 > 0$ in the current instant while pooling is chosen thereafter. That is,

$$v(x^0, z^0) + V_x(x^0, 0)d(x^0, z^0) > v(x^0, 0) + V_x(x^0, 0)d(x^0, 0).$$

Due to linearity of this equation, it also has to hold with $z^0 = \bar{z}$, and therefore also expressed as derivative,

$$v_z(x^0, 0) + V_x(x^0, 0)d_z(x^0, 0) > 0. \tag{31}$$

Since this is a linear equation in $x^0$, to be consistent with (28), it must be that (31) in fact holds for any $x^0 > \bar{x}^*$, including $x^0 = x^p = \lambda$. In that case, however, (31) simplifies to $v_z(x^p, 0) + V_x(x^p, 0)d_z(x^p, 0) > 0$, which is false, since $V_x(x, 0) > 0$, $d_z(x, 0) < 0$ and $v_z(x^p, 0) = -\kappa(1 - R_L)(c - (1 - \lambda)) < 0$ by Assumption 2.

**Case 4: There exists $x < \bar{x}^* \leq x^s$ where pooling is optimal.** Let $\mathbf{V}(x)$ be our conjectured value function left of $\bar{x}^*$. By design, $\mathbf{V}(x)$ solves

$$r\mathbf{V}(x) = v(x, \bar{z}) + \mathbf{V}'(x)d(x, \bar{z})$$

where $d(x, \bar{z}) = \alpha^{\bar{z}}(x^s - x)$ and $\mathbf{V}'(x)$ solves

$$(r + \alpha^{\bar{z}})\mathbf{V}'(x) = v_x(x, \bar{z}) + \mathbf{V}''(x)d(x, \bar{z}).$$

This ODE can be solved explicitly, giving[20]

$$\mathbf{V}'(x) = r(1 - R_L)\left(\frac{\rho}{r + \alpha^p} - \frac{\rho - \bar{z}}{r + \alpha^{\bar{z}}}\right)\left(\frac{x^s - x}{x^s - \bar{x}^*}\right)^{-\beta} + r(1 - R_L)\frac{\rho - \bar{z}}{r + \alpha^{\bar{z}}}$$

where $\beta = 1 + \frac{r}{\alpha^{\bar{z}}}$. The coefficient on the first term is positive, since we assumed $\rho\lambda\kappa < r + \kappa + \delta$. Thus, $\mathbf{V}'(x)$ is bounded above by

$$\mathbf{V}'(x) \leq \mathbf{V}'(\bar{x}^*) = r(1 - R_L)\frac{\rho}{r + \alpha^p}. \tag{32}$$

[20]Note that $v_x(x, \bar{z})$ is a constant in $x$.

Could it ever be that the planner prefers pooling in this region? If so, we would have an $x < \overline{x}^*$ with

$$v_z(x,0) + \mathbf{V}'(x)d_z(x,0) < 0$$

which due to (32) and the fact that $d_z(x,0) = -\kappa\lambda(1-x) < 0$ implies that

$$v_z(x,0) + \mathbf{V}'(\overline{x}^*)d_z(x,0) < 0.$$

Using the expressions in (22) and (23) we then see that this cannot hold as the left hand side is zero at $\overline{x}^*$ (by definition), and has a negative slope throughout,

$$v_{xz} + \mathbf{V}'(\overline{x}^*)d_{xz} = r(1-R_L)\left[-1 + \frac{\rho\lambda\kappa}{r+\alpha^p}\right] < 0$$

where again we used $\rho\lambda\kappa < r + \kappa + \delta$. This is a contradiction: there cannot be an $x < \overline{x}^*$ where pooling is optimal. □

### A.8.2 Optimal bank operation policy $\theta^*(x)$

Next we focus on the optimal policy $\theta^*(x)$ for banks to operate. We prove the following result.

**Lemma 3.** *If it is strictly optimal to have banks operate at $\overline{x}^*$, the optimal policy describing when banks operate is bang-bang, that is,*

$$\theta^*(x) = \begin{cases} 0 & x < \underline{x}^* \\ 1 & x > \underline{x}^* \end{cases} \tag{33}$$

*The threshold $\underline{x}^*$ is the supremum of all $x \in [0,\lambda]$ that solve*

$$v(x, z^*(x)) + \mathbf{V}'(x)\left(\kappa\left(\lambda - x\right) - \kappa(1-x)z^*(x)\lambda\right) < 0 \tag{34}$$

*where $\mathbf{V}(x)$ is the value function associated with the optimal screening policy $z^*(x)$.*

*Proof.* Let $\underline{x}^*$ be defined as in (34) and let $\mathbf{V}(x)$ be the value function conditional on banks operating with screening policy $z^*(x)$. If it is optimal for the planner to follow the bang-bang policy (33), then its value function for $x \geq \underline{x}^*$ is given by $\mathbf{V}(x)$, whereas for $x < \underline{x}^*$ the value function solves

$$r\mathbf{V}(x) = \mathbf{V}'(x)\delta(\lambda - x)$$

which can be solved to express the marginal value in state $x$ as

$$\mathbf{V}'(x) = \mathbf{V}'(\underline{x}^*)\left(\frac{\lambda - x}{\lambda - \underline{x}^*}\right)^{-1-r/\delta}.$$

Observe that this is increasing in $x$. To prove that the bang-bang policy (33) is indeed optimal, we need to prove that

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)d(x,z,1) \le \max_{z\in[0,\bar{z}]} \mathbf{V}'(x)d(x,z,0) \tag{35}$$

for $x < \underline{x}^*$, where

$$d(x,z,\theta) \equiv \theta\kappa(1-x)(1-z)\lambda - \theta\kappa x(1-\lambda) + \delta(\lambda - x)$$

is the speed at which the pool improves given $(x,z,\theta)$. Simplifying (35), we obtain

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)\left[\kappa\lambda(1-z) - \kappa x(1-z\lambda)\right] \le 0.$$

The left hand side of this inequality has a negative cross-partial in $(x,z)$, since $v_{xz} < 0$ and $\mathbf{V}'(x)(1-x) \propto (\lambda-x)^{-r/\delta}\frac{1-x}{\lambda-x}$ increases in $x$. Thus, given that $z=\bar{z}$ is optimal for $x=\underline{x}^*$, it is also optimal for any $x < \underline{x}^*$.

The problem then reduces from (35) to showing that for $x < \underline{x}^*$

$$F(\lambda - x) \equiv v(x,\bar{z}) + \mathbf{V}'(x)\left[\kappa\lambda(1-\bar{z}) - \kappa x(1-\bar{z}\lambda)\right] < 0. \tag{36}$$

To see this, we first show that $F(y)$ is quasi-concave (only has a single local maximum) and therefore can at most have two roots. $F(y)$ is of the form

$$F(y) = -F_0 y + F_1 y^{-\alpha-1}(y - y_0) + const$$

where $\alpha = r/\delta > 0$, $F_0 = r(R_H - 1) + r(1-\bar{z})(1-R_L) > 0$, $F_1 = \kappa(1-\lambda\bar{z})\mathbf{V}'(\underline{x}^*)(\lambda-\underline{x}^*)^{1+\alpha} > 0$ , $y_0 = \lambda - x^s > 0$. $F$ can only ever have a single local maximum as long as these parameters are positive:

$$F'(y) = 0 \quad \Leftrightarrow \quad y^{-\alpha-2}\left[(1+\alpha)y_0 - \alpha y\right] = F_1/F_0$$

The left hand side of this equation is strictly decreasing for $y \in (0, (1+\alpha)y_0/\alpha)$ with range $(0,\infty)$ and thus admits a unique solution for any $F_1/F_0 > 0$. This establishes that $F(y)$ is quasi-concave.

Since $F(y)$ is quasi-concave, it admits at most two roots, $y_1 < y_2$, in between which $F(y)$ is positive, and negative outside of $[y_1, y_2]$. Root $y_2$ must correspond to $\lambda - \underline{x}^*$: if $y_1$ were to correspond to $\lambda - \underline{x}^*$, $\underline{x}^*$ would not be the supremum of $x$ with $F(\lambda - x) < 0$ since for any $\epsilon > 0$ small enough, $F(\lambda - (\underline{x}^* - \epsilon)) > 0$. But if $y_2 = \lambda - \underline{x}^*$, then $F(\lambda - x) < 0$ for any $x < \underline{x}^*$, which proves (36). □

### A.9 Derivation of law of motion for credit boom shock

tbd

### A.10 Constrained efficiency with slow thawing

tbd

### A.11 Optimal policy with government banks

## B Slow thawing during convergence to screening steady state

tbd

## C Boom-bust cycle

tbd